The anomalous dimension of twist-3 operators in $\mathcal{N} = 4$ SYM theory

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Deep Inelastic Scattering

[Gross, Wilczek '73]

$$\mathcal{O}^{q}_{\mu_1,\ldots,\mu_j} = \bar{q}\,\gamma_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\,q + \text{symmetrisation} - \text{traces}$$

Twist = Canonical dimension - Lorentz spin j [Gross, Wilczek '73]

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Operators mix under renormalization → Matrix of anomalous dimensions

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$$\Gamma = \begin{pmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{pmatrix}$$

$$\gamma_{qq} = 2C_F\left[4S_1(j) - 3 - \frac{2}{j(j+1)}\right] \qquad \gamma_{qg} = -8T_R\frac{j^2+j+2}{j(j+1)(j+2)}$$

$$\gamma_{gq} = -4C_F\frac{j^2+j+2}{(j-1)j(j+1)} \qquad \gamma_{gg} = \left[8C_A\left(S_1(j) - \frac{1}{j(j-1)} - \frac{1}{(j+1)(j+2)} - \frac{11}{12}\right) + \frac{8}{3}T_R\right]$$

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$$S_1(j) = \sum_{k=1}^{j}\frac{1}{k} = \Psi(1) - \Psi(j+1)$$

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Quantum Chromodynamics: Wilson twist-2 operators

$$\gamma_{qq} = C_F \left[8S_1(j) \ - \frac{4}{j} + \frac{4}{j+1} - 6 \right]$$

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Quantum Chromodynamics: Wilson twist-2 operators

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Origin:

Quantum Chromodynamics: Wilson twist-2 operators

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Origin: $\mathcal{N} = 1$ Supersymmetric Yang-Mills theory

Wilson twist-2 operators: [Gross, Wilczek '73]

$$\mathcal{O}^{g}_{\mu_1,\ldots,\mu_j} = G^{a}_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\ldots\mathcal{D}_{\mu_{j-1}}G^{a}_{\rho\mu_j}$$

$$\mathcal{O}^{\lambda}_{\mu_1,\ldots,\mu_j} = \bar{\lambda}^{a}_{i}\gamma_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\lambda^{a,\,i}$$

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$$\mathcal{O}^{\phi}_{\mu_1,\ldots,\mu_j} = \bar{\phi}^a_r\mathcal{D}_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\phi^{a,\,r}$$

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Anomalous dimension matrix in leading order: [Lipatov '00]

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Anomalous dimension matrix in leading order: [Lipatov '00]

$$\Gamma = \begin{pmatrix} \gamma_{gg} & \gamma_{g\lambda} & \gamma_{g\phi} \\ \gamma_{\lambda g} & \gamma_{\lambda\lambda} & \gamma_{\lambda\phi} \\ \gamma_{\phi g} & \gamma_{\phi\lambda} & \gamma_{\phi\phi} \end{pmatrix}$$

Wilson twist-2 operators:

[Gross, Wilczek '73]

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Anomalous dimension matrix in leading order:

[Lipatov '00]

$$\gamma^{(0)}_{gg} = -4S_1(j) + \frac{4}{j-1} - \frac{4}{j} + \frac{4}{j+1} - \frac{4}{j+2} \qquad \gamma^{(0)}_{\lambda g} = \frac{8}{j} - \frac{16}{j+1} + \frac{16}{j+2}$$

$$\gamma^{(0)}_{\phi g} = \frac{12}{j+1} - \frac{12}{j+2} \qquad \gamma^{(0)}_{g\lambda} = \frac{4}{j-1} - \frac{4}{j} + \frac{2}{j+1} \qquad \gamma^{(0)}_{\lambda\phi} = \frac{8}{j} \qquad \gamma^{(0)}_{\phi\lambda} = \frac{6}{j+1}$$

$$\gamma^{(0)}_{\lambda\lambda} = -4S_1(j) + \frac{4}{j} - \frac{4}{j+1} \qquad \gamma^{(0)}_{\phi\phi} = -4S_1(j) \qquad \gamma^{(0)}_{g\phi} = \frac{4}{j-1} - \frac{4}{j}$$

