

# Mixing scenarios, dark matter and lepton universality with three generations of heavy neutral leptons

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# Outline

- 1 Seesaw type I model for three generations of Majorana neutrino
- 2 General cosmological restrictions
- 3 Minimal mixing scenario and beyond
- 4 Restrictions for  $N_2 - N_3$  HNL of the second and third generations
- 5 Perturbative calculations with Majorana fermions
- 6 Lepton universality violation
- 7 Summary



# Extended lepton sector of the SM

**Heavy neutral leptons (HNL)**, or Majorana fermions with sterile flavor states,  $SU(2)_L \times U(1)_Y$  singlets

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \partial_\mu \gamma^\mu \nu_R - \left( F \bar{l}_L \nu_R \tilde{H} + \frac{M_M}{2} \overline{\nu^c}_R \nu_R + h.c. \right),$$

где  $l_L = (\nu_L, e_L)^T$  – left SM doublet,  $\nu_R$  – Majorana flavor states,  $H$  – Higgs doublet ( $\tilde{H} = i\tau_2 H^*$ ),  $F$  – Yukawa matrix,  $M_M$  Majorana mass matrix. After spontaneous symmetry breaking  $M_D = F\langle H \rangle = Fv$  ( $v = 174$  GeV)

$$\frac{1}{2} (\bar{\nu}_L \overline{\nu^c}_R) \begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.,$$



For physically interesting scenarios in the seesaw type I models it is needed to combine very small active (standard) neutrino masses  $\sim F^2 v^2 / M_M$  with moderately heavy  $M_{HNL}$  within the LHC and next colliders energy reach, and not too small mixing  $\sim \sqrt{m_\nu / M_{HNL}}$  providing observable signals at the luminosity frontier. This is achieved either by fine-tuning of the mixing matrices in a specific scenarios with additional symmetries, or in the framework of Casas-Ibarra diagonalisation where the mixing can be enhanced. First sort of models gives quasi-Dirac neutrinos processed by the standard calculation technique, which are not fully consistent with the second sort of models not using the "Dirac limit", evaluating with Majorana Feynman rules. Collider studies are performed using so-called "model independent approach" or "phenomenological seesaw type I model" with one generation of HNL and mixing independent of HNL mass.

**It is interesting to consider explicit forms of mixing for three HNL generations beyond the "Dirac limit" in view of the available data.**



# From flavor states to mass states

The full  $6 \times 6$  mass matrix  $\mathcal{M} = \mathcal{U} \mathcal{D} \mathcal{U}^T$ , where  $\mathcal{U}$  - unitary,  $\mathcal{D}$  - diagonal non-negative. **Mass** and **flavor** states

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \mathcal{U} P_L \begin{pmatrix} \nu \\ N \end{pmatrix}, \quad \mathcal{U} = \exp \begin{pmatrix} 0 & \theta \\ -\theta^\dagger & 0 \end{pmatrix} \cdot \begin{pmatrix} U_\nu & 0 \\ 0 & U_N^* \end{pmatrix}$$

$$\nu_L \simeq \left(1 - \frac{1}{2} \theta \theta^\dagger\right) U_\nu P_L \nu + \theta U_N^* P_L N,$$

$$\nu_R^c \simeq -\theta^\dagger U_\nu P_L \nu + \left(1 - \frac{1}{2} \theta^\dagger \theta\right) U_N^* P_L N.$$

Exponent is decomposed in  $\theta$  - "Casas-Ibarra diagonalization"

NPB 618 (2001) 171 (hep-ph/0103065)



