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O.V.Haneychuk, V.Yu.Shirokova, K.V.Stepanyantz

Moscow State University, Physical Faculty,
Department of Theoretical Physics

Three-loop β-functions and a class of the NSVZ schemes for MSSM obtained with the help of the higher covariant derivative regularization

based on the paper O.H.,V.Shirokova, K.Stepanyantz

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The Minimal Supersymmetric Standard Model (MSSM) is a softly broken N = 1 supersymmetric theory with the gauge group

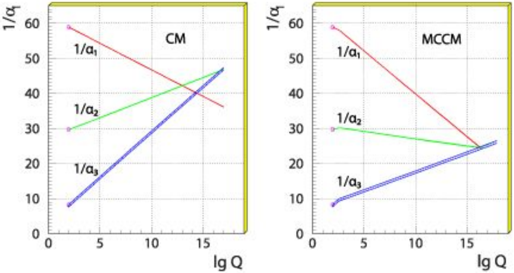
$$G = SU(3) \times SU(2) \times U(1)_Y.$$

As a result, the theory contains three gauge coupling constants

$$\alpha_3 = \frac{{e_3}^2}{4\pi}; \quad \alpha_2 = \frac{{e_2}^2}{4\pi}; \quad \alpha_1 = \frac{5}{3}\cdot\frac{{e_1}^2}{4\pi},$$

where the factor $\frac{5}{3}$ is included into the definition of α_1 in order to have the gauge coupling unification condition in the form $\alpha_1 = \alpha_2 = \alpha_3$.

The chiral matter superfields include three generations of quarks and leptons, two Higgs fields H_u and H_d,and their superpartners as components.



Supersymmetry based theories have an important property which can be used to simplify calculation of the β-functions encoding the evolution of the running gauge coupling constants. In such theories, β-functions can be related to the anomalous dimensions of the chiral matter superfields with the help of the exact Novikov, Shifman, Vainshtein, and Zakharov (NSVZ) β-function.

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1 - C_2\alpha/2\pi)}.$$

Here α and λ are the gauge and Yukawa coupling constants, respectively, and we use the notation

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA} = \dim G.$$

NSVZ relations and choice of regularization

The use of the NSVZ equations greatly simplifies calculations in higher orders, since NSVZ relations relate β-functions to anomalous dimensions in all previous orders. For example, knowing the two-loop anomalous dimensions, the NSVZ equations make it possible to obtain three-loop β-functions.

However, not every renormalization prescription belongs to the so-called NSVZ schemes, that is, schemes in which the NSVZ relation is satisfied. In particular, the most popular $\overline{\text{DR}}$-scheme, which implies that a theory is regularized by dimensional reduction and divergences are removed by modified minimal subtraction, is not NSVZ.

I. Jack, D. R. T. Jones and C. G. North, Phys. Lett. B **386** (1996) 138; I. Jack, D. R. T. Jones and C. G. North, Nucl. Phys. B **486** (1997) 479; I. Jack, D. R. T. Jones and A. Pickering, Phys. Lett. B **435** (1998) 61;

It was shown, that, starting from the three-loop approximation, the NSVZ relation is no longer satisfied for this renormalization prescription and its restoration requires a special finite renormalization in each order.

L.V.Avdeev, O.V.Tarasov, Phys.Lett. B **112** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett B **386** (1996) 138; Nucl.Phys. B **486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

NSVZ relations and choice of regularization

Here we will obtain the three-loop β-functions for the MSSM using the higher covariant derivative regularization.

A. A. Slavnov, Nucl. Phys. B **31** (1971) 301; A. A. Slavnov, Theor.Math.Phys. **13** (1972) 1064 [Teor. Mat. Fiz. 13 (1972) 174].

Renormalization group functions (RGFs) defined in terms of the bare couplings satisfy the NSVZ relations in all orders under this regularization.

K. V. Stepanyantz, Nucl. Phys. B **909** (2016) 316; K. V. Stepanyantz, JHEP **1910** (2019) 011; K. Stepanyantz, Eur. Phys. J. C **80** (2020) no.10, 911.

RGFs defined in terms of the renormalized couplings can also obey the NSVZ equations when the HD+MSL renormalization prescription is used, which implies that theory is regularized by Higher Derivatives and divergences are removed by Minimal Subtractions of Logarithms.

A. L. Kataev and K. V. Stepanyantz, Nucl. Phys. B **875** (2013) 459.

In the $\overline{\text{DR}}$-scheme the corresponding result for the β-functions has already been obtained earlier.