$\mathcal{N}=4$ SYM theory: One loop

Wilson twist-2 operators: [Gross, Wilczek '73]

$$\mathcal{O}^{g}_{\mu_1,\ldots,\mu_j} = G^a_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\ldots\mathcal{D}_{\mu_{j-1}}G^a_{\rho\mu_j} \qquad \tilde{\mathcal{O}}^{g}_{\mu_1,\ldots,\mu_j} = G^a_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\ldots\mathcal{D}_{\mu_{j-1}}\tilde{G}^a_{\rho\mu_j}$$

$$\mathcal{O}^{\lambda}_{\mu_1,\ldots,\mu_j} = \bar{\lambda}^a_i\gamma_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\lambda^{a,\,i} \qquad \tilde{\mathcal{O}}^{\lambda}_{\mu_1,\ldots,\mu_j} = \bar{\lambda}^a_i\gamma_5\gamma_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\lambda^{a,\,i}$$

$$\mathcal{O}^{\phi}_{\mu_1,\ldots,\mu_j} = \bar{\phi}^a_r\mathcal{D}_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\phi^{a,\,r}$$

Anomalous dimension matrix in leading order: [Lipatov '00]

$$\gamma^{(0)}_{gg} = -4S_1(j) + \frac{4}{j-1} - \frac{4}{j} + \frac{4}{j+1} - \frac{4}{j+2} \qquad \gamma^{(0)}_{\lambda g} = \frac{8}{j} - \frac{16}{j+1} + \frac{16}{j+2}$$

$$\gamma^{(0)}_{\phi g} = \frac{12}{j+1} - \frac{12}{j+2} \qquad \gamma^{(0)}_{g\lambda} = \frac{4}{j-1} - \frac{4}{j} + \frac{2}{j+1} \qquad \gamma^{(0)}_{\lambda\phi} = \frac{8}{j} \qquad \gamma^{(0)}_{\phi\lambda} = \frac{6}{j+1}$$

$$\gamma^{(0)}_{\lambda\lambda} = -4S_1(j) + \frac{4}{j} - \frac{4}{j+1} \qquad \gamma^{(0)}_{\phi\phi} = -4S_1(j) \qquad \gamma^{(0)}_{g\phi} = \frac{4}{j-1} - \frac{4}{j}$$

$$\widetilde{\Gamma} = \begin{pmatrix} \tilde{\gamma}_{gg} & \tilde{\gamma}_{g\lambda} \\ \tilde{\gamma}_{\lambda g} & \tilde{\gamma}_{\lambda\lambda} \end{pmatrix}$$

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$$\mathcal{O}^{\lambda}_{\mu_1,\dots,\mu_j} = \bar{\lambda}^{a}_{i}\gamma_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\lambda^{a,\,i} \qquad \tilde{\mathcal{O}}^{\lambda}_{\mu_1,\dots,\mu_j} = \bar{\lambda}^{a}_{i}\gamma_5\gamma_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\lambda^{a,\,i}$$

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$$\gamma^{(0)}_{\phi g} = \frac{12}{j+1} - \frac{12}{j+2} \qquad \gamma^{(0)}_{g\lambda} = \frac{4}{j-1} - \frac{4}{j} + \frac{2}{j+1} \qquad \gamma^{(0)}_{\lambda\phi} = \frac{8}{j} \qquad \gamma^{(0)}_{\phi\lambda} = \frac{6}{j+1}$$

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$$\Gamma^{(0)} = \begin{pmatrix} \gamma_{gg}^{(0)} & \gamma_{g\lambda}^{(0)} & \gamma_{g\phi}^{(0)} \\ \gamma_{\lambda g}^{(0)} & \gamma_{\lambda\lambda}^{(0)} & \gamma_{\lambda\phi}^{(0)} \\ \gamma_{\phi g}^{(0)} & \gamma_{\phi\lambda}^{(0)} & \gamma_{\phi\phi}^{(0)} \end{pmatrix} \qquad \widetilde{\Gamma}^{(0)} = \begin{pmatrix} \tilde{\gamma}_{gg}^{(0)} & \tilde{\gamma}_{g\lambda}^{(0)} \\ \tilde{\gamma}_{\lambda g}^{(0)} & \tilde{\gamma}_{\lambda\lambda}^{(0)} \end{pmatrix}$$

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$$\Downarrow$$

$$\begin{pmatrix} S_1(j-2) & 0 & 0 \\ 0 & S_1(j) & 0 \\ 0 & 0 & S_1(j+2) \end{pmatrix}$$

$$\widetilde{\Gamma}^{(0)} = \begin{pmatrix} \tilde{\gamma}_{gg}^{(0)} & \tilde{\gamma}_{g\lambda}^{(0)} \\ \tilde{\gamma}_{\lambda g}^{(0)} & \tilde{\gamma}_{\lambda\lambda}^{(0)} \end{pmatrix}$$

