# Twenty-First Lomonosov Conference on Particle Physics

August 24–30, 2023, Moscow State University

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# Quantum corrections and exact results in supersymmetric theories regularized by higher covariant derivatives

## The higher covariant derivative regularization

In order to deal with the ultraviolet divergences, one should regularize a theory. A proper choice of a regularization can significantly simplify calculations and can even reveal some interesting features of quantum corrections. Here we will discuss the application of the higher covariant derivative method to the regularization of supersymmetric theories. It is a consistent regularization which does not break supersymmetry.

The higher covariant derivative regularization was proposed by A.A.Slavnov

A.A.Slavnov, Nucl.Phys. **B31**, (1971), 301;
Theor.Math.Phys. **13** (1972) 1064.

By construction, it includes insertion of the Pauli–Villars determinants for removing residual one-loop divergencies

A.A.Slavnov, Theor.Math.Phys. **33**, (1977), 977.

This regularization can be formulated in a manifestly supersymmetric way in terms of $\mathcal{N}=1$ superfields

V.K.Krivoshchekov, Theor.Math.Phys. **36** (1978) 745;
P.West, Nucl.Phys. B268, (1986), 113.

and, therefore, does not break supersymmetry in higher orders.

The exact Novikov, Shifman, Vainshtein, and Zakharov (NSVZ) $\beta$-function

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

can naturally be obtained in the case of using the higher covariant derivative regularization. It relates the $\beta$-function and the anomalous dimension of the matter superfields in $\mathcal{N}=1$ supersymmetric gauge theories,

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1-C_2\alpha/2\pi)}.$$

Here $\alpha$ and $\lambda$ are the gauge and Yukawa coupling constants, respectively, and we use the notation

$$\mbox{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2\delta^{AB}; \qquad r \equiv \delta_{AA} = \mbox{dim}\, G.$$

# Explicit calculations and the problem of constructing an NSVZ scheme

Three- and four-loop calculations in $\mathcal{N} = 1$ supersymmetric theories made with dimensional reduction supplemented by modified minimal subtraction (i.e. in the so-called $\overline{\text{DR}}$-scheme)

L.V.Avdeev, O.V.Tarasov, Phys.Lett. **112 B** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett **B386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

revealed that the NSVZ relation in the $\overline{\text{DR}}$-scheme holds only in the one- and two-loop approximations, where the $\beta$-function is scheme independent.

However, in the three- and four-loop approximations it is possible to restore the NSVZ relation with the help of a specially tuned finite renormalization of the gauge coupling constant. Note that a possibility of making this finite renormalization is highly nontrivial.

This implies that the NSVZ relation holds only in some special renormalization schemes, which are usually called "NSVZ schemes", and the $\overline{\text{DR}}$-scheme is not NSVZ.

Some all-loop NSVZ schemes can be constructed with the help of the higher covariant derivative regularization.

A.L.Kataev and K.S., Nucl.Phys. **B875** (2013) 459; K.S., Eur. Phys. J. C **80** (2020) no.10, 911.

Renormalizable $\mathcal{N}=1$ supersymmetric gauge theories with matter superfields at the classical level are described by the action

$$S=\frac{1}{2e_0^2}\,\mbox{Re}\,\mbox{tr}\int d^4x\,d^2\theta\,W^aW_a+\frac{1}{4}\int d^4x\,d^4\theta\,\phi^{*i}(e^{2V})_i{}^j\phi_j$$
$$+\Big\{\int d^4x\,d^2\theta\,\Big(\frac{1}{4}m_0^{ij}\phi_i\phi_j+\frac{1}{6}\lambda_0^{ijk}\phi_i\phi_j\phi_k\Big)+\mbox{c.c.}\Big\}.$$

We assume that the gauge group is simple, and the chiral matter superfields $\phi_i$ lie in its representation $R$. The bare gauge and Yukawa coupling constants are denoted by $e_0$ and $\lambda_0^{ijk}$, respectively. The strength of the gauge superfield $V$ is defined by the equation

$$W_a\equiv\frac{1}{8}\bar D^2\Big(e^{-2V}D_a e^{2V}\Big).$$

For quantizing the theory it is convenient to use the background field method. Moreover, it is necessary to take into account nonlinear renormalization of the quantum gauge superfield