I. Jack, D. R. T. Jones and A. F. Kord, Annals Phys. **316** (2005), 213.

RGFs in terms of the bare and renormalized couplings

The matter superfields ϕ_a are renormalized as

$$\phi_a = (\sqrt{Z})_a{}^b \phi_{b,R},$$

where $\phi_{b,R}$ are the renormalized superfields. Subscript a here numerates sets of chiral matter superfilds ϕ_a either belonging to a certain irreducible representation of the simple subgroup G_K or having certain charge q_{aK} with respect to $G_K = U(1)$. The renormalization of each coupling constant α_K and of the superfields ϕ_a is encoded in the corresponding β-function and anomalous dimension, respectively. For RGFs defined in terms of the bare couplings α_0 and λ_0 the derivatives are taken with respect to the dimensionful regularization parameter Λ at fixed values of the renormalized couplings.

$$\beta_K(\alpha_0, \lambda_0) \equiv \frac{d\alpha_{0K}}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}}; \qquad \gamma_a{}^b(\alpha_0, \lambda_0) \equiv -\frac{d\ln Z_a{}^b}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}},$$

For RGFs standardly defined in terms of the renormalized couplings α and λ the differentiations are made with respect to the renormalization point μ at fixed values of bare couplings.

$$\widetilde{\beta}_K(\alpha, \lambda) \equiv \frac{d\alpha_K}{d\ln\mu}\Big|_{\alpha_0,\lambda_0=\text{const}}; \qquad \widetilde{\gamma_a}{}^b(\alpha, \lambda) \equiv \frac{d\ln Z_a{}^b}{d\ln\mu}\Big|_{\alpha_0,\lambda_0=\text{const}}.$$

It should be noted that for $a \neq b$ $\gamma_a{}^b(\alpha_0, \lambda_0) = 0$ and $\gamma_a{}^b(\alpha, \lambda) = 0$, except when a and b correspond to different generations of the same superfield.

Generalization of the NSVZ equations

RGFs defined in terms of the bare couplings depend on a regularization, but do not depend on a renormalization prescription.

A. L. Kataev and K. V. Stepanyantz, Nucl. Phys. B **875** (2013) 459.

In the case of using the higher covariant derivative regularization they satisfy the NSVZ equations which are generalized for the theory with multiple gauge couplings as follows:

$$\frac{\beta_K(\alpha_0,\lambda_0)}{\alpha_{0K}{}^2} = -\frac{1}{2\pi(1-C_2(G_K)\alpha_{0K}/2\pi)}\Big[3C_2(G_K) - \sum_a T_{aK}(1-\gamma_a{}^a(\alpha_0,\lambda_0))\Big],$$

where

$$C_2(G_K)\delta^{A_K B_K} = f^{A_K C_K D_K} f^{B_K C_K D_K}; \quad T_K(R_{aK})\delta^{A_K B_K} = (T_a^{A_K} T_a^{B_K})_{i_K}{}^{i_K};$$

$$T_{aK} = \begin{cases} \delta_{i_1}{}^{i_1}\cdot\ldots\cdot\delta_{i_{K-1}}{}^{i_{K-1}} T_K(R_{ak})\delta_{i_{K+1}}{}^{i_{K+1}}\cdot\ldots\cdot\delta_{i_n}{}^{i_n}, & G_K \text{ is simple;}\\ \delta_{i_1}{}^{i_1}\cdot\ldots\cdot\delta_{i_{K-1}}{}^{i_{K-1}} q_{aK}^2\delta_{i_{K+1}}{}^{i_{K+1}}\cdot\ldots\cdot\delta_{i_n}{}^{i_n}, & G_K = U(1). \end{cases}$$

Values of T_{aK} for all MSSM superfields are given in the table.

superfield	Q_1,Q_2,Q_3	U_1,U_2,U_3	D_1,D_2,D_3	L_1,L_2,L_3	E_1,E_2,E_3	H_u	H_d
$SU(3)$	1	1/2	1/2	0	0	0	0
$SU(2)$	3/2	0	0	1/2	0	1/2	1/2
$U(1)_Y$	1/6	4/3	1/3	1/2	1	1/2	1/2