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$$\begin{pmatrix} S_1(j-1) & 0 \\ 0 & S_1(j+1) \end{pmatrix}$$

$\mathcal{N}=4$ SYM theory: One loop

Anomalous dimension matrix in leading order:

[Lipatov '00]

$$\Gamma^{(0)} = \begin{pmatrix} \gamma_{gg}^{(0)} & \gamma_{g\lambda}^{(0)} & \gamma_{g\phi}^{(0)} \\ \gamma_{\lambda g}^{(0)} & \gamma_{\lambda\lambda}^{(0)} & \gamma_{\lambda\phi}^{(0)} \\ \gamma_{\phi g}^{(0)} & \gamma_{\phi\lambda}^{(0)} & \gamma_{\phi\phi}^{(0)} \end{pmatrix}$$

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Eigenvalues of anomalous dimension matrix are expressed through the same function $\gamma_{uni}^{(0)}(j+2) = S_1(j)$ with shifted argument

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$$\begin{pmatrix} S_1(j-2) & 0 & 0 \\ 0 & S_1(j) & 0 \\ 0 & 0 & S_1(j+2) \end{pmatrix} \qquad \begin{pmatrix} S_1(j-1) & 0 \\ 0 & S_1(j+1) \end{pmatrix}$$

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$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$\begin{pmatrix} S_1(j-2) & 0 & 0 \\ 0 & S_1(j) & 0 \\ 0 & 0 & S_1(j+2) \end{pmatrix} \qquad \begin{pmatrix} S_1(j-1) & 0 \\ 0 & S_1(j+1) \end{pmatrix}$$

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Origin: All multiplicatively renormalizable operators in $\mathcal{N}=4$ SYM theory belong to the same supermultiplet

$\mathcal{N}=4$ SYM theory: One loop

Anomalous dimension matrix in leading order: [Lipatov '00]

$$\Gamma^{(0)} = \begin{pmatrix} \gamma^{(0)}_{gg} & \gamma^{(0)}_{g\lambda} & \gamma^{(0)}_{g\phi} \\ \gamma^{(0)}_{\lambda g} & \gamma^{(0)}_{\lambda\lambda} & \gamma^{(0)}_{\lambda\phi} \\ \gamma^{(0)}_{\phi g} & \gamma^{(0)}_{\phi\lambda} & \gamma^{(0)}_{\phi\phi} \end{pmatrix} \qquad \widetilde{\Gamma}^{(0)} = \begin{pmatrix} \tilde{\gamma}^{(0)}_{gg} & \tilde{\gamma}^{(0)}_{g\lambda} \\ \tilde{\gamma}^{(0)}_{\lambda g} & \tilde{\gamma}^{(0)}_{\lambda\lambda} \end{pmatrix}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{pmatrix} S_1(j-2) & 0 & 0 \\ 0 & S_1(j) & 0 \\ 0 & 0 & S_1(j+2) \end{pmatrix} \qquad \begin{pmatrix} S_1(j-1) & 0 \\ 0 & S_1(j+1) \end{pmatrix}$$

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$$\mathcal{O}^{T}_{\mu_1,\ldots,\mu_j} = \mathcal{O}^{g}_{\mu_1,\ldots,\mu_j} + \mathcal{O}^{\lambda}_{\mu_1,\ldots,\mu_j} + \mathcal{O}^{\phi}_{\mu_1,\ldots,\mu_j}$$

[Maldacena '97] [Witten '98] [Gubser, Klebanov, Polyakov '98]

Maximally extended supersymmetric gauge theory $\Leftrightarrow$ IIB string on $AdS_5 \times S^5$

AdS/CFT-correspondence

[Maldacena '97] [Witten '98] [Gubser, Klebanov, Polyakov '98]

