Relativistic description of fully heavy tetraquark spectroscopy

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Introduction

◇ "Ordinary" hadrons:

- baryons qqq,
- mesons $\mathrm{q\bar{q}}$.

◇ Exotic hadrons:

- tetraquarks $\mathrm{qq\bar{q}\bar{q}}$,
- pentaquarks $\mathrm{qqqq\bar{q}}$, etc.

◇ Searches for the $\mathrm{X_{cc\bar{c}\bar{c}}}$, $\mathrm{X_{bb\bar{b}\bar{b}}}$ are conducted on the Large Hadron Collider (LHC) by the LHCb, ATLAS and CMS Collaborations.

Model description I

- $m_c = 1.55$ GeV, $m_b = 4.88$ GeV.
- Quark content:
  - symmetric – $cc\bar{c}\bar{c}$, $cb\bar{c}\bar{b}$, $bb\bar{b}\bar{b}$,
  - asymmetric – $cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$.

Model description II

◇ Diquark-antidiquark bound state:
$\{(Q_1Q_2)-(\overline{Q}_3\overline{Q}_4)\}$.

◇ Ground state diquarks:

- scalar (S) — $J=0$,
- axialvector (A) — $J=1$.

◇ Diquark content:

- only axialvector – $cc\bar{c}\bar{c}$, $cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$,
- both axialvector and scalar – $cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $cb\bar{c}\bar{b}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$.

Relativistic quark model I

- Relativistic Schrödinger-type quasipotential equation:

$$\left(\frac{b^2(M)}{2\mu_R(M)} - \frac{\mathbf{p}^2}{2\mu_R(M)}\right)\Psi_{T,d}(\mathbf{p}) = \int \frac{d^3 q}{(2\pi)^3} \; V(\mathbf{p},\mathbf{q};M)\Psi_{T,d}(\mathbf{q})$$

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

- Diquark-antidiquark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M) = \frac{< d(\mathcal{P})|J_\mu|d(\mathcal{Q}) >}{2\sqrt{E_d}\sqrt{E_d}} \frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k}) \frac{< d'(\mathcal{P}')|J_\nu|d'(\mathcal{Q}') >}{2\sqrt{E_{d'}}\sqrt{E_{d'}}}$$

$$+ \Psi^*_d(\mathcal{P})\Psi^*_{d'}(\mathcal{P}')[J_{d;\mu}J^\mu_{d'}V^V_{\text{conf.}}(\mathbf{k}) + V^S_{\text{conf.}}(\mathbf{k})]\Psi_d(\mathcal{Q})\Psi_{d'}(\mathcal{Q}')$$

◇ Diquark-antidiquark interaction quasipotential in configuration space:

$$
\begin{aligned}
V(r) = &\Bigg[V_{\text{Coul.}}(r) + V_{\text{conf.}}(r) + \frac{1}{E_1 E_2}\left\{ \mathbf{p}\left[V_{\text{Coul.}}(r) + V^V_{\text{conf.}}(r)\right]\mathbf{p} - \frac{1}{4}\Delta V^V_{\text{conf.}}(r) + V'_{\text{Coul.}}(r)\frac{\mathbf{L}^2}{2r}\right\}\Bigg]_a \\
&+ \Bigg[\Bigg\{ \frac{1}{2}\left[\frac{1}{E_1(E_1+M_1)} + \frac{1}{E_2(E_2+M_2)}\right]\frac{V'_{\text{Coul.}}(r)}{r} - \frac{1}{2}\left[\frac{1}{M_1(E_1+M_1)} + \frac{1}{M_2(E_2+M_2)}\right]\frac{V'_{\text{conf.}}(r)}{r} \\
&+ \frac{\mu_d}{4}\left[\frac{1}{M_1^2} + \frac{1}{M_2^2}\right]\frac{V'^V_{\text{conf.}}(r)}{r} + \frac{1}{E_1 E_2}\left[V'_{\text{Coul.}}(r) + \frac{\mu_d}{4}\left(\frac{E_1}{M_1} + \frac{E_2}{M_2}\right) V'^V_{\text{conf.}}(r)\right]\frac{1}{r}\Bigg\}\mathbf{L}(\mathbf{S}_1 + \mathbf{S}_2) \\
&+ \Bigg\{ \frac{1}{2}\left[\frac{1}{E_1(E_1+M_1)} - \frac{1}{E_2(E_2+M_2)}\right]\frac{V'_{\text{Coul.}}(r)}{r} - \frac{1}{2}\left[\frac{1}{M_1(E_1+M_1)} - \frac{1}{M_2(E_2+M_2)}\right]\frac{V'_{\text{conf.}}(r)}{r} \\
&+ \frac{\mu_d}{4}\left[\frac{1}{M_1^2} - \frac{1}{M_2^2}\right]\frac{V'^V_{\text{conf.}}(r)}{r} + \frac{1}{E_1 E_2}\frac{\mu_d}{4}\left(\frac{E_1}{M_1} - \frac{E_2}{M_2}\right)\frac{V'^V_{\text{conf.}}(r)}{r}\Bigg\}\mathbf{L}(\mathbf{S}_1 - \mathbf{S}_2)\Bigg]_b \\
&+ \Bigg[\frac{1}{3E_1E_2}\left\{\frac{1}{r}V'_{\text{Coul.}}(r) - V''_{\text{Coul.}}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\left(\frac{1}{r}V'^V_{\text{conf.}}(r) - V''^V_{\text{conf.}}(r)\right)\right\} \times \left[\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right]\Bigg]_c \\
&+ \Bigg[\frac{2}{3E_1E_2}\left\{\Delta V_{\text{Coul.}}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^V_{\text{conf.}}(r)\right\}\mathbf{S}_1\mathbf{S}_2\Bigg]_d
\end{aligned}
$$

Interactions I

◇ Interaction $\mathrm{V(r)}$:

- $\langle \mathrm{L_T\, S_T\, J_T} |\, \mathrm{V(r)}\, | \mathrm{L'_T\, S'_T\, J'_T} \rangle \equiv \langle \mathrm{L\, S\, J} |\, \mathrm{V(r)}\, | \mathrm{L'\, S'\, J'} \rangle.$
- $\mathrm{V(r)}$:
  - $\Big[\dots \Big]_{\mathrm{a}} \equiv \mathrm{V_{spin-ind}},$
  - $\mathrm{L} \cdot (\mathrm{S_{d_1}} + \mathrm{S_{d_2}}) \equiv \mathrm{LS_+},$
  - $\mathrm{L} \cdot (\mathrm{S_{d_1}} - \mathrm{S_{d_2}}) \equiv \mathrm{LS_-},$
  - $\frac{3}{\mathrm{r}^2} \cdot \Big(\mathrm{S_{d_1} r} \Big) \cdot \Big(\mathrm{S_{d_2} r} \Big) - \mathrm{S_{d_1}} \cdot \mathrm{S_{d_2}} \equiv \mathrm{T},$
  - $\mathrm{S_{d_1}} \cdot \mathrm{S_{d_2}} \equiv \mathrm{SS}.$

Interactions II

- Symmetric compositions:
  - $\{\mathrm{L},\, \mathrm{J}\} = \{\mathrm{L}',\, \mathrm{J}'\}$,
  - LS_+ – diagonal,
  - $\mathrm{LS}_- \equiv 0$,
  - T – non-diagonal,
  - SS – diagonal.
- Non-diagonal elements arise only for a few states. They are very small numerically and can be ignored. Thus, effectively:
  - $\{\mathrm{L},\, \mathrm{S},\, \mathrm{J}\} = \{\mathrm{L}',\, \mathrm{S}',\, \mathrm{J}'\}$,
  - T – diagonal.

  and there is no mixing between any states.