# Neutral and charged currents interaction with $W^\pm$ and $Z$

$$\mathcal{L}_{CC}^\nu = -\frac{g}{\sqrt{2}} \bar{l} \gamma_\mu U_{PMNS} \nu_{iL} W^\mu + h.c. \quad (1a)$$

$$\mathcal{L}_{NC}^\nu = \frac{g}{2c_W} \bar{\nu}_{iL} \gamma_\mu U_{PMNS}^\dagger U_{PMNS} \nu_{jL} Z^\mu + h.c. \quad (1b)$$

$$\mathcal{L}_{CC}^N = -\frac{g}{\sqrt{2}} \bar{l} \gamma_\mu \theta U_N^* N_{kL} W^\mu + h.c. \quad (1c)$$

$$\begin{aligned} \mathcal{L}_{NC}^N = & -\frac{g}{2c_W} \bar{N}_{iL} \gamma_\mu U_N^T \theta^\dagger \theta U_N^* N_{jL} Z^\mu + \\ & + \left( -\frac{g}{2c_W} \bar{\nu}_{iL} \gamma_\mu U_{PMNS}^\dagger \left( I - \frac{1}{2} \theta^\dagger \theta \right) \theta U_N^* N_{jL} Z^\mu + h.c. \right) \end{aligned} \quad (1d)$$

HNL mixing with left SM neutrino (active neutrino) is described by  $\Theta \equiv \theta U_N^*$  matrix.

Solving the diagonalization equations in the  $\mathcal{O}(\theta^2)$  order

$$I = \Omega^T \Omega = [-i\sqrt{\hat{m}^{-1}} U_\nu^\dagger M_D U_N^* \sqrt{\hat{M}^{-1}}]^T \cdot [-i\sqrt{\hat{m}^{-1}} U_\nu^\dagger M_D U_N^* \sqrt{\hat{M}^{-1}}],$$

where  $\Omega$  – arbitrary orthogonal matrix, may include additional parameters to enhance the mixing,  $U_\nu$  и  $U_N$  diagonalize  $\nu_{e,\mu,\tau}$  and  $N_{1,2,3}$  sectors.

$$\Theta = iU_\nu \sqrt{\hat{m}} \Omega \sqrt{\hat{M}^{-1}}, \text{ где } \hat{m} = \text{diag}(m_1, m_2, m_3), \hat{M} = \text{diag}(M_1, M_2, M_3)$$

PMNS matrix non-unitary:  $U_{\text{PMNS}} = (1 - \frac{1}{2}\theta^\dagger\theta + \mathcal{O}(\theta^4)) U_\nu$



## Non-minimal $\mathcal{O}(\theta^3)$ decomposition

Take into account the terms of the order of  $\mathcal{O}(\theta M_D)$  when  
( $\hat{M} = \text{diag}(M_1, M_2, M_3)$ )

$$M_N = U_N^* \hat{M} U_N^\dagger = (\theta^{-1} - \frac{1}{3} \theta^\dagger) M_D = M_M + \theta^\dagger M_D$$

whereas, within  $\mathcal{O}(\theta^2)$  approximation for the see-saw mechanism, it is assumed that  $M_N = M_M$ . For non-minimal decomposition of the **exp** matrix, the condition must be met

$$\Omega^{-1} = \Omega^T + \frac{1}{3} \hat{M}^{-1} (\Omega^{-1})^* \hat{m},$$

which is a condition for the self-consistency of the diagonalization procedure, taking into account the  $\mathcal{O}(\theta M_D)$  terms.





# Neutrino Minimal Standard Model ( $\nu$ MSM)

In the following  $\nu$ MSM model will be favorable

- explains neutrino oscillation data

	NH	IH
$m_1$	small	$\sqrt{\Delta m_{31}^2} \simeq 0.049 \text{ eV}$
$m_2$	$\sqrt{ \Delta m_{21}^2 } \simeq 0.009 \text{ eV}$	$\sqrt{\Delta m_{32}^2} \simeq 0.050 \text{ eV}$
$m_3$	$\sqrt{\Delta m_{31}^2} \simeq 0.049 \text{ eV}$	small

- no very distinctive mass scales
- $N_1$  is the dark matter particle
- baryonic asymmetry is generated by means of  $N_2 - N_3$  oscillations if masses of  $N_2 \sim N_3 \gg N_1$

T.Asaka, S.Blanchet and M.Shaposhnikov, Phys.Lett. B631 (2005) 151 (hep-ph/0503065)