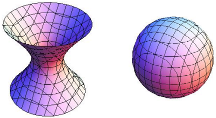
Maximally extended supersymmetric gauge theory $\Leftrightarrow$ IIB string on $AdS_5 \times S^5$

spin	
1	Gauge field $\mathcal{A}_\mu$
1/2	4 fermions λ^i
0	3 complex scalars Φ^r

Conformal Field Theory

[Maldacena '97] [Witten '98] [Gubser, Klebanov, Polyakov '98]

spin	Maximally extended supersymmetric gauge theory	⇔	IIB string on $AdS_5 \times S^5$
1	Gauge field $\mathcal{A}_\mu$		
1/2	4 fermions λ^i		
0	3 complex scalars Φ^r		
	Conformal Field Theory		AdS - Anti-de Sitter space with negative curvature



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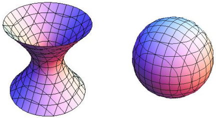

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Conformal Field Theory

AdS - Anti-de Sitter space with negative curvature

The same symmetry

[Maldacena '97] [Witten '98] [Gubser, Klebanov, Polyakov '98]

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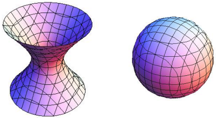

The same symmetry

Operators – $\mathcal{O}_A(x) = \mathrm{Tr}\, \mathcal{A} \ldots \Psi \ldots \Phi$ $\Leftrightarrow$ $|\mathcal{O}_A\rangle$ – String states

Dimension – $\Delta = 2 + \sqrt{4 + m^2 R^2}$ $\Leftrightarrow$ m – Mass

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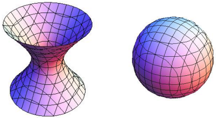

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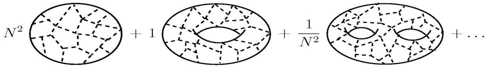

$1/N$ $\Leftrightarrow$ g_{st}

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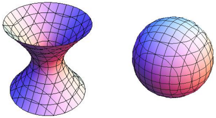

The same symmetry

Operators – $\mathcal{O}_A(x) = \text{Tr}\,\mathcal{A}\ldots\Psi\ldots\Phi \quad \Leftrightarrow \quad |\mathcal{O}_A\rangle$ – String states

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$$1/N \quad \Leftrightarrow \quad g_{st}$$

$$\lambda = g_{YM}^2 N \quad \Leftrightarrow \quad \lambda = R^4/\alpha'^2$$

$$\langle \mathcal{O}_A(x)\mathcal{O}_B(y)\rangle \sim \frac{\delta_{A,B}}{(x-y)^{2\Delta(\lambda,\frac{1}{N})}} \quad \Leftrightarrow \quad \mathcal{H}_{String}|\mathcal{O}_A\rangle = E_A\Big(\frac{1}{\sqrt{\lambda}}, g_S\Big)|\mathcal{O}_A\rangle$$

$$\Delta\left(\lambda, \frac{1}{N}\right) \quad \lambda \ll 1 \quad = \quad E\left(\frac{1}{\sqrt{\lambda}}, g_S\right) \quad \lambda \gg 1$$

BMN-operators: [Berenstein, Maldacena and Nastase '02]

$$\text{Tr}\, Z^J = \text{Tr}\, ZZZZZZZZZZZZ\ldots$$
$$\text{Tr}\, XZ^J = \text{Tr}\, XZZ\ldots = \text{Tr}\, ZXZ\ldots$$
$$\sum_k \text{Tr}\; XZ^k XZ^{J-k}$$

$X=\Phi^1, Y=\Phi^2$ and $Z=\Phi^3$ – scalar fields

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BMN-limit: $N, J \to \infty,\ \lambda' = \dfrac{g_{YM}^2 N}{J^2}$

Two dimensional quantum field theory in flat space

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Computations of anomalous dimension of BMN-operators: $\mathrm{Tr}\; XZXZ$

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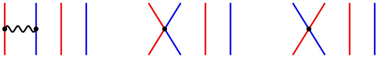

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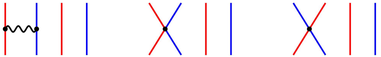

$$\text{Tr}\, XZXZ_{\text{Ren}} = A_1(\lambda)\, \text{Tr}\, XZXZ + A_2(\lambda)\, \text{Tr}\, ZXXZ$$

Integrability in $\mathcal{N}=4$ SYM

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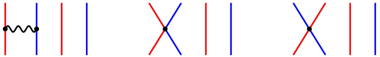
Computations of anomalous dimension of BMN-operators: $\mathrm{Tr}\; XZXZ$



$$\mathrm{Tr}\, XZXZ_{\mathsf{Ren}} = A_1(\lambda)\,\mathrm{Tr}\, XZXZ + A_2(\lambda)\,\mathrm{Tr}\, ZXXZ$$