O. Piguet and K. Sibold, Nucl.Phys. **B197** (1982) 257; 272;
I.V.Tyutin, Yad.Fiz. **37** (1983) 761.

This can be done with the help of the replacement $e^{2V} \to e^{2\mathcal{F}(V)}e^{2\boldsymbol{V}}$, where $\boldsymbol{V}$ and $V$ are the background and quantum gauge superfields, respectively, and the function $\mathcal{F}(V)$ includes an infinite set of parameters needed for describing the nonlinear renormalization. In the lowest order

J.W.Juer and D.Storey, Phys.Lett. **119B** (1982) 125; Nucl. Phys. **B216** (1983) 185.

$$\mathcal{F}(V)^A = V^A + e_0^2\, y_0\, G^{ABCD}\, V^B V^C V^D + \ldots,$$

where $y_0$ is one of the constants entering this set, and $G^{ABCD}$ is a certain function of the structure constants.

For constructing the regularized theory we first add to its action terms with higher derivatives,

$$S_{\text{reg}} = \frac{1}{2e_0^2}\,\text{Re}\,\text{tr}\int d^4x\, d^2\theta\, W^a \Big(e^{-2\boldsymbol{V}}e^{-2\mathcal{F}(V)}\Big)_{Adj} R\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big)_{Adj}$$
$$\times\Big(e^{2\mathcal{F}(V)}e^{2\boldsymbol{V}}\Big)_{Adj} W_a + \frac{1}{4}\int d^4x\, d^4\theta\, \phi^{*i}\Big[F\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big)e^{2\mathcal{F}(V)}e^{2\boldsymbol{V}}\Big]_i{}^j\phi_j$$
$$+\Big[\int d^4x\, d^2\theta\,\Big(\frac{1}{4}m_0^{ij}\phi_i\phi_j + \frac{1}{6}\lambda_0^{ijk}\phi_i\phi_j\phi_k\Big) + \text{c.c.}\Big],$$

where the regulator functions $R(x)$ and $F(x)$ should rapidly increase at infinity and satisfy the condition $R(0) = F(0) = 1$. A similar function should be introduced in the gauge fixing term.

# The Pauli–Villars determinants in the non-Abelian case

For regularizing residual one-loop divergences we insert into the generating functional two Pauli–Villars determinants,

$$Z = \int D\mu\, \mathsf{Det}(PV, M_\varphi)^{-1} \mathsf{Det}(PV, M)^c \times \exp\Big\{ i\Big( S_{\mathsf{reg}} + S_{\mathsf{gf}} + S_{\mathsf{FP}} + S_{\mathsf{NK}} + S_{\mathsf{sources}} \Big)\Big\},$$

where $D\mu$ is the functional integration measure, and

$$\mathsf{Det}(PV, M_\varphi)^{-1} \equiv \int D\varphi_1\, D\varphi_2\, D\varphi_3\, \exp(iS_\varphi);$$

$$\mathsf{Det}(PV, M)^{-1} \equiv \int D\Phi\, \exp(iS_\Phi).$$

Here we use chiral commuting Pauli–Villars superfields.
The superfields $\varphi_{1,2,3}$ belong to the adjoint representation and cancel one-loop divergences coming from gauge and ghost loops. The superfields $\Phi_i$ lie in a representation $R_{\mathsf{PV}}$ and cancel divergences coming from a loop of the matter superfields if $c = T(R)/T(R_{\mathsf{PV}})$. The masses of these superfields are

$$M_\varphi = a_\varphi \Lambda; \qquad M = a\Lambda,$$

where the coefficients $a_\varphi$ and $a$ do not depend on couplings.

# Different definitions of renormalization group functions

It is important to distinguish the renormalization group functions (RGFs) defined in terms of the bare couplings $\alpha_0$ and $\lambda_0$,

$$\beta(\alpha_0,\lambda_0) \equiv \frac{d\alpha_0}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}};\qquad \gamma_x(\alpha_0,\lambda_0) \equiv -\frac{d\ln Z_x}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}},$$

and RGFs standardly defined in terms of the renormalized couplings $\alpha$ and $\lambda$,

$$\beta(\alpha,\lambda) \equiv \frac{d\alpha}{d\ln\mu}\Big|_{\alpha_0,\lambda_0=\text{const}};\qquad \gamma_x(\alpha,\lambda) \equiv \frac{d\ln Z_x}{d\ln\mu}\Big|_{\alpha_0,\lambda_0=\text{const}}.$$

A.L.Kataev and K.S., Nucl.Phys. **B875** (2013) 459.

RGFs defined in terms of the bare couplings do not depend on a renormalization prescription for a fixed regularization, but depend on a regularization.

RGFs defined in terms of the renormalized couplings depend both on a regularization and on a renormalization prescription.