The exact NSVZ expressions for the MSSM β-functions can be written in the form

$$\frac{\beta_3(\alpha_0,\lambda_0)}{{\alpha_{03}}^2} = -\frac{1}{2\pi(1-3\alpha_{03}/2\pi)}\Big[3 + \text{tr}\Big(\gamma_Q(\alpha_0,Y_0) + \frac{1}{2}\gamma_U(\alpha_0,Y_0) + \frac{1}{2}\gamma_D(\alpha_0,Y_0)\Big)\Big];$$

$$\frac{\beta_2(\alpha_0,\lambda_0)}{{\alpha_{02}}^2} = -\frac{1}{2\pi(1-\alpha_{02}/\pi)}\Big[-1 + \text{tr}\Big(\frac{3}{2}\gamma_Q(\alpha_0,Y_0) + \frac{1}{2}\gamma_L(\alpha_0,Y_0)\Big) + \frac{1}{2}\gamma_{H_u}(\alpha_0,Y_0) + \frac{1}{2}\gamma_{H_d}(\alpha_0,Y_0)\Big];$$

$$\frac{\beta_1(\alpha_0,\lambda_0)}{{\alpha_{01}}^2} = -\frac{3}{5}\frac{1}{2\pi}\Big[-11 + \text{tr}\Big(\frac{1}{6}\gamma_Q(\alpha_0,Y_0) + \frac{4}{3}\gamma_U(\alpha_0,Y_0) + \frac{1}{3}\gamma_D(\alpha_0,Y_0) + \frac{1}{2}\gamma_L(\alpha_0,Y_0) + \gamma_E(\alpha_0,Y_0)\Big) + \frac{1}{2}\gamma_{H_u}(\alpha_0,Y_0) + \frac{1}{2}\gamma_{H_d}(\alpha_0,Y_0)\Big],$$

where traces imply summation over generation indices.

M. A. Shifman, Int. J. Mod. Phys. A **11** (1996), 5761; D. Korneev, D. Plotnikov, K. Stepanyantz and N. Tereshina, The NSVZ relations for $\mathcal{N}=1$ supersymmetric theories with multiple gauge couplings, JHEP **10** (2021) 046.

The two-loop anomalous dimension for MSSM

The expression for the two-loop anomalous dimension of the matter superfields defined in terms of the bare couplings for a theory with a single gauge coupling constant regularized by higher covariant derivatives can be written in the form

$$\gamma_i{}^j(\alpha_0,\lambda_0) = -\frac{d\ln Z_i{}^j}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\frac{\alpha_0}{\pi}C(R)_i{}^j + \frac{1}{4\pi^2}\lambda^*_{0imn}\lambda_0^{jmn}$$
$$+\frac{\alpha_0^2}{2\pi^2}\Big[[C(R)^2]_i{}^j - 3C_2C(R)_i{}^j(\ln a_\varphi + 1 + \frac{A}{2}) + T(R)C(R)_i{}^j(\ln a + 1 + \frac{A}{2})\Big]$$
$$-\frac{\alpha_0}{8\pi^3}\lambda^*_{0lmn}\lambda_0^{jmn}C(R)_i{}^l(1-B+A) + \frac{\alpha_0}{4\pi^3}\lambda^*_{0imn}\lambda_0^{jml}C(R)_l{}^n(1+B-A)$$
$$-\frac{1}{16\pi^4}\lambda^*_{0iac}\lambda_0^{jab}\lambda^*_{0bde}\lambda_0^{cde} + O(\alpha_0^3,\alpha_0^2\lambda_0^2,\alpha_0\lambda_0^4,\lambda_0^6),$$

where

$$C(R)_i{}^j \equiv (T^AT^A)_i{}^j; \quad tr(T^AT^B) = \delta^{AB}T(R); \quad T(Adj) \equiv C_2,$$
$$A \equiv \int\limits_0^\infty dx\ln x\frac{d}{dx}\Big(\frac{1}{R(x)}\Big); \quad B \equiv \int\limits_0^\infty dx\ln x\frac{d}{dx}\Big(\frac{1}{F^2(x)}\Big).$$

The regulator functions $R(x)$ and $F(x)$ (the same for all subgroups G_K) rapidly grow at infinity and are equal to 1 at $x=0$.