Interactions III

- Asymmetric compositions:
  - $\{\mathrm{L},\ \mathrm{J}\} = \{\mathrm{L}',\ \mathrm{J}'\}$,
  - LS_+ – diagonal,
  - LS_- – non-diagonal,
  - T – non-diagonal,
  - SS – diagonal.
- Significant mixing between the $\mathrm{n_r}\,{}^{\mathrm{S}}\mathrm{L_J}$ and $\mathrm{n_r}\,{}^{\mathrm{S}'}\mathrm{L_J}$ states arises.

Mixing I

◇ Notations:

- $M_{L=a,\,J=b} \equiv M_{a,b}$,
- $M_{L=a,\,J=b}(S=c,\; S'=d) \equiv M_{a,b}(c,d)$,
- $\Delta M_{a,b}(c,d) = \big[M_{a,b}(c,d)\big]_{\text{full}} - \big[M_{a,b}(c,d)\big]_{\text{spin-ind}}$.

◇ P-wave:

- $J=1$: $\quad M_{1,1} = \text{eig}\begin{pmatrix} M_{1,1}(0,0) & \Delta M_{1,1}(0,1) & \Delta M_{1,1}(0,2) \\ \Delta M_{1,1}(1,0) & M_{1,1}(1,1) & \Delta M_{1,1}(1,2) \\ \Delta M_{1,1}(2,0) & \Delta M_{1,1}(2,1) & M_{1,1}(2,2) \end{pmatrix}$
- $J=2$: $\quad M_{1,2} = \text{eig}\begin{pmatrix} M_{1,2}(1,1) & \Delta M_{1,2}(1,2) \\ \Delta M_{1,2}(2,1) & M_{1,2}(2,2) \end{pmatrix}$

Mixing II

◇ D-wave:

- $J = 1$: $\quad M_{2,1} = \operatorname{eig}\begin{pmatrix} M_{2,1}(1,1) & \Delta M_{2,1}(1,2) \\ \Delta M_{2,1}(2,1) & M_{2,1}(2,2) \end{pmatrix}$
- $J = 2$: $\quad M_{2,2} = \operatorname{eig}\begin{pmatrix} M_{2,2}(0,0) & \Delta M_{2,2}(0,1) & \Delta M_{2,2}(0,2) \\ \Delta M_{2,2}(1,0) & M_{2,2}(1,1) & \Delta M_{2,2}(1,2) \\ \Delta M_{2,2}(2,0) & \Delta M_{2,2}(2,1) & M_{2,2}(2,2) \end{pmatrix}$
- $J = 3$: $\quad M_{2,3} = \operatorname{eig}\begin{pmatrix} M_{2,3}(1,1) & \Delta M_{2,3}(1,2) \\ \Delta M_{2,3}(2,1) & M_{2,3}(2,2) \end{pmatrix}$

Asymmetric compositions mass spectra I

◇ $A\bar{A}$-configuration:

Table 1: Masses of the ground states (1S) and radial (up to 3S) and orbital (up to 1D) excitations of the asymmetric ($cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$) fully heavy tetraquarks in the $A\bar{A}$-configuration.

$d\bar{d}'$	nL	n_r	L	S	J	J^P	$M_{cc\bar{c}\bar{b},\, bc\bar{c}\bar{c}}$	$M_{cc\bar{b}\bar{b},\, bb\bar{c}\bar{c}}$	$M_{bb\bar{b}\bar{c},\, cb\bar{b}\bar{b}}$
$A\bar{A}$	1S	0	0	0	0	0^+	9,606	12,848	16,102
				1	1	1^+	9,611	12,852	16,104
				2	2	2^+	9,620	12,859	16,108
	1P	0	1	1	0	0^-	9,875	13,106	16,326
				0	1	1^-	9,871	13,103	16,325
				1			9,877	13,108	16,326
				2			9,881	13,111	16,329
				1	2	2^-	9,875	13,106	16,327
				2			9,882	13,112	16,329
				2	3	3^-	9,881	13,110	16,330
	2S	1	0	0	0	0^+	10,063	13,282	16,481
				1	1	1^+	10,064	13,282	16,481
				2	2	2^+	10,064	13,283	16,481
	1D	0	2	2	0	0^+	10,113	13,330	16,513
				1	1	1^+	10,111	13,328	16,513
				2			10,114	13,331	16,514
				0	2	2^+	10,108	13,324	16,513
				1			10,113	13,330	16,514
				2			10,117	13,334	16,515
				1	3	3^+	10,111	13,327	16,515
				2			10,116	13,332	16,516
				2	4	4^+	10,114	13,329	16,516
	2P	1	1	1	0	0^-	10,265	13,468	16,631
				0	1	1^-	10,258	13,461	16,629
				1			10,264	13,468	16,630
				2			10,270	13,472	16,633
				1	2	2^-	10,260	13,463	16,630
				2			10,268	13,470	16,632
				2	3	3^-	10,263	13,466	16,631
	3S	2	0	0	0	0^+	10,442	13,629	16,765
				1	1	1^+	10,442	13,629	16,765
				2	2	2^+	10,440	13,628	16,764