## General cosmological restriction: lifetime

HNL of the first generation – dark matter candidate – does not decay on the cosmological time scale  $\tau_{N_1} \geq H_0^{-1} \simeq 4 \times 10^{17}$  sec. The one-loop mediated decay  $N \rightarrow \gamma, \nu$  can be a distinctive signal with photon energy  $E_\gamma = M_1/2$ , then the lifetime limit  $N_1 \rightarrow 3\nu$  is enhanced by astro-gamma observations [1, 2, 3]. In the following  $\tau_{N_1} > 10^{25}$  sec.

$$\Gamma(N_1 \rightarrow \gamma, \nu) = \frac{9\alpha_{EM} G_F^2 M_1^5}{256\pi^4} \sum_{\alpha} |\Theta_{\alpha 1}|^2.$$

$$\tau_{N_1} = 3 \times 10^{22} \left( \frac{M_1}{1 \text{ keV}} \right)^{-4} \left( \sum_{\alpha} \frac{(m_D)_{\alpha 1}}{1 \text{ eV}} \right)^{-1} \text{ sec.}$$

Useful variable for  $\Omega$ -independent observables

$$(m_D)_{\alpha l} = \left| \sum_k \sqrt{m_k} U_{\alpha k} \Omega_{kl} \right|^2$$

$$\sum_{\alpha} (m_D)_{\alpha l} = \left| \sum_k \sqrt{m_k} U_{\alpha k} \delta_{k1} \right|^2 = m_1, \quad \sum_{\alpha} (m_D)_{\alpha l} = \left| \sum_k \sqrt{m_k} U_{\alpha k} \delta_{k3} \right|^2 = m_3$$



# General cosmological restriction: dark matter energy fraction

Active-sterile neutrino mixing  $\Theta$  is small and HNL DM particles have never been in thermal equilibrium. Main HNL production source is *Dodelson-Widrow mechanism* [4] of active-sterile neutrino oscillations. In the case of nonresonant production the cosmological energy fraction

$$\Omega_N h^2 \simeq 0.1 \sum_{l=1}^3 \sum_{\alpha=e,\nu,\tau} \left( \frac{|\Theta_{\alpha l}|^2}{10^{-8}} \right) \left( \frac{M_l}{1 \text{ keV}} \right)^2.$$

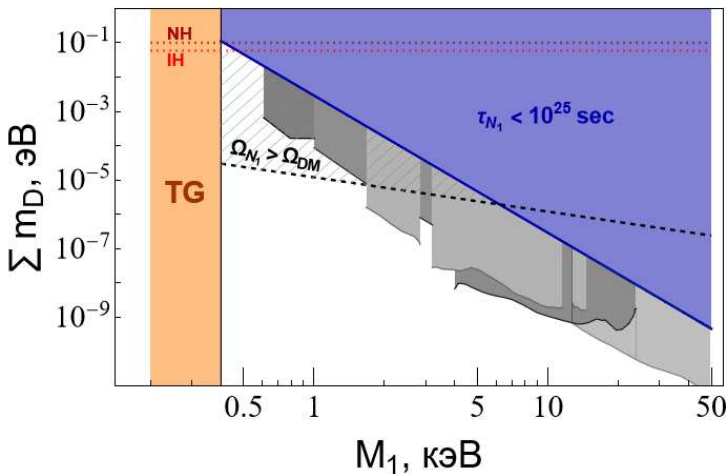
In particular for  $N_1$  fermions

$$\Omega_{N_1} h^2 \simeq \left( \frac{\sum_{\alpha} (m_D)_{\alpha 1}}{10^{-4} \text{ eV}} \right) \left( \frac{M_1}{1 \text{ keV}} \right) \leq \Omega_{DM} h^2 = 0.12.$$

$$\sum_{\alpha} (m_D)_{\alpha 1} < \overline{(m_D)}_{DM} = 10^{-5} \left( \frac{M_1}{1 \text{ keV}} \right)^{-1} \text{ eV}$$



# Exclusion contours for $N_1$ DM particle



Cosmological restrictions on  $(m_D)_{\alpha l} = \left| \sum_k \sqrt{m_k} U_{\alpha k} \Omega_{kl} \right|^2$  parameter for  $N_1$  DM, summed over flavor index  $\alpha = e, \mu, \tau$ .  $\Omega$ -independent plot. Gray regions excluded by satellite experiments XMM, Chandra, HEAO-1, etc. recalculated to  $\Sigma m_D$  from the data summary in 0811.2385.

