Relativistic partial wave analysis of inclusive meson production $A + B \Longrightarrow V + X \Longrightarrow 1 + 2 + X$ and determination of the spin quantization axis via the cross sections $A + B \Longrightarrow V + X$.

Machavariani Alexander[a,b]

[a] **HEPI TSU, Tbilisi**
[b] **VBLHEP JINR, Dubna**

♣ What is Relativistic partial wave analysis?

Spin operator $\widehat{S}$ is defined in rest frame of the system of the particles, where the total momentum $\mathbf{P} = 0$

$$\widehat{S}^* = (0, \widehat{\mathbf{S}}^*)$$

Wigner rotation: from the one-particle rest frame with $\mathbf{p} = 0$, $S^* \equiv (0, \widehat{\mathbf{S}}^*)$ an arbitrary frame

$$|\mathbf{p}, \mathbf{S} >= \widehat{R}\widehat{L}\widehat{R}^{-1} \, |\mathbf{p} = \mathbf{0}, \mathbf{S}^* >$$

where

$$p = (\sqrt{m^2 + \mathbf{p}^2}, \mathbf{p}) = \widehat{R}\widehat{L}\widehat{R}^{-1}(m, 0); \qquad \widehat{S} = (\widehat{S}_o, \mathbf{S}) = \widehat{R}\widehat{L}\widehat{R}^{-1}(0, \mathbf{S}^*)$$

where $\widehat{R}$ and $\widehat{L}$ are 3D rotation and Lorentz transformation operators into an arbitrary frame, where $\mathbf{p} \neq 0$ and spin is directed along spin quantization axis.

♣ Let's consider inclusive production of the vector meson resonance $J+1$

$$A + B \Longrightarrow V + X \Longrightarrow 1 + 2 + X$$

$$A, B = \gamma,\ h\ (hadrons),\ \overline{h}, d, heavy\ ions, ...$$

$$V = \rho, \phi, ...$$

$$1 + 2 = lepton\ pairs,\ hadron\ pairs, ...$$

♣♣ Resonance decay amplitudes
$\gamma^* \leftarrow V$. Therefore the following approximation is often used for $\mathcal{A}^M_{1+2\leftarrow V}$

$$\mathcal{A}^M_{e^+ + e^- \leftarrow V} = g_{\ell\bar{\ell}\leftarrow V}\xi_\mu{}^*(\mathbf{P}_V, M)\overline{u}(\mathbf{p}_\ell)\gamma_\mu v(\mathbf{p}_{\bar{\ell}})$$

$\xi^{\mu *}(\mathbf{P}_V, M)$ is the polarization vector of the spin 1 particle and coupling constant $g_V \equiv g_{\ell\bar{\ell}\leftarrow V}$ is constant defined within vector meson dominance model.

$$\mathcal{A}^M_{h\overline{h}-\leftarrow V} = g_{h\overline{h}-V}\xi_\mu{}^*(\mathbf{P}_V, M)(p_h + p_{\overline{h}})_\mu$$

where $g_{h\overline{h}-V} \equiv g_{h\overline{h}-V}\Big((p_h + p_{\overline{h}})^2\Big)$. is form-factor of $h\overline{h} - V$-system.

♣ Cross-sections

Helicity basis simplifies construction of cross sections because helicity of any particle Lorentz invariant and Wigner transformations are exactly taken into account

Reactions $A + B \Longrightarrow V + X \Longrightarrow 1 + 2 + X$ are described by one particle (resonance) exchange diagram, therefore

$$\frac{d\sigma_{1+2+X\leftarrow A+B}}{d\Omega_{1+2+X\leftarrow A+B}} =$$

$$= \sum_{MM'} \frac{d\widetilde{\sigma}^{MM'}_{1+2\leftarrow V}}{d\Omega_{1+2\leftarrow V}} \frac{1}{(m_V - \sqrt{s_{12}})^2 + \Gamma_V^2/4} \frac{d\sigma^{MM'}_{V+X\leftarrow A+B}}{d\Omega_{V+X\leftarrow A+B}}$$

where M, M' are magnetic quantum number of V-meson resonance

$$\frac{d\sigma^{MM'}_{V+X\leftarrow A+B}}{d\Omega_{V+X\leftarrow A+B}} \sim \prod_{i}^{X} \mathcal{A}^{M}_{V+X\leftarrow A+B}(\mathcal{A}^{*}_{V+X\leftarrow A+B})^{M'}(2\pi)^3\frac{d\mathbf{P_{12}}}{(2\pi)^3}\prod_{i=1}^{X} d\widetilde{\mathbf{p}}_i$$

$$(2\pi)^4\delta^3(\mathbf{P}_A+\mathbf{P}_B-\mathbf{P}_{12}-\mathbf{P}_X)\delta(P^o_A+P^o_B-P^o_{12}-P^o_X)$$

where $\mathbf{P}_{12}=\mathbf{p}_1+\mathbf{p}_2$ and $P^o_{12}=p^o_1+p^o_2$,

$$\frac{d\widetilde{\sigma}^{MM'}_{1+2\leftarrow V}}{d\Omega_{1+2\leftarrow V}} = \mathcal{A}^{M}_{1+2\leftarrow V}(\mathcal{A}^{*}_{1+2\leftarrow V})^{M'} d\widetilde{\mathbf{p_1}} d\widetilde{\mathbf{p_2}}\delta^3(\mathbf{p_1}^* + \mathbf{p}^*_2)$$