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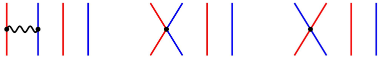
Two dimensional quantum field theory in flat space

Computations of anomalous dimension of BMN-operators: Tr $XZXZ$



$$\text{Tr}\, XZXZ_{\text{Ren}} = A_1(\lambda)\, \text{Tr}\, XZXZ + A_2(\lambda)\, \text{Tr}\, ZXXZ$$
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Relation with Heisenberg spin chain [Minahan and Zarembo '02]

$$\mathrm{Tr}\, Z^J = \mathrm{Tr}\, ZZZZZZZZZZZZ\ldots \quad \Leftrightarrow \quad |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\rangle$$

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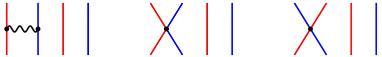
$$\sum_k \mathrm{Tr}\, XZ^kXZ^{J-k} \quad \Leftrightarrow \quad |\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\rangle$$

$X=\Phi^1, Y=\Phi^2$ and $Z=\Phi^3$ – scalar fields $\Leftrightarrow$ spin up and spin down

BMN-limit: $N, J \to \infty,\ \lambda' = \dfrac{g_{YM}^2 N}{J^2}$

Two dimensional quantum field theory in flat space

Computations of anomalous dimension of BMN-operators: $\mathrm{Tr}\, XZXZ$



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BMN-limit: $N, J \to \infty,\ \lambda' = \dfrac{g_{YM}^2 N}{J^2}$

Two dimensional quantum field theory in flat space

Computations of anomalous dimension of BMN-operators: $\mathrm{Tr}\, XZXZ$

$$\mathcal{D}(\lambda)\begin{pmatrix}\mathrm{Tr}\, XZXZ\\ \mathrm{Tr}\, ZXXZ\end{pmatrix} = \mathbb{A}(\lambda)\begin{pmatrix}\mathrm{Tr}\, XZXZ\\ \mathrm{Tr}\, ZXXZ\end{pmatrix} \quad \Leftrightarrow \quad \mathcal{H}(\lambda)\begin{pmatrix}|\downarrow\uparrow\downarrow\uparrow\rangle\\ |\uparrow\downarrow\downarrow\uparrow\rangle\end{pmatrix} = \mathbb{E}(\lambda)\begin{pmatrix}|\downarrow\uparrow\downarrow\uparrow\rangle\\ |\uparrow\downarrow\downarrow\uparrow\rangle\end{pmatrix}$$

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Relation with Heisenberg spin chain [Minahan and Zarembo '02]

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Exact solution: INTEGRABILITY (Bethe-Anzatz) [Bethe '31, Faddeev... '80]

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$\mathcal{H}_2$ and $\mathcal{H}_3$ were computed: TEST [Beisert, Staudacher '03]

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All-loop Bethe anzatz: $\lambda = g^2 N$ [Beisert and Staudacher, Kazakov '04-'06]

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The same Bethe-anzatz from the field theory and from the string

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Solution: In leading order – system of non-linear equations

Explicit solution: $Q_M(u) = C_M\prod_{k=1}^{M}(u-u_k) = {}_3F_2[-M, M+1, \frac{1}{2}-iu; 1, 1; 1]$

At higher orders – system of linear equations

Twist-2 in $\mathcal{N} = 4$ SYM: Calculations

Twist-2 operators: $\mathcal{O}_{\mathrm{twist}-2} = \mathrm{Tr} Z \mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} \ldots \mathcal{D}_{\mu_M} Z$

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Set of Q-functions of the complex spectral parameter u and relations between them:

$$Q_{a|i}\left(u+\frac{i}{2}\right) - Q_{a|i}\left(u-\frac{i}{2}\right) = Q_{a|0}(u)\, Q_{0|i}(u)$$

Why do we need it?