# Possible $\Omega$ forms

- $\Omega = I$  for normal hierarchy (NH) or anti-diagonal orthogonal  $\Omega$  analogous to  $\Omega = I$  for inverse hierarchy (IH);
- $\Omega \in SO(3, \mathbb{R})$  parametrized by Euler angles  $\alpha_j$

$$\Omega = \mathbf{X}_1 \mathbf{Z}_2 \mathbf{X}_3 = \begin{pmatrix} c_2 & -c_3 s_2 & s_2 c_3 \\ c_1 s_2 & c_1 c_2 c_3 - s_1 s_3 & -c_3 s_1 - c_1 c_2 s_3 \\ s_1 s_2 & c_1 s_3 + c_2 c_3 s_1 & c_1 c_3 - c_2 s_1 s_3 \end{pmatrix}$$

- $\Omega \in SO(3, \mathbb{C})$  like above, complex-valued angles  $\omega_j = \alpha_j + i\beta_j$ .



# Surfaces for the real-valued $\Omega$ parameters.

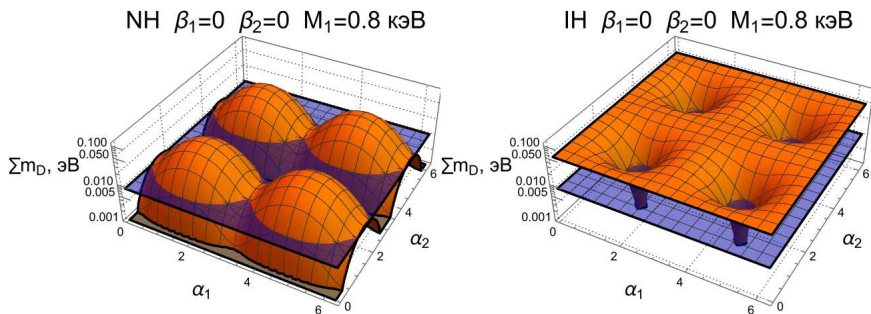
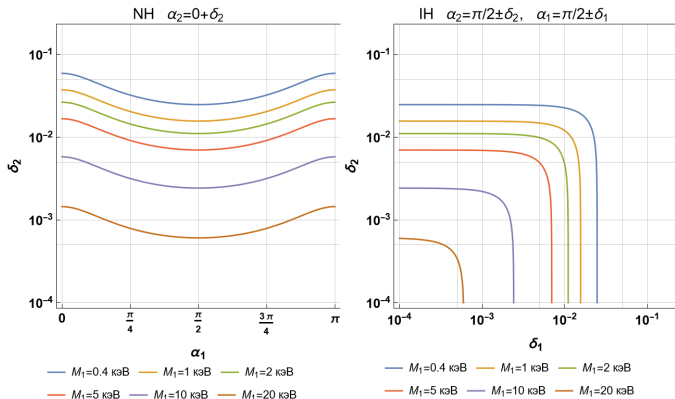


Рис.: Surfaces for  $\sum_{\alpha} (m_D)_{\alpha 1}$  vs  $\alpha_1$  и  $\alpha_2$  for normal hierarchy (left) and inverse hierarchy (right plot). Blue horizontal plane -  $\overline{(m_D)}_{X\text{-ray}}$  at  $M_1 = 0.8$  keV.

# Contours for the real-valued $\Omega$ parameters



**Рис.:** Exclusion contours for  $\alpha_1$  и  $\alpha_2$  for various masses  $M_1$  DM. Combination of  $\tau_{N_1}$  and energy fraction limits is taken.  $M_1$  masses in keV.