Anisotropy (alignment) of V-meson resonance decay using the V-meson spin density matrix

$$\rho^{MM'}_{A+B\Longrightarrow V+X} = \frac{d\sigma^{MM'}_{A+B\rightarrow V+X}/d\Omega^*_{A+B\Longrightarrow V+X}}{\int d\Omega_{V+X\leftarrow A+B}\sum_M d\sigma^{MM}_{A+B\rightarrow V+X}/d\Omega^*_{A+B\rightarrow V+X}};$$

Experiments in 2022-2023 Years: In CERN collaborations H1, H1 SV, ZEUS, NMC, E665, HERMES, CompaSS, DELPHI (CERN). STAR RHIC. $\sqrt{s}=11.5,\ 19.6,\ 27,\ 39,\ 62.4,\ 200 GeV$ in Z^o decay

Investigations in CERN was started in 1960-1970 Years.

♣ Alternate partial wave decomposition of cross sections within L, S, J quantum numbers and without Wigner transformation

Dirac bispinor of one-particle state is solution of Lorentz invariant Dirac equation

$$u(\mathbf{p}, \mathbf{s}_Z) = \frac{p^\mu \gamma_\mu + m}{\sqrt{2m(m + E_{\mathbf{p}})}} u(\mathbf{p} = 0, \mathbf{s}_Z)$$

$$u(\mathbf{p} = 0, \mathbf{s}_Z) = \begin{pmatrix} \chi(\mathbf{s}_Z) \\ 0 \end{pmatrix} .$$

where $E_{\mathbf{p}} \equiv p_o = \sqrt{\mathbf{p}^2 + m^2}$. These functions are automatically defined in an arbitrary coordinate system and they do not need a Wigner transformation from one frame of reference to another.

Therefore one can use Lorentz invariant vertex functions

$$\mathcal{A}^M_{1+2\leftarrow V} = \overline{v}(\mathbf{p}_2, s_{2Z})\xi^\mu(0, M)\Big\{G_V(\mathbf{p}^*_{12}, P^{*o}{}_{12}; \mathbf{S}^*_V)\gamma_\mu$$

$$+G_T(\mathbf{p}^*_{12}, P^{*o}{}_{12}; \mathbf{S}^*_V)\frac{i\sigma_{\mu\nu}(p_1 + p_2)^\nu}{\mathsf{m}_1 + \mathsf{m}_2}\Big\}u(\mathbf{p_1}, s_{1Z}),$$

Tensor spherical harmonics

$$\mathcal{Y}^{LS}_{JM}(\widehat{\mathbf{p}}^*_{12}) = \sum_{M_L} < LM_L SM_S|JM > Y_{LM_L}(\widehat{\mathbf{p}}^*_{12})\chi_{SM_S}(s_{1Z}s_{2Z}),$$

Expansion over $\mathcal{Y}^{LS}_{JM}(\widehat{\mathbf{p}}^*_{12})$

$$< (\mathbf{p}^*_{12}, SM_S|\mathcal{A}_{1+2\leftarrow V} >= \sum_{J,L} \mathcal{Y}^{LS}_{JM}(\mathbf{p}^*_{12}; M_S)\mathsf{F}^{SL}_J(\sqrt{s_{12}})$$

$$\mathsf{F}^{SL}_J(\sqrt{s_{12}}) = \int d\widehat{\mathbf{p}}^*_{12}\Big[\mathcal{Y}^{LS}_{JM}(\mathbf{p}^*_{12}; M_S)\Big]^+\mathcal{A}_{1+2\leftarrow V}(\mathbf{p}^*_{12}, SM_S)$$

$$< (\mathbf{p}^*_{12}, SM_S|\mathcal{A}_{1+2\leftarrow V} >= \sum_{J,L} \mathcal{Y}^{LS}_{JM}(\mathbf{p}^*_{12}; M_S)\mathsf{F}^{SL}_J(\sqrt{s_{12}})$$

$$\chi_{SM_S}(s_{1Z} s_{2Z}) =$$

$$< \frac{1}{2} s_{1Z} \frac{1}{2} s_{1Z} | S M_S > v^+(\mathbf{p}_2 = 0, s_{2Z}) u(\mathbf{p}_1 = 0, s_{1Z}),$$

Existing formula for cross section with Wigner rotations and Jacob-Wick decompositions:

$$\frac{d\sigma_{1+2+X\leftarrow A+B}}{d\Omega^*_{1+2+X}} \sim \frac{(g_{1+2\leftarrow V})^2}{(M_V^2 - s_{12})^2 + M_V^2\Gamma_V^2}$$

$$\left[\sum_{M,M'} D^{*1}_{M\lambda_1-\lambda_2}(\phi^*,\theta^*,-\phi^*)\frac{d\sigma^{M,M'}_{V+X\leftarrow A+B}}{d\Omega^*_{V+X\leftarrow A+B}} D^1_{M'\lambda_1-\lambda_2}(\phi^*,\theta^*,-\phi^*)\right]$$