- Check integrability
- Study of analytical properties
- Back to QCD

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$\mathcal{N} = 4$ SYM: Anomalous dimension of twist-2 operators

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Summing large logarithms $(\alpha_s \ln^k x) \sim 1$ in all orders

- Advantages: All-loop results
- Price: Near negative integer values $M = -N + \omega$

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Analytical continuation

Anomalous dimension $\gamma(M)$ for positive (even) integer M

$$\gamma(M) = \sum_{\ell=1} g^{2\ell} \gamma_{2\ell}(M) \qquad \gamma_2(M) \sim S_1(M) = \Psi(1) - \Psi(M+1)$$

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$$S_1(M) = \sum_{k=1}^{M}\frac{1}{k} = \sum_{k=1}^{\infty}\frac{1}{k} - \sum_{k=M+1}^{\infty}\frac{1}{k} = \sum_{k=0}^{\infty}\frac{1}{k+1} - \sum_{k=0}^{\infty}\frac{1}{k+M+1}$$

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BFKL equation predicts all poles at $M = -1 + \omega$

$$\gamma(-1+\omega) = \sum_{\ell=1} b_\ell \frac{g^{2\ell}}{\omega^\ell} = \sum_{\ell=1} b_\ell \left(\frac{g^2}{\omega}\right)^\ell$$

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Double-logarithmic equation predicts all poles at $M = -2+\omega$

$$\gamma(-2+\omega) = \sum_{\ell=1} d_\ell\,\frac{g^{2\ell}}{\omega^{2\ell-1}} = \sum_{\ell=1} d_\ell\left(\frac{g^2}{\omega^2}\right)^\ell\omega$$

Anomalous dimension for twist-2 operators known up to seven loops

$$\gamma(M) = \sum_{\ell} \gamma_\ell(M)\, g^{2\ell}, \qquad S_{i_1,\ldots,i_k}(M) = \sum_{k=1}^{M} \frac{sign(i_1)^k}{k^{|i_1|}} S_{i_1,\ldots,i_k}(k)$$

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Investigation of analytical properties

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Analytical continuation to $M = -2 + \omega$ [arXiv:1104.4100]

$$\gamma(-2+\omega) = \sum_{\ell=1} \omega \left(\frac{g^2}{\omega^2}\right)^{\ell} \left(\boxed{d_{0,\ell}} + \boxed{d_{1,\ell}\,\omega} \ldots \right) \Rightarrow \sum_{\ell=1} \sum_{k=0} \omega \left(\frac{g^2}{\omega^2}\right)^{\ell} d_{k,\ell}\, \omega^k$$

Double-logs equation is known up to $(g^2/\omega^2)^k \omega^{2\ell}$ if we know ℓ orders

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BFKL equation is known in next-to-next-to-leading-logarithm (NNLLA)

Analytical continuation to $M = -2 + \omega$ [arXiv:1104.4100]

$$\gamma(-2+\omega) = \sum_{\ell=1} \omega \left(\frac{g^2}{\omega^2}\right)^{\ell} \left(\boxed{d_{0,\ell}} + \boxed{d_{1,\ell}\,\omega} \ldots \right) \Rightarrow \sum_{\ell=1} \sum_{k=0} \omega \left(\frac{g^2}{\omega^2}\right)^{\ell} d_{k,\ell}\, \omega^k$$

Double-logs equation is known up to $(g^2/\omega^2)^k \omega^{2\ell}$ if we know ℓ orders

$$\Big(\gamma + 2\,\omega\Big)\gamma = \sum_{\ell=1} \sum_{k=0} \omega^k \left(g^2\right)^{\ell} \hat{d}_{k,\ell} = -16\, g^2 + \cdots$$

Twist-3 operators: $\mathcal{O}_{\mathrm{twist-2}} = \mathrm{Tr}\, Z\, \mathcal{D}_{\mu_1} \ldots \mathcal{D}_{\mu_k} Z\, \mathcal{D}_{\mu_{k+1}} \ldots \mathcal{D}_{\mu_M} Z$

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Asymptotic Bethe Ansatz: $\lambda = g^2 N_c \qquad x_k^{\pm} = x(u_k \pm \frac{i}{2})$

$$\left(\frac{x_k^+}{x_k^-}\right)^3 = \prod_{\substack{j=1\\ j\neq k}}^{M} \frac{x_k^- - x_j^+}{x_k^+ - x_j^-}\,\frac{1-\lambda/x_k^+ x_j^-}{1-\lambda/x_k^- x_j^+}\exp^{\left(2\,i\,\theta(u_k,u_j)\right)} \qquad \gamma(M) = 2\,\lambda \sum_{k=1}^{M}\left(\frac{i}{x_k^+} - \frac{i}{x_k^-}\right)$$