A. Kazantsev and K. Stepanyantz, JHEP **06** (2020) 108.

The two-loop anomalous dimension for MSSM

The generalization of this expression for the case of a theory with several coupling constants can be presented in the form

$$\gamma_a{}^b(\alpha_0,\lambda_0) = -\frac{d\ln Z_a{}^b}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\sum_K \frac{\alpha_{0K}}{\pi} C(R_{aK})\delta_a{}^b + \frac{1}{4\pi^2}(\lambda_0^*\lambda_0)_a{}^b$$

$$+\sum_{KL}\frac{\alpha_{0K}\alpha_{0L}}{2\pi^2}C(R_{aK})C(R_{aL})\delta_a{}^b - \sum_K \frac{3\alpha_{0K}^2}{2\pi^2}C_2(G_K)C(R_{aK})(\ln a_{\varphi,K}+1+\frac{A}{2})\delta_a{}^b$$

$$+\sum_K \frac{\alpha_{0K}^2}{2\pi^2}C(R_{aK})\sum_c \mathbf{T}_{cK}(\ln a_K + 1 + \frac{A}{2})\delta_a{}^b - \sum_K \frac{\alpha_{0K}}{8\pi^3}(\lambda_0^*\lambda_0)_a{}^b C(R_{aK})$$

$$\times(1-B+A) + \sum_K \frac{\alpha_{0K}}{4\pi^3}(\lambda_0^* C_K\lambda_0)_a{}^b(1+B-A) - \frac{1}{16\pi^4}(\lambda_0^*[\lambda_0^*\lambda_0]\lambda_0)_a{}^b$$

$$+O(\alpha_0^3,\alpha_0^2\lambda_0^2,\alpha_0\lambda_0^4,\lambda_0^6),$$

where we use the notations

$$(T_a^{A_K}T_a^{A_K})_{i_K}{}^{j_K} = C(R_{aK})\delta_{i_K}{}^{j_K}; \quad (\lambda_0^*\lambda_0)_a{}^b\delta_{i_a}{}^{j_b} = \sum_{cd}\lambda^*_{0 i_a m_c n_d}\lambda_0^{j_b m_c n_d};$$

$$(\lambda_0^* C_K\lambda_0)_a{}^b\delta_{i_a}{}^{j_b} = \sum_{cd}\lambda^*_{0 i_a m_c n_d} C(R_{dK})\lambda_0^{j_b m_c n_d};$$

$$(\lambda_0^*[\lambda_0^*\lambda_0]\lambda_0)_a{}^b\delta_{i_a}{}^{j_b} = \sum_{cdefg}\lambda^*_{0 i_a k_e l_f}\lambda_0^{j_b k_e p_g}\lambda^*_{0 p_g m_c n_d}\lambda_0^{l_f m_c n_d}.$$

The two-loop anomalous dimension for MSSM

The two-loop anomalous dimensions in the case of using the higher covariant derivative regularization are too large. As an example we present here the one for the left quarks. In terms of the bare couplings (without the one-loop contribution) it is given by the expression

$$
\begin{aligned}
\gamma_Q(\alpha_0, Y_0)^T =& \frac{1}{2\pi^2}\Big[\frac{1}{3600}\alpha_{01}^2 + \frac{9}{16}\alpha_{02}^2 + \frac{16}{9}\alpha_{03}^2 + \frac{1}{40}\alpha_{01}\alpha_{02} + \frac{2}{45}\alpha_{01}\alpha_{03} \\
&+ 2\alpha_{02}\alpha_{03} - \frac{9}{2}\alpha_{02}^2(\ln a_{\varphi,SU(2)} + 1 + \frac{A}{2}) - 12\alpha_{03}^2(\ln a_{\varphi,SU(3)} + 1 + \frac{A}{2}) \\
&+ \frac{11}{100}\alpha_{01}^2(\ln a_{U(1)} + 1 + \frac{A}{2}) + \frac{21}{4}\alpha_{02}^2(\ln a_{SU(2)} + 1 + \frac{A}{2}) \\
&+ 8\alpha_{03}^2(\ln a_{SU(3)} + 1 + \frac{A}{2})\Big] + \frac{1}{8\pi^2}Y_{0U}Y_{0U}^\dagger\Big[\frac{\alpha_{01}}{\pi}\Big(\frac{1}{5} + \frac{13}{60}(B-A)\Big) \\
&+ \frac{3}{4}\frac{\alpha_{02}}{\pi}(B-A) + \frac{4}{3}\frac{\alpha_{03}}{\pi}(B-A)\Big] \\
&+ \frac{1}{8\pi^2}Y_{0D}Y_{0D}^\dagger\Big[\frac{\alpha_{01}}{\pi}\Big(\frac{1}{10} + \frac{7}{60}(B-A)\Big) + \frac{3}{4}\frac{\alpha_{02}}{\pi}(B-A) + \frac{4}{3}\frac{\alpha_{03}}{\pi}(B-A)\Big] \\
&- \frac{1}{(8\pi^2)^2}\Big[(Y_{0U}Y_{0U}^\dagger)^2 + (Y_{0D}Y_{0D}^\dagger)^2 + \frac{3}{2}tr(Y_{0U}Y_{0U}^\dagger)Y_{0U}Y_{0U}^\dagger \\
&+ \frac{3}{2}tr(Y_{0D}Y_{0D}^\dagger)Y_{0D}Y_{0D}^\dagger + \frac{1}{2}tr(Y_{0E}Y_{0E}^\dagger)Y_{0D}Y_{0D}^\dagger\Big] + O(\alpha_0^3,\alpha_0^2Y_0^2,\alpha_0Y_0^4,Y_0^6).
\end{aligned}
$$