# The HD+MSL scheme

Both definitions of RGFs give the same functions in the HD+MSL-scheme, when a theory is regularized by Higher Derivatives, and divergences are removed by Minimal Subtractions of Logarithms. This means that the renormalization constants include only powers of $\ln\Lambda/\mu$, where $\mu$ is a renormalization point.

$$\beta(\alpha,\lambda)\Big|_{\text{HD+MSL}} = \beta(\alpha_0\to\alpha,\lambda_0\to\lambda);$$
$$\gamma_x(\alpha,\lambda)\Big|_{\text{HD+MSL}} = \gamma_x(\alpha_0\to\alpha,\lambda_0\to\lambda).$$

According to

K.S., JHEP **10** (2019), 011; JHEP **01** (2020), 192; Eur. Phys. J. C **80** (2020) no.10, 911.

1. NSVZ equation is valid for RGFs defined in terms of the bare couplings in the case of using the higher covariant derivative regularization for an arbitrary renormalization prescription.

2. For RGFs defined in terms of the renormalized couplings some NSVZ schemes are given by the HD+MSL prescription. (MSL can supplement various versions of the higher covariant derivative regularization.)

1. First, one proves the ultraviolet finiteness of the triple vertices with two external lines of the Faddeev–Popov ghosts and one external line of the quantum gauge superfield.
2. Next, it is necessary to rewrite the NSVZ relation in the equivalent form

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\gamma_c(\alpha_0,\lambda_0)$$
$$-2C_2\gamma_V(\alpha_0,\lambda_0) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r\Big).$$

K.S., Nucl.Phys. **B909** (2016) 316.

3. After this we prove that the $\beta$-function is determined by integrals of double total derivatives with respect to loop momenta and present a method for constructing this integrals.

K.S., JHEP **10** (2019) 011.

4. Then the NSVZ equation is obtained by summing singular contributions.
5. Finally, an NSVZ scheme is constructed.

K.S., Eur.Phys.J. **C80** (2020) 10, 911.

# The two-loop anomalous dimension of the matter superfields with the higher derivative regularization

For theories with a single gauge coupling the two-loop anomalous dimension defined in terms of the bare couplings for $\mathcal{N}=1$ supersymmetric theories regularized by higher derivatives has been calculated in

A.E.Kazantsev, K.S., JHEP **2006** (2020) 108.

$$(\gamma_\phi)_i{}^j(\alpha_0,\lambda_0) = -\frac{\alpha_0}{\pi}C(R)_i{}^j + \frac{1}{4\pi^2}\lambda^*_{0imn}\lambda_0^{jmn} + \frac{\alpha_0^2}{2\pi^2}\left[C(R)^2\right]_i{}^j - \frac{1}{16\pi^4}$$
$$\times\lambda^*_{0iac}\lambda_0^{jab}\lambda^*_{0bde}\lambda_0^{cde} - \frac{3\alpha_0^2}{2\pi^2}C_2C(R)_i{}^j\Big(\ln a_\varphi + 1 + \frac{A}{2}\Big) + \frac{\alpha_0^2}{2\pi^2}T(R)C(R)_i{}^j$$
$$\times\Big(\ln a + 1 + \frac{A}{2}\Big) - \frac{\alpha_0}{8\pi^3}\lambda^*_{0lmn}\lambda_0^{jmn}C(R)_i{}^l(1-B+A) + \frac{\alpha_0}{4\pi^3}\lambda^*_{0imn}\lambda_0^{jml}$$
$$\times C(R)_l{}^n(1-A+B) + O\Big(\alpha_0^3, \alpha_0^2\lambda_0^2, \alpha_0\lambda_0^4, \lambda_0^6\Big),$$

where

$$A = \int\limits_0^\infty dx\,\ln x\,\frac{d}{dx}\frac{1}{R(x)};\qquad B = \int\limits_0^\infty dx\,\ln x\,\frac{d}{dx}\frac{1}{F^2(x)}\qquad a = \frac{M}{\Lambda};\qquad a_\varphi = \frac{M_\varphi}{\Lambda}.$$

# Obtaining the three-loop $\beta$-function from the NSVZ equation

If the anomalous dimension of the matter superfields defined in terms of the bare couplings has been calculated in $L$-loops with the higher derivative regularization, then it is possible to construct the $(L+1)$-loop $\beta$-function from the NSVZ equation without loop calculations. For example, in the three-loop approximation