Suggesting expression for vector mesons $J=1$, $L=0,1,2$, $S=0,1$.

$$\frac{d\sigma_{1+2+X\leftarrow A+B}}{d\Omega^*_{1+2+X\leftarrow A+B}} \sim \sum_{MM'}\left(\sum_{LS}\left[\mathcal{Y}^{LS}_{JM}(\mathbf{p}^*_{12})\mathsf{F}^{SL}_J(\sqrt{s_{12}})\right]^+\right.$$

$$\left.\frac{d\sigma^{M,M'}_{V+X\leftarrow A+B}}{d\Omega^*_{V+X\leftarrow A+B}}\frac{\sum_{L'S'}\left[\mathcal{Y}^{L'S'}_{JM'}(\mathbf{p}^*_{12})\mathsf{F}^{S'L'}_J(\sqrt{s_{12}})\right]}{(M_V^2 - s_{12})^2 + M_V^2\Gamma_V^2}\right),$$

where spin-orbital interaction, coupling of S and D partial waves - non-centrality of interaction of particles 1 and 2 are exactly taken into account.

Analogue with nonrelativistic formulas for Hydrogen-type atom.

♣ Determination of the direction of the spin $\mathbf{S}_V$ of the V-meson resonance from the cross sections $V+X \leftarrow A+B$

Exists a number of theoretical indication about importance of choice of spin quantization axis in particle physic. But experimentally this axis was not yet fixed.

$$\left[\frac{d\sigma^{MM'}_{V+X\leftarrow A+B}}{d\Omega^*_{V+X\leftarrow A+B}}\right]$$

$$=< M|\mathcal{M}_{V+X\leftarrow A+B}\Big(s, t_A, t_B, (\mathbf{P}^*_A\mathbf{S}^*_V), (\mathbf{P}^*_B\mathbf{S}^*_V), (\mathbf{P}^*_A\times\mathbf{P}^*_B\mathbf{S}^*_V)\Big)|M' >$$

$\mathcal{M}_{V+X\leftarrow A+B}$ is invariant under Lorentz and $3D$ rotations. using

$$(\mathbf{nS}_V)^{2k} = (\mathbf{nS}_V)^2 = \frac{2}{3}\mathsf{E} + \sum_{i,j=1}^{3} \mathbf{n}_i\mathbf{n}_j Q_{ij}; \qquad (\mathbf{nS}_V)^{2k+1} = (\mathbf{nS}_V); \qquad (i,j =$$

Q_{ij} operator of quadruple momentum

$$\left[\frac{d\sigma^{MM'}_{V+X\leftarrow A+B}}{d\Omega^*_{V+X\leftarrow A+B}}\right] = \mathsf{a}_1\delta_{M,M'} + \mathsf{a}_2 < M|(\mathbf{e_X}\mathbf{S}^*_V)^2|M' >$$
$$+\mathsf{a}_3 < M|(\mathbf{e_y}\mathbf{S}^*_V)^2|M' > +\mathsf{a}_4 < M|(\mathbf{e_z}\mathbf{S}^*_V)^2|M' > .$$

$\mathbf{e_X}$, $\mathbf{e_Y}$, $\mathbf{e_Z}$ are unit vectors which are constructed from $\mathbf{P_A}^*$ and $\mathbf{P_B}^*$

FINALLY

$$\frac{d\sigma^{MM'}_{V+X\leftarrow A+B}}{d\Omega^*_{V+X\leftarrow A+B}} = \mathsf{b}_1\delta_{M,M'} + cos^2(\Theta)\mathsf{b}_2 + sin^2(\Theta)\cos^2(\Phi)\mathsf{b}_3 +$$
$$+sin^2(\Theta)\sin^2(\Phi)\mathsf{b}_4 + ...$$

where

$$\mathbf{S}^*_V = |\mathbf{S}^*_V|\Big(cos(\Phi)sin(\Theta), sin(\Phi)sin(\Theta), cos(\Theta)\Big)$$

Combining $d\sigma^{MM'}_{V+X\leftarrow A+B}/d\Omega^*_{V+X\leftarrow A+B}$ with different $M, M' = X, Y, Y$ one can extract Θ and Φ .

CONCLUSION:

♣ It is suggested the recipe of determination of direction of the V-meson spin quantization axis via the cross section of reaction $1+2+X \leftarrow V+X \leftarrow A+B$.

This is first recipe of determination of location of spin quantization axis.

♣♣ It is suggested procedure of partial wave decomposition of the cross sections of reactions $1+2+X \leftarrow V+X \leftarrow A+B$ which allows to take into account exactly orbital moment and spin of V-meson resonance from the experimental data.

In high energy physic dependence on LS was ignored. Only during the last year are appearing first experimental paper about importance of LS