Twist-3 in $\mathcal{N}=4$ SYM

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$$\left(u+\frac{i}{2}\right)^3 Q(u+i) + \left(u-\frac{i}{2}\right)^3 Q(u-i) = \left[2\,u^3 - \left(M^2 + 2\,M + \tfrac{3}{2}\right)u\right]Q(u)$$

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Quantum Spectral Curve: Result only for fixed M

Twist-3 in $\mathcal{N} = 4$ SYM: Reconstruction

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Reconstruction of the anomalous dimension from results for fixed M:

$$\gamma(M) = x_1 B_1(M) + x_2 B_2(M) + \ldots + x_n B_n(M)$$

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Reconstruction: LLL-algorithm

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Loops	1	2	3	4	5	6	7
Nested sums	1	4	16	64	256	1024	4096

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The Lenstra-Lenstra-Lovász (LLL) lattice basis reduction algorithm

Polynomial time lattice reduction algorithm invented by
Arjen Lenstra, Hendrik Lenstra and László Lovász in 1982
The LLL algorithm outputs an LLL-reduced (short, nearly orthogonal) lattice basis under the Euclidean norm

- start from the system of equations:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 0 \\ \frac{15}{32}x_1 + \frac{27}{32}x_2 + \frac{33}{32}x_3 + \frac{39}{32}x_4 + \frac{21}{32}x_5 &= \frac{9}{16} \end{aligned}$$

- take matrix (divide to the greatest common divisor – GCD function):

$$\mathsf{SE} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 5 & 9 & 11 & 13 & 7 & -6 \end{pmatrix}$$

- multiply SE to some huge integer number, for example 8^8
- create unity matrix $\mathbb{I}$ with rank equal to the length of row in SE
- append transpose SE to the right side of the unity matrix $\mathbb{I}$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 8^8 & 5\times 8^8 \\ 0 & 1 & 0 & 0 & 0 & 0 & 8^8 & 9\times 8^8 \\ 0 & 0 & 1 & 0 & 0 & 0 & 8^8 & 11\times 8^8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 8^8 & 13\times 8^8 \\ 0 & 0 & 0 & 0 & 1 & 0 & 8^8 & 7\times 8^8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -6\times 8^8 \end{pmatrix}$$

- apply `LatticeReduce` to this matrix

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LLL-algorithm: System of linear diophantine equations

- start from the system of equations:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 0 \\ \frac{15}{32}x_1 + \frac{27}{32}x_2 + \frac{33}{32}x_3 + \frac{39}{32}x_4 + \frac{21}{32}x_5 &= \frac{9}{16} \end{aligned}$$

- take matrix (divide to the greatest common divisor – `GCD` function):

$$\mathsf{SE} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 5 & 9 & 11 & 13 & 7 & -6 \end{pmatrix}$$

- multiply SE to some huge integer number, for example 8^8
- create unity matrix $\mathbb{I}$ with rank equal to the length of row in SE
- append transpose SE to the right side of the unity matrix $\mathbb{I}$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 8^8 & 5\times 8^8 \\ 0 & 1 & 0 & 0 & 0 & 0 & 8^8 & 9\times 8^8 \\ 0 & 0 & 1 & 0 & 0 & 0 & 8^8 & 11\times 8^8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 8^8 & 13\times 8^8 \\ 0 & 0 & 0 & 0 & 1 & 0 & 8^8 & 7\times 8^8 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -6\times 8^8 \end{pmatrix}$$

- apply `LatticeReduce` to this matrix

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- apply `LatticeReduce` to this matrix

As result we obtain the following matrix:

$$\mathrm{RSE} = \begin{pmatrix} -1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & -8^8 & -8^8 \\ -1 & 0 & 0 & 0 & 0 & -1 & -8^8 & 8^8 \end{pmatrix}$$

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Origin: Application of LLL-algorithm to solution of Diophantine equation

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -a_1 & -b_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -a_2 & -b_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & -a_3 & -b_3 \\ 0 & 0 & 0 & 1 & 0 & 0 & -a_4 & -b_4 \\ 0 & 0 & 0 & 0 & 1 & 0 & -a_5 & -b_5 \\ 0 & 0 & 0 & 0 & 0 & 1 & -a_6 & -b_6 \end{pmatrix}$$

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We need about ten times less values of $\gamma(M)$ for fixed M