Other two-loop anomalous dimensions (defined both in terms of the bare and renormalized couplings) can be found in

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The two-loop anomalous dimension for MSSM

For the anomalous dimension standardly defined in terms of the renormalized couplings, the corresponding result can be written as follows:

$$\tilde{\gamma}_Q(\alpha, Y)^T = -\frac{\alpha_1}{60\pi} - \frac{3\alpha_2}{4\pi} - \frac{4\alpha_3}{3\pi} + \frac{1}{8\pi^2}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) + \frac{1}{2\pi^2}\Big[\frac{1}{3600}\alpha_1^2 + \frac{9}{16}\alpha_2^2$$
$$+\frac{16}{9}\alpha_3^2 + \frac{1}{40}\alpha_1\alpha_2 + \frac{2}{45}\alpha_1\alpha_3 + 2\alpha_2\alpha_3 + \frac{11}{100}\alpha_1^2\left(\ln a_{U(1)} + 1 + \frac{A}{2} + g_{Q1} - b_{1,1}\right)$$
$$+\frac{3}{4}\alpha_2^2\left(-6\ln a_{\varphi,SU(2)} + 7\ln a_{SU(2)} + 1 + \frac{A}{2} + g_{Q2} - b_{1,2}\right)$$
$$-4\alpha_3^2\left(3\ln a_{\varphi,SU(3)} - 2\ln a_{SU(3)} + 1 + \frac{A}{2} + g_{Q3} - b_{1,3}\right)\Big]$$
$$+\frac{1}{8\pi^2} Y_U Y_U^\dagger\Big[\frac{\alpha_1}{\pi}\left(\frac{1}{5} + \frac{13}{60}(B - A + 2g_{QU} - 2j_{U1})\right) +$$
$$+\frac{3}{4}\frac{\alpha_2}{\pi}(B - A + 2g_{QU} - 2j_{U2}) + \frac{4}{3}\frac{\alpha_{03}}{\pi}(B - A + 2g_{QU} - 2j_{U3})\Big]$$
$$+\frac{1}{8\pi^2} Y_D Y_D^\dagger\Big[\frac{\alpha_1}{\pi}\left(\frac{1}{10} + \frac{7}{60}(B - A + 2g_{QD} - 2j_{D1})\right) + \frac{3}{4}\frac{\alpha_2}{\pi}(B - A + 2g_{QD} - 2j_{D2})$$
$$+\frac{4}{3}\frac{\alpha_3}{\pi}(B - A + 2g_{QD} - 2j_{D3})\Big] - \frac{1}{(8\pi^2)^2}\Big[(Y_U Y_U^\dagger)^2(1 + 3g_{QU} - 3j_{UU}) + (Y_D Y_D^\dagger)^2$$
$$\times(1 + 3g_{QD} - 3j_{DD}) + \frac{3}{2}tr(Y_U Y_U^\dagger) Y_U Y_U^\dagger(1 + 2g_{QU} - 2j_{UtU}) + \frac{3}{2}tr(Y_D Y_D^\dagger) Y_D Y_D^\dagger$$
$$\times(1 + 2g_{QD} - 2j_{DtD}) + \frac{1}{2}tr(Y_E Y_E^\dagger) Y_D Y_D^\dagger(1 + 2g_{QD} - 2j_{DtE})$$
$$+\frac{1}{2}\left(Y_U Y_U^\dagger Y_D Y_D^\dagger + Y_D Y_D^\dagger Y_U Y_U^\dagger\right)(g_{QU} + g_{QD} - j_{UD} - j_{DU})\Big] + O(\alpha^3, \alpha^2 Y^2, \alpha Y^4, Y^6).$$

The result for the three-loop MSSM β-functions defined in terms of the bare couplings was obtained by using the NSVZ equations for MSSM and is presented here only for α_{03}. Other three-loop β-functions (defined both in terms of the bare and renormalized couplings) can be found in