# Contours for the complex-valued $\Omega$ parameters

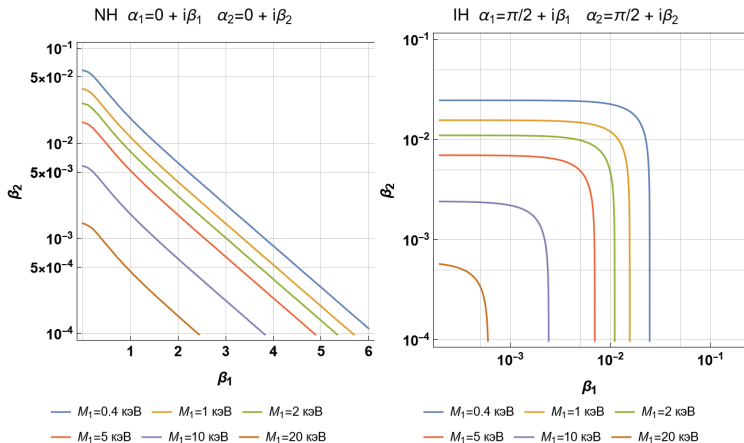


Рис.: Exclusion contours for imaginary parts  $\beta_1$  and  $\beta_2$  of Euler angles  $\omega_1$  and  $\omega_2$  which parametrize  $\Omega$  at fixed real parts of  $\alpha_1$  and  $\alpha_2$  (NH left plot and IH right plot).





## "Minimal mixing" scenario

"Minimal mixing", does not include redundant unknown parameters reflecting the general properties of constraints in the case of real-valued  $\Omega$ .

$$\Theta_{\min}^{(\text{NH})} = \begin{pmatrix} iU_{e1}\sqrt{\frac{m_1}{M_1}} & iU_{e2}\sqrt{\frac{m_2}{M_2}} & iU_{e3}\sqrt{\frac{m_3}{M_3}} \\ iU_{\mu1}\sqrt{\frac{m_1}{M_1}} & iU_{\mu2}\sqrt{\frac{m_2}{M_2}} & iU_{\mu3}\sqrt{\frac{m_3}{M_3}} \\ iU_{\tau1}\sqrt{\frac{m_1}{M_1}} & iU_{\tau2}\sqrt{\frac{m_2}{M_2}} & iU_{\tau3}\sqrt{\frac{m_3}{M_3}} \end{pmatrix}, \quad \Omega_{\min}^{(\text{NH})} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Theta_{\min}^{(\text{IH})} = \begin{pmatrix} iU_{e3}\sqrt{\frac{m_3}{M_1}} & iU_{e2}\sqrt{\frac{m_2}{M_2}} & iU_{e1}\sqrt{\frac{m_1}{M_3}} \\ iU_{\mu3}\sqrt{\frac{m_3}{M_1}} & iU_{\mu2}\sqrt{\frac{m_2}{M_2}} & iU_{\mu1}\sqrt{\frac{m_1}{M_3}} \\ iU_{\tau3}\sqrt{\frac{m_3}{M_1}} & iU_{\tau2}\sqrt{\frac{m_2}{M_2}} & iU_{\tau1}\sqrt{\frac{m_1}{M_3}} \end{pmatrix}, \quad \Omega_{\min}^{(\text{IH})} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



## "Exponential mixing" scenario

Cosmological restrictions favor block-diagonal  $\Omega$  and the mass scale  $M_1 \sim 1 - 10$  keV.

$$\Omega_{\text{NH}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega) & -\sin(\omega) \\ 0 & \xi \sin(\omega) & \xi \cos(\omega) \end{pmatrix} \quad \Omega_{\text{IH}} = \begin{pmatrix} 0 & \cos(\omega) & -\sin(\omega) \\ 0 & \xi \sin(\omega) & \xi \cos(\omega) \\ 1 & 0 & 0 \end{pmatrix}$$

with  $\omega$  complex-valued. Mixing enhancement controlled by  $X_\omega = e^{\text{Im}(\omega)} \gg 1$ ,  $\text{Im}(\omega) > 1$ .

Specifically for  $\nu$ MSM second and third generations of *HNL*  $M_2 \simeq M_3 \gg M_1$ . Quasidegenerate in mass for generation of baryonic asymmetry [5].