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha_0}{4\pi^2}\Big\{ -3C_2^2 + \frac{1}{r}C_2\,\mbox{tr}\,C(R) + \frac{2}{r}\,\mbox{tr}\left[C(R)^2\right]\Big\}$$
$$-\frac{1}{8\pi^3 r}C(R)_j{}^i\lambda^*_{0imn}\lambda_0^{jmn} + \frac{\alpha_0^2}{8\pi^3}\Big\{ -3C_2^3 + \frac{1}{r}C_2^2\,\mbox{tr}\,C(R) - \frac{2}{r}\,\mbox{tr}\left[C(R)^3\right] + \frac{2}{r}$$
$$\times C_2\,\mbox{tr}\left[C(R)^2\right]\Big(3\ln a_\varphi + 4 + \frac{3A}{2}\Big) - \frac{2}{r^2}\,\mbox{tr}\,C(R)\,\mbox{tr}\left[C(R)^2\right]\Big(\ln a + 1 + \frac{A}{2}\Big)\Big\}$$
$$-\frac{\alpha_0 C_2}{16\pi^4 r}C(R)_j{}^i\lambda^*_{0imn}\lambda_0^{jmn} + \frac{\alpha_0}{16\pi^4 r}\left[C(R)^2\right]_j{}^i\lambda^*_{0imn}\lambda_0^{jmn}\Big(1 + A - B\Big) - \frac{\alpha_0}{8\pi^4 r}$$
$$\times C(R)_j{}^i C(R)_l{}^n\lambda^*_{0imn}\lambda_0^{jml}\Big(1 - A + B\Big) + \frac{1}{32\pi^5 r}C(R)_j{}^i\lambda^*_{0iac}\lambda_0^{jab}\lambda^*_{0bde}\lambda_0^{cde}$$
$$+O\Big(\alpha_0^3,\,\alpha_0^2\lambda_0^2,\,\alpha_0\lambda_0^4,\,\lambda_0^6\Big).$$

This result has been confirmed by some explicit three-loop calculation, see, e.g.,


V.Yu.Shakhmanov, K.S., Nucl.Phys., **B920**, (2017), 345;
A.E.Kazantsev, V.Yu.Shakhmanov, K.S., JHEP 1804 (2018) 130.

The above results can be applied to the particular case of $\mathcal{N}=2$ supersymmetric theories which can certainly be formulated in $\mathcal{N}=1$ superspace. In this formulation one supersymmetry is manifest, while the other is hidden. The action in the massless limit is written as

$$S=\frac{1}{2e_0^2}\mbox{Re}\,\mbox{tr}\int d^4x\,d^2\theta\,W^aW_a+\frac{1}{2e_0^2}\mbox{tr}\int d^4x\,d^4\theta\,\Phi^+e^{2V}\Phi e^{-2V}$$
$$+\frac{1}{4}\int d^4x\,d^4\theta\Big(\phi^+e^{2V}\phi+\widetilde{\phi}^+e^{-2V^T}\widetilde{\phi}\Big)+\Big(\frac{i}{\sqrt{2}}\int d^4x\,d^2\theta\,\widetilde{\phi}^T\Phi\phi+\mbox{c.c.}\Big).$$

The chiral matter superfields $\boldsymbol{\phi_i}=\Big(\Phi^A,\,\phi_i,\,\widetilde{\phi}^i\Big)$ belong to the reducible representation $R=Adj+R_0+\bar{R}_0$. Taking into account that

$$\frac{i}{\sqrt{2}}\int d^4x\,d^2\theta\,\widetilde{\phi}^T\Phi\phi=\frac{ie_0}{\sqrt{2}}(T^A)_i{}^j\int d^4x\,d^2\theta\,\widetilde{\phi}^i\Phi^A\phi_j$$

we see that the Yukawa couplings are related to the gauge coupling constant by the equations

$$(\lambda_0)_i{}^{jA}=(\lambda_0)_i{}^{Aj}=(\lambda_0)^j{}_i{}^A=(\lambda_0)^A{}_i{}^j=(\lambda_0)^{jA}{}_i=(\lambda_0)^{Aj}{}_i=\frac{ie_0}{\sqrt{2}}(T^A)_i{}^j,$$

where $(T^A)_i{}^j$ are the generators of the representation $R_0$.

It appears that for an arbitrary $\mathcal{N}=1$ renormalization prescription the anomalous dimensions and the higher order contributions to the $\beta$-function do not vanish starting from the two- and three-loop approximations, respectively.

S.S.Aleshin, K.S., Phys. Rev. D **107** (2023) no.10, 105006.