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$$\frac{\beta_3(\alpha_0,Y_0)}{\alpha_{03}^2} = -\frac{1}{2\pi}\Big[3 - \frac{11}{20}\frac{\alpha_{01}}{\pi} - \frac{9}{4}\frac{\alpha_{02}}{\pi} - \frac{7}{2}\frac{\alpha_{03}}{\pi} + \frac{1}{4\pi^2}tr(Y_{0U}Y_{0U}^\dagger + Y_{0D}Y_{0D}^\dagger) +$$
$$+ \frac{1}{2\pi^2}\Big[\frac{137}{1200}\alpha_{01}^2 + \frac{27}{16}\alpha_{02}^2 + \frac{1}{6}\alpha_{03}^2 + \frac{3}{40}\alpha_{01}\alpha_{02} - \frac{11}{60}\alpha_{01}\alpha_{03} - \frac{3}{4}\alpha_{02}\alpha_{03} -$$
$$- \frac{27}{2}\alpha_{02}^2(\ln a_{\varphi,SU(2)} + 1 + \frac{A}{2}) - 72\alpha_{03}^2(\ln a_{\varphi,SU(3)} + 1 + \frac{A}{2}) +$$
$$+ \frac{363}{100}\alpha_{01}^2(\ln a_{U(1)} + 1 + \frac{A}{2}) + \frac{63}{4}\alpha_{02}^2(\ln a_{SU(2)} + 1 + \frac{A}{2}) +$$
$$+ 48\alpha_{03}^2(\ln a_{SU(3)} + 1 + \frac{A}{2})\Big] + \frac{1}{8\pi^3}\Big(tr(Y_{0U}Y_{0U}^\dagger)\Big[\alpha_{01}\Big(\frac{3}{20} + \frac{13}{30}(B-A)\Big) +$$
$$+ \alpha_{02}\Big(\frac{3}{4} + \frac{3}{2}(B-A)\Big) + \alpha_{03}\Big(3 + \frac{8}{3}(B-A)\Big)\Big] +$$
$$+ tr(Y_{0D}Y_{0D}^\dagger)\Big[\alpha_{01}\Big(\frac{3}{20} + \frac{7}{30}(B-A)\Big) + \alpha_{02}\Big(\frac{3}{4} + \frac{3}{2}(B-A)\Big) +$$
$$+ \alpha_{03}\Big(3 + \frac{8}{3}(B-A)\Big)\Big]\Big) - \frac{1}{(8\pi^2)^2}\Big(\frac{3}{2}tr\Big((Y_{0U}Y_{0U}^\dagger)^2\Big) + \frac{3}{2}tr\Big((Y_{0D}Y_{0D}^\dagger)^2\Big) +$$
$$+ 3tr^2(Y_{0U}Y_{0U}^\dagger) + 3tr^2(Y_{0D}Y_{0D}^\dagger) + tr(Y_{0E}Y_{0E}^\dagger)tr(Y_{0D}Y_{0D}^\dagger) + tr(Y_{0D}Y_{0D}^\dagger Y_{0U}Y_{0U}^\dagger)\Big)\Big]$$
$$+O(\alpha_0^3,\alpha_0^2Y_0^2,\alpha_0Y_0^4,Y_0^6).$$

Next, we substitute this expression into the corresponding renormalization group equation and find the relation between the bare couplings and the renormalized ones, which allows us to obtain the three-loop MSSM β-functions defined in terms of the renormalized couplings.