Reconstruction: LLL-algorithm

$$\gamma_2(M) = S_1(M/2) \qquad \gamma_4(M) = -8\,S_3(M/2) - 16\,S_1(M/2)S_2(M/2)$$

$$\gamma_6 = 8\Big(2S_2S_3 + S_5 + 4S_{3,2} + 4S_{4,1} - 8S_{3,1,1} + S_1(4S_2^2 + 2S_4 + 8S_{3,1})\Big)$$

$\gamma_2(M) = S_1(M/2)$ $\gamma_4(M) = -8\,S_3(M/2) - 16\,S_1(M/2)S_2(M/2)$

Nested harmonic sums with only positive indices

Loops	1	2	3	4	5	6	7
Nested sums	1	4	16	64	256	1024	4096

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Starting from 6 loops the harmonic sums with negative indices are appeared

$$\hat{S}_{2,2,\underline{1}}(M) = S_{2,2,1}(M) + S_{2,-2,1}(M) + S_{-2,2,1}(M) + S_{-2,-2,1}(M)$$

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Basis extends up to ~ 2000 harmonic sums

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Modular arithmetic

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Modular arithmetic

$$\gamma(M) = \sum_i g^{2i}\,\gamma_{2i}(M) = \sum_i g^{2i} \sum_{\vec{i}=2i-1} C_{i_1,\ldots,i_k}\; S_{i_1,\ldots,i_k}$$

$$= \sum_i g^{2i} \sum_{\ell=0}^{2i-1} \boxed{S_1^\ell} \sum_{\substack{\vec{i}=2i-1-\ell \\ i_1\neq 1}} C^\ell_{i_1,\ldots,i_k}\; S_{i_1,\ldots,i_k}$$

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Modular arithmetic: anomalous dimension $\gamma(M)$ modulo $S_1^\ell(M)$

$$\gamma(M) = \sum_i g^{2i}\,\gamma_{2i}(M) = \sum_i g^{2i} \sum_{\vec{i}=2i-1} C_{i_1,\dots,i_k}\ S_{i_1,\dots,i_k}$$

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Anomalous dimension for twist-3 operators is known up to seven loops

[VV '23]

$$\gamma(M) = \sum_{\ell} \gamma_\ell(M)\, g^{2\ell}, \qquad S_{i_1,\ldots,i_k}(M) = \sum_{k=1}^{M} \frac{\left(\text{sign}(i_1)\right)^k}{k^{|i_1|}} S_{i_1,\ldots,i_k}(k)$$

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Analytical continuation to $M = -1 + \omega$

$$\gamma(-1+\omega) = \sum_{\ell=1} \sum_{k=0} C_{\ell,k}\, \omega^k g^{2\ell}$$

Twist-3: analytical properties

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No BFKL-like poles at all

Analytical continuation to $M=-2+\omega$

$$\gamma(-2+\omega)=-8\frac{g^2}{\omega}+g^4\left(-\frac{8}{\omega^3}+\frac{16\zeta_2}{\omega}\right)+g^6\left(-\frac{8}{\omega^5}+\frac{48\zeta_2}{\omega^3}+\frac{48\zeta_3}{\omega^2}\right)$$
$$+g^8\left(-\frac{8}{\omega^7}+\frac{80\zeta_2}{\omega^5}+\frac{80\zeta_3}{\omega^4}\right)+g^{10}\left(-\frac{8}{\omega^9}+\frac{112\zeta_2}{\omega^7}-\frac{912\zeta_3}{\omega^6}\right)+\cdots$$

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Analytical continuation to $M = -1 + \omega$

$$\gamma(-1+\omega) = \sum_{\ell=1}\sum_{k=0} C_{\ell,k}\, \omega^k g^{2\ell}$$

No BFKL-like poles at all

Analytical continuation to $M = -2 + \omega$

$$\gamma(-2+\omega) = -8\frac{g^2}{\omega} + g^4\left(-\frac{8}{\omega^3} + \frac{16\zeta_2}{\omega}\right) + g^6\left(-\frac{8}{\omega^5} + \frac{48\zeta_2}{\omega^3} + \frac{48\zeta_3}{\omega^2}\right)$$
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Resummation

$$\gamma(-2+\omega) = -8\frac{g^2}{\omega}\left(\frac{1}{1-t} - \zeta_2\frac{1+3t^2}{(1-t)^2}\omega^2\right)$$

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https://rscf.ru/en/project/23-22-00311