However, the anomalous dimensions of chiral matter superfields vanish for such renormalization prescriptions that

1. The renormalization prescription does not break the $\mathcal{N}=2$ relation between the gauge and Yukawa couplings,

$$\frac{d}{d\ln\mu}\Big(\frac{\lambda^A{}_i{}^j}{e}\Big)=0.$$

2. The renormalization prescription is compatible with the structure of quantum corrections.

3. Moreover, all contributions to the $\beta$-function beyond the one-loop approximation vanish if the conditions 1 and 2 are satisfied and the renormalization prescription is NSVZ. Then

$$\gamma_\Phi=0;\qquad (\gamma_\phi)_i{}^j=0;\qquad \frac{\beta(\alpha)}{\alpha^2}=-\frac{1}{\pi}\Big(C_2-T(R_0)\Big).$$

Note that for $\mathcal{N}=2$ supersymmetric theories $\overline{\text{DR}}$-scheme is NSVZ, at least, in the lowest loops.

# Exact results for the $P = \frac{1}{3}Q$ theories

Some interesting properties of quantum corrections exist in the so-called $P = \frac{1}{3}Q$ theories, which by definition satisfy the relation

$$\lambda^*_{imn}\lambda^{jmn} - 4\pi\alpha C(R)_i{}^j = \frac{2\pi\alpha}{3} Q\delta_i^j,$$

where $Q \equiv T(R) - 3C_2$. Really, it was demonstrated

I.Jack, D.R.T.Jones, C.G.North, Nucl. Phys. B **473** (1996), 308.

that in these theories in the first two orders of the perturbation theory the ratio of the Yukawa couplings to the gauge coupling is RG invariant,

$$\frac{d}{d\ln\mu}\Big(\frac{\lambda^{ijk}}{e}\Big) = 0.$$

This condition can equivalently be rewritten as a relation between anomalous dimensions of quantum superfields in each order of the perturbation theory

M.D.Kuzmichev, K.S., Phys.Lett. **844** (2023), 138094.

## Exact results for the $P = \frac{1}{3}Q$ theories

Due to the nonrenormalization of the superpotential

$$\lambda^{ijk} = \lambda_0^{mnp} \big(\sqrt{Z_\phi}\big)_m{}^i \big(\sqrt{Z_\phi}\big)_n{}^j \big(\sqrt{Z_\phi}\big)_p{}^k,$$

and due to the nonrenormalization of the triple gauge-ghost vertices

$$Z_\alpha^{-1/2} Z_c Z_V = 1.$$

Using these equations we rewrite the condition for RG invariance of the ratio $\lambda^{ijk}/e$ in the form

$$0 = \frac{\lambda_0^{mnp}}{e_0} \frac{d}{d\ln\mu} \Big( Z_c^{-1} Z_V^{-1} \big(\sqrt{Z_\phi}\big)_m{}^i \big(\sqrt{Z_\phi}\big)_n{}^j \big(\sqrt{Z_\phi}\big)_p{}^k \Big).$$

This equation is evidently valid if

$$\frac{d}{d\ln\mu} \Big( (Z_c)^{-2/3} (Z_V)^{-2/3} (Z_\phi)_i{}^j \Big) = 0.$$

Therefore, the anomalous dimensions of quantum superfields in the $P = \frac{1}{3}Q$ theories in each order of the perturbation theory should satisfy the equation

$$2\big(\gamma_c + \gamma_V\big)\, \delta_i^j = 3\big(\gamma_\phi\big)_i{}^j.$$

# Exact results for the $P = \frac{1}{3}Q$ theories

The one-loop expressions for the anomalous dimensions entering the above equation are written as

$$\gamma_c^{(1)} = -\frac{\alpha C_2(1-\xi)}{6\pi}; \qquad \gamma_V^{(1)} = \frac{\alpha C_2(1-\xi)}{6\pi} + \frac{Q\alpha}{4\pi};$$
$$(\gamma_\phi^{(1)})_i{}^j = -\frac{\alpha}{\pi} C(R)_i{}^j + \frac{1}{4\pi^2}\lambda^*_{imn}\lambda^{jmn} = \frac{Q\alpha}{6\pi}\delta_i^j.$$

We see that they really satisfy the above relation, which is therefore valid in the one-loop approximation. Also it is possible to demonstrate that for a certain renormalization prescriptions this relation is valid in the two-loop approximation.