$$
\begin{aligned}
\frac{\tilde\beta_3(\alpha,Y)}{\alpha_3^2} &= -\frac{1}{2\pi}\Big[3-\frac{11}{20}\frac{\alpha_1}{\pi}-\frac{9}{4}\frac{\alpha_2}{\pi}-\frac{7}{2}\frac{\alpha_3}{\pi}+\frac{1}{4\pi^2}tr(Y_UY_U^\dagger+Y_DY_D^\dagger)\\
&+\frac{1}{2\pi^2}\Big[\frac{137}{1200}\alpha_1^2+\frac{27}{16}\alpha_2^2+\frac{1}{6}\alpha_3^2+\frac{3}{40}\alpha_1\alpha_2-\frac{11}{60}\alpha_1\alpha_3-\frac{3}{4}\alpha_2\alpha_3+\frac{363}{100}\alpha_1^2\\
&\times\Big(\ln a_{U(1)}+1+\frac{A}{2}+b_{2,31}-b_{1,1}\Big)+\frac{9}{4}\alpha_2^2\Big(7\ln a_{SU(2)}-6\ln a_{\varphi,SU(2)}+1+\frac{A}{2}+b_{2,32}-b_{1,2}\Big)\\
&+24\alpha_3^2\Big(2\ln a_{SU(3)}-3\ln a_{\varphi,SU(3)}+1+\frac{A}{2}+\frac{7}{16}b_{2,33}-\frac{7}{16}b_{1,3}\Big)\Big]+\frac{1}{8\pi^3}\Big(tr(Y_UY_U^\dagger)\\
&\times\Big[\alpha_1\Big(\frac{3}{20}+\frac{13}{30}(B-A+2b_{2,3U}-2j_{U1})\Big)+\alpha_2\Big(\frac{3}{4}+\frac{3}{2}(B-A+2b_{2,3U}-2j_{U2})\Big)+\alpha_3\\
&\times\Big(3+\frac{8}{3}(B-A+2b_{2,3U}-2j_{U3})\Big)\Big]+tr(Y_DY_D^\dagger)\Big[\alpha_1\Big(\frac{3}{20}+\frac{7}{30}(B-A+2b_{2,3D}-2j_{D1})\Big)\\
&+\alpha_2\Big(\frac{3}{4}+\frac{3}{2}(B-A+2b_{2,3D}-2j_{D2})\Big)+\alpha_3\Big(3+\frac{8}{3}(B-A+2b_{2,3D}-2j_{D3})\Big)\Big]\Big)\\
&-\frac{1}{(8\pi^2)^2}\Big(\frac{3}{2}tr\Big((Y_UY_U^\dagger)^2\Big)(1+4b_{2,3U}-4j_{UU})+\frac{3}{2}tr\Big((Y_DY_D^\dagger)^2\Big)(1+4b_{2,3D}-4j_{DD})\\
&+3tr^2(Y_UY_U^\dagger)(1+2b_{2,3U}-2j_{UtU})+3tr^2(Y_DY_D^\dagger)(1+2b_{2,3D}-2j_{DtD})\\
&+tr(Y_EY_E^\dagger)tr(Y_DY_D^\dagger)(1+2b_{2,3D}-2j_{DtE})+tr(Y_DY_D^\dagger Y_UY_U^\dagger)\\
&\times(1+2b_{2,3U}+2b_{2,3D}-2j_{UD}-2j_{UD}))\Big]+O(\alpha^3,\alpha^2Y^2,\alpha Y^4,Y^6).
\end{aligned}
$$

Under certain renormalization prescription both three-loop β-functions and two-loop anomalous dimensions exactly reproduce the $\overline{\text{DR}}$ expressions obtained earlier, for example,

$$\tilde{\gamma}_Q(\alpha,Y)^T = -\frac{\alpha_1}{60\pi} - \frac{3\alpha_2}{4\pi} - \frac{4\alpha_3}{3\pi} + \frac{1}{8\pi^2}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) + \frac{1}{2\pi^2}\left[\frac{199}{3600}\alpha_1^2 + \frac{15}{16}\alpha_2^2 - \frac{2}{9}\alpha_3^2 + \frac{1}{40}\alpha_1\alpha_2 + \frac{2}{45}\alpha_1\alpha_3 + 2\alpha_2\alpha_3\right] + \frac{1}{8\pi^3}Y_U Y_U^\dagger \cdot \frac{\alpha_1}{5} + \frac{1}{8\pi^3}Y_D Y_D^\dagger \cdot \frac{\alpha_1}{10} - \frac{1}{(8\pi^2)^2}\left[(Y_U Y_U^\dagger)^2 + (Y_D Y_D^\dagger)^2 + \frac{3}{2}tr(Y_U Y_U^\dagger)Y_U Y_U^\dagger + \frac{3}{2}tr(Y_D Y_D^\dagger)Y_D Y_D^\dagger + \frac{1}{2}tr(Y_E Y_E^\dagger)Y_D Y_D^\dagger\right] + O(\alpha^3,\alpha^2 Y^2,\alpha Y^4,Y^6)$$