◇ $S\overline{A}$, $A\overline{S}$-configuration:

Table 2: Masses of the ground states (1S) and radial (up to 3S) and orbital (up to 1D) excitations of the asymmetric ($cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$) fully heavy tetraquarks in the $S\overline{A}$, $A\overline{S}$-configuration.

$d\bar{d}'$	nL	n_r	L	S	J	J^P	$M_{cc\bar{c}\bar{b},bc\bar{c}\bar{c}}$	$M_{bb\bar{b}\bar{c},cb\bar{b}\bar{b}}$
$S\overline{A}$, $A\overline{S}$	1S	0	0	1	1	1^+	9,608	16,099
	1P	0	1		0	0^-	9,873	16,320
					1	1^-	9,872	16,321
					2	2^-	9,871	16,322
	2S	1	0		1	1^+	10,057	16,474
	1D	0	2		1	1^+	10,108	16,507
					2	2^+	10,107	16,508
					3	3^+	10,105	16,509
	2P	1	1		0	0^-	10,262	16,624
					1	1^-	10,260	16,624
					2	2^-	10,254	16,624
	3S	2	0		1	1^+	10,434	16,758

Threshold analysis: general I

- If energetically possible, the tetraquark will fall-apart into a meson pair through the quark rearrangement.

$$\Delta = \mathrm{M}_{\mathrm{QQ'\overline{QQ}'}} - \mathrm{M}^{\mathrm{lowest}}_{\mathrm{threshold}}$$

- If $\Delta < 0$, state is stable against fall-apart strong decays.
- The smaller $\Delta > 0$, the narrower is the state.

Threshold analysis: general II

- Many masses lie well above thresholds with $\Delta > 100$ MeV.
- Few masses lie in the $[-70 < \Delta < 100]$ MeV interval.
- Such behavior is seen for all quark compositions and all excitations.
- It is consistent with the lack of significant advances in experimental searches.

Threshold analysis: asymmetric

◇ The most promising to be stable states:

Table 3: Ground states (1S) and radial (up to 3S) and orbital (up to 1D) excitations of the most promising to be stable asymmetric ($cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$, $bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$) fully heavy tetraquarks and the corresponding meson–meson thresholds.

$QQ'\overline{QQ'}$	$d\bar{d}'$	nL	S	J^P	M	M_{thr}	Δ	meson pair
$cc\bar{c}\bar{b}$, $bc\bar{c}\bar{c}$	$A\bar{A}$	1P	1	2^-	9,875	9,831	44	$\chi_{c2}(1P)B_c^{\pm}$
			2	2^-	9,882		51	
			2	3^-	9,881	9,888	-7	$\chi_{c2}(1P)B_c^{*\pm}$
		1D	1	3^+	10,111	10,117	-6	$\psi_3(3842)B_c^{\pm}$
			2	3^+	10,116		-1	
			2	4^+	10,114	10,175	-61	$\psi_3(3842)B_c^{*\pm}$
	$S\bar{A}$, $A\bar{S}$	1P	1	2^-	9,871	9,831	40	$\chi_{c2}(1P)B_c^{\pm}$
		1D	1	3^+	10,105	10,117	-12	$\psi_3(3842)B_c^{\pm}$
$bb\bar{b}\bar{c}$, $cb\bar{b}\bar{b}$	$A\bar{A}$	1P	2	3^-	16,330	16,244	86	$\chi_{b2}(1P)B_c^{*\pm}$
		1D	1	3^+	16,515	16,496	19	$\Upsilon_2(1D)B_c^{*\pm}$
			2	3^+	16,516		20	
	$S\bar{A}$, $A\bar{S}$		1	3^+	16,509		13	