Due to the nonrenormalization of the triple gauge-ghost vertices

$$\beta = 2\alpha(\gamma_c + \gamma_V),$$

so that for the $P = \frac{1}{3}Q$ theories the RG invariance of the ratio $\lambda^{ijk}/e$ leads to the relation

I.Jack, D.R.T.Jones, C.G.North, Nucl. Phys. B **473** (1996), 308.

$$(\gamma_\phi)_i{}^j = \frac{\beta}{3\alpha}\,\delta_i^j.$$

# Exact results for the $P = \frac{1}{3}Q$ theories

The above equation and the NSVZ relation

$$\beta(\alpha,\lambda) = \frac{\alpha^2\Big(Q - C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1 - C_2\alpha/2\pi)}$$

give the exact expressions for the $\beta$-function and for the anomalous dimension

$$\beta(\alpha) = \frac{\alpha^2 Q}{2\pi(1+\alpha Q/6\pi)};\qquad (\gamma_\phi)_i{}^j(\alpha) = \frac{\alpha Q}{6\pi(1+\alpha Q/6\pi)}\delta_i^j \equiv \gamma_\phi \delta_i^j.$$

At present they are not proved in all orders by a direct calculation. The first step towards this derivation is to rewrite the expression for the anomalous dimension in such a form that a contribution to it in a certain loop will be related to RGFs in the previous loop only, e.g.,

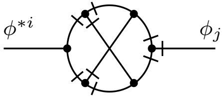
$$\gamma_\phi(\alpha) = \frac{\alpha Q}{6\pi}\Big(1 - \gamma_\phi(\alpha)\Big),$$

However, a more general expression can be written

$$\gamma_\phi(\alpha) = \frac{\alpha Q}{6\pi}\Big(1 - \frac{2}{3}x\,(\gamma_c(\alpha) + \gamma_V(\alpha)) - (1-x)\gamma_\phi(\alpha)\Big),$$

where $x$ is an arbitrary real number. At present, we are not able to pick out such a value of $x$ needed for the perturbative derivation of this equation.

However, essential problems appear in the three-loop approximation. Really, the three-loop anomalous dimension of the matter superfields is contributed by the superdiagram




given by the expression

$$(\Delta\gamma_\phi)_i{}^j = \frac{3\zeta(3)}{64\pi^6}\,\lambda^*_{ikl}\lambda^{kpq}\lambda^{lrs}\lambda^*_{mpr}\lambda^*_{nqs}\lambda^{jmn},$$

which cannot be simplified with the help of the $P = \frac{1}{3}Q$ condition. Therefore, the above relations between RGFs are not satisfied for all group structures (if the Yukawa coupling renormalization is related to the renormalization of the chiral matter superfields) and, in particular, in the $\overline{\text{DR}}$ scheme. Nevertheless, in principle, it is possible to absorb the above structure into a finite redefinition of $\lambda^{ijk}$. However,

I.Jack, D.R.T.Jones, C.G.North, Nucl. Phys. B **473** (1996), 308.

it is impossible to remove other terms proportional to $\zeta(3)$. Therefore, to achieve the RG invariance of the ratio $\lambda^{ijk}/e$, one should either impose some additional constraints to the Yukawa couplings (certainly, if possible) or to suggest that the above exact equations are valid only for a certain class of superdiagrams, e.g., for the planar supergraphs.

# Conclusion

- The higher covariant derivative regularization allows revealing some interesting features of supersymmetric theories and deriving some all-loop results.
- RGFs defined in terms of the bare couplings satisfy the NSVZ relation in theories regularized by higher derivatives in all loops.
- Some all-order NSVZ schemes are given by the HD+MSL prescription.
- Validity of the NSVZ equation with the higher covariant derivative regularization allows to essentially simplify some multiloop calculations.
- The $\mathcal{N}=2$ non-renormalization theorems for $\mathcal{N}=2$ supersymmetric theories formulated in $\mathcal{N}=1$ superspace are valid if a renormalization prescription is compatible with a structure of quantum corrections, do not break the relation between Yukawa and gauge couplings, and is NSVZ.
- The RG invariance of the ratio $\lambda^{ijk}/e$ in the $P=\frac{1}{3}Q$ theories is equivalent to a certain relation between the anomalous dimensions of the quantum superfields, which should be valid in each order of the perturbative theory.
- Up to the two-loop approximations this relation is valid for certain renormalization prescriptions, but in higher orders it is presumably valid for some special classes of diagrams (e.g., for the planar supergraphs) or under some additional restictions on the Yukawa couplings.

Thank you for the attention!