$$\frac{\tilde{\beta}_3(\alpha,Y)}{\alpha_3^2} = -\frac{1}{2\pi}\left[3 - \frac{11}{20}\frac{\alpha_1}{\pi} - \frac{9}{4}\frac{\alpha_2}{\pi} - \frac{7}{2}\frac{\alpha_3}{\pi} + \frac{1}{4\pi^2}tr(Y_U Y_U^\dagger + Y_D Y_D^\dagger)\right. + \frac{1}{2\pi^2}\left[\frac{851}{300}\alpha_1^2 + \frac{27}{8}\alpha_2^2 - \frac{347}{24}\alpha_3^2 + \frac{3}{40}\alpha_1\alpha_2 - \frac{11}{60}\alpha_1\alpha_3 - \frac{3}{4}\alpha_2\alpha_3\right] + \frac{1}{8\pi^3}\left(tr(Y_U Y_U^\dagger)\right. \times \left[\frac{11}{30}\alpha_1 + \frac{3}{2}\alpha_2 + \frac{13}{3}\alpha_3\right] + tr(Y_D Y_D^\dagger)\left.\left[\frac{4}{15}\alpha_1 + \frac{3}{2}\alpha_2 + \frac{13}{3}\alpha_3\right]\right) - \frac{1}{(8\pi^2)^2}\left(3tr\left((Y_U Y_U^\dagger)^2\right)\right. \left.\left. + 3tr\left((Y_D Y_D^\dagger)^2\right) + \frac{9}{2}tr^2(Y_U Y_U^\dagger) + \frac{9}{2}tr^2(Y_D Y_D^\dagger) + \frac{3}{2}tr(Y_E Y_E^\dagger)tr(Y_D Y_D^\dagger) + 2tr(Y_D Y_D^\dagger Y_U Y_U^\dagger)\right)\right] + O(\alpha^3,\alpha^2 Y^2,\alpha Y^4,Y^6).$$

A class of the NSVZ schemes for MSSM

The NSVZ equations is scheme-dependent in the considered approximation, but it is possible to find restrictions to the finite constants contained in the RGFs defined in terms of the renormalized couplings under which the NSVZ equations remain valid:

$$b_{2,11} = \frac{1}{398}(g_{Q1} + 128g_{U1} + 8g_{D1} + 27g_{L1} + 216g_{E1} + 9g_{H_u1} + 9g_{H_d1});$$

$$b_{2,12} = \frac{1}{6}(g_{Q2} + 3g_{L2} + g_{H_u2} + g_{H_d2}); \quad b_{2,13} = \frac{1}{11}(g_{Q3} + 8g_{U3} + 2g_{D3});$$

$$b_{2,21} = \frac{1}{6}(g_{Q1} + 3g_{L1} + g_{H_u1} + g_{H_d1}); \quad b_{2,22} = \frac{1}{50}(27g_{Q2} + 9g_{L2} + 3g_{H_u2} + 3g_{H_d2} + 8b_{1,2});$$

$$b_{2,23} = g_{Q3}; \quad b_{2,31} = \frac{1}{11}(g_{Q1} + 8g_{U1} + 2g_{D1}); \quad b_{2,32} = g_{Q2};$$

$$b_{2,33} = \frac{1}{7}(8g_{Q3} + 4g_{U3} + 4g_{D3} - 9b_{1,3}); \quad b_{2,1U} = \frac{1}{26}(g_{QU} + 16g_{UU} + 9g_{H_uU});$$

$$b_{2,1D} = \frac{1}{14}(g_{QD} + 4g_{DD} + 9g_{H_dD}); \quad b_{2,1E} = \frac{1}{6}(g_{LE} + 4g_{EE} + g_{H_dE});$$

$$b_{2,2U} = \frac{1}{2}(g_{QU} + g_{H_uU}); \quad b_{2,2D} = \frac{1}{2}(g_{QD} + g_{H_dD}); \quad b_{2,2E} = \frac{1}{2}(g_{LE} + g_{H_dE});$$

$$b_{2,3U} = \frac{1}{2}(g_{QU} + g_{UU}); \quad b_{2,3D} = \frac{1}{2}(g_{QD} + g_{DD}).$$

Conclusion

Using the Slavnov higher derivative regularization, we found the two-loop anomalous dimensions for all MSSM chiral matter superfields and the three-loop MSSM gauge β-functions defined both in terms of the bare and renormalized couplings. We used an arbitrary supersymmetric renormalization prescription, that is a prescription to renormalize all superfields as a whole.

Firstly we obtained the two-loop anomalous dimensions defined in terms of the bare couplings and calculated the three-loop MSSM β-functions with the help of the NSVZ equations which are valid in all orders with this regularization. Next we obtained RGFs standardly defined in terms of the renormalized couplings and constracted a class of the renormalization prescriptions under which renormalized RGFs continue to satisfy the NSVZ relations.

As a test of the calculation correctness, we checked that for a certain choice of a subtraction scheme the results (for both the two-loop anomalous dimensions and the three-loop β-functions) coincide with the ones obtained earlier in the $\overline{\text{DR}}$-scheme.

Thank you for the attention!