Experimental data I

- In 2020 the LHCb Collaboration announced the discovery of the narrow resonance X(6900).
- Several other broad structures peaking at about 6.4 and 7.2 GeV were reported.
- In 2022 CMS and ATLAS Collaborations confirmed X(6900) and hinted on a few more states, including structures at 6.4 and 7.2 GeV.

Experimental data II

◇ Current observation status and our predictions:

Table 4: Exotic X states observed and hinted by the LHCb, ATLAS and CMS Collaborations in di-J/ψ and $J/\psi\,\psi(2S)$ invariant mass spectra and our candidates.

Collaboration		State	M, MeV	Γ, MeV	Our candidates: nL	S	J^{PC}	M, MeV
LHCb		X(6600)	$6400 \div 6600$		1S	2	2^{++}	6367
ATLAS	m_0, model A		$6410 \pm 80^{+80}_{-30}$	$590 \pm 350^{+120}_{-200}$				
ATLAS	m_0, model B		$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$				
ATLAS	m_1, model A		$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$				
CMS	BW_1, no interference		$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	2S	0	0^{++}	6782
CMS	BW_1, interference		6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$				
LHCb	NRSPS, no interference	X(6900)	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	2S	2	2^{++}	6868
LHCb	NRSPS, interference		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$				
ATLAS	m_2, model A		$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	1D	0	2^{++}	6921
ATLAS	m_2, model B		$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$		2	0^{++}	6899
ATLAS	m_3, model β		$6960 \pm 50 \pm 30$	$510 \pm 170^{+110}_{-100}$		2	1^{++}	6904
CMS	BW_2, no interference		$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 18$		2	2^{++}	6915
CMS	BW_2, interference		6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}				
LHCb		X(7200)	$7200 \div 7400$		3S	0	0^{++}	7259
ATLAS	m_3, model α		$7220 \pm 30^{+10}_{-30}$	$90 \pm 60^{+60}_{-30}$				
CMS	BW_3, no interference		$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$		2	2^{++}	7333
CMS	BW_3, interference		7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}				

Conclusion I

- Masses of ground and excited states of fully heavy tetraquarks were calculated.
- The finite diquark size was taken into account.

Conclusion II

- Calculations for the asymmetric compositions were carried out.
- Mixing between the states with the same $\{\mathrm{L_T}, \mathrm{J_T}\}$, but different $\mathrm{S_T}$ via the LS_{-} and T interactions was taken into account.

Conclusion III

- Asymmetric tetraquark states which are the most convenient for the experimental detection were identified.
- Masses of resonances in the di-J/ψ production detected at the LHCb, ATLAS and CMS agree with our predictions for the ground and excited $X_{cc\overline{cc}}$ states.

Publications

◇ This talk is based on the following publications:

- Masses of the $QQ\overline{QQ}$ tetraquarks in the relativistic diquark-antidiquark picture, Physical Review D, 2020, vol. 102, №11, p. 114030;
- Heavy Tetraquarks in the Relativistic Quark Model, Universe, 2021, vol. 7, №4, p. 94;
- Fully heavy tetraquark spectroscopy in relativistic quark model, Memoirs of the Faculty of Physics, 2022, №4, p. 2241512;
- Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model, Symmetry, 2022, vol. 14, №12, p. 2504;
- Relativistic description of the mass spectra of fully heavy tetraquarks, Memoirs of the Faculty of Physics, 2023, №4, p. 2341504.

Thank you
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