## Restrictions for $\nu$ MSM-type model: $N_2 - N_3$ sector

- Accelerator experiments with *missing energy reconstruction* (PIENU, TRIUMPH, KEK, NA62, E949) and *displaced vertices detection* (PS-191, CHARM, NuTeV, DELPHI) in total give upper bounds for the mixing variables ( $\alpha = e, \mu, \tau$ )

$$U_\alpha^2 = \sum_{I=1}^3 |\Theta_{\alpha I}|^2 = \begin{cases} \frac{m_1}{M_1} |U_{\alpha 1}|^2 + |\Theta_{\alpha 2}^{(NH)}|^2 + |\Theta_{\alpha 3}^{(NH)}|^2, & \text{NH} \\ \frac{m_3}{M_1} |U_{\alpha 3}|^2 + |\Theta_{\alpha 2}^{(IH)}|^2 + |\Theta_{\alpha 3}^{(IH)}|^2, & \text{IH} \end{cases}$$

- Lifetime restriction for  $N_2$  и  $N_3$ ,  $\tau_N < 0.02$  sec, when there is no overproduction of the light elements ( ${}^4\text{He}$ ,  ${}^2\text{H}$ ) in the primary plasma, [6] (socalled **primary nucleosynthesis** or **Big Bang nucleosynthesis**, **BBN**). Gives a bound from below on  $U_\alpha^2$ .



# Perturbative calculations with Majorana fermions

General form

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \overline{\lambda}_a (i\hat{\partial} - M_a) \lambda_a + \overline{\Psi}_b (i\hat{\partial} - m_b) \Psi_b + \\ & + \frac{1}{2} g_{abc}^i \overline{\lambda}_a \Gamma_i \lambda_b \Phi_c + k_{abc}^i \overline{\lambda}_a \Gamma_i \Psi_b \Phi_c^* + (k_{abc}^i)^* \overline{\Psi}_a \Gamma_i \lambda_b \Phi_c + \\ & + h_{abc}^i \Psi_a \Gamma_i \Psi_b \Phi_c, \end{aligned} \quad (2)$$

where  $\lambda/\Psi$  are Majorana/Dirac fermions,  $\Phi$  - boson.

H.Haber and G.Kane, *The Search for Supersymmetry: Probing Physics Beyond the Standard Model*, Phys.Rept. 117 (1985) 75-263 (general basis)

A.Denner, H.Eck, O.Hahn and J.Kublbeck, *Feynman rules for fermion number violating interactions*, Nucl.Phys.B 387 (1992) 467-481 (fermion flow technique implemented in FeynCalc package)



neutral current interaction

$$\mathcal{L}_{\nu N} = -\frac{g}{2c_w} \left[ (U^\dagger \Theta)_{iJ} \bar{\nu}_i \gamma^\mu P_L N_J + (U^\dagger \Theta)_{iJ}^* \bar{N}_J \gamma^\mu P_L \nu_i \right] Z_\mu$$

for Majorana case can be rewritten in the form

$$\begin{aligned} \bar{N}_J \gamma^\mu P_L \nu_i &= (\bar{N}_J \gamma^\mu P_L \nu_i)^T = (-1) \nu_i^T (\gamma^\mu P_L)^T \bar{N}_J^T = (-1) (-\bar{\tilde{\nu}}_i C) (\gamma^\mu P_L) \\ &= \bar{\tilde{\nu}}_i \underbrace{C (\gamma^\mu P_L)^T C^{-1}}_{=-\gamma^\mu P_R} \tilde{N}_J = \left\{ \begin{array}{l} \bar{\tilde{\nu}}_i = \bar{\nu}_i \\ \tilde{N}_J = N_J \end{array} \right\} = -\bar{\nu}_i \gamma^\mu P_R N_J \quad (3) \end{aligned}$$

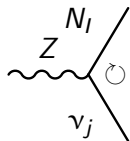
and

$$\mathcal{L}_{\nu N} = -\frac{g}{2c_w} \bar{\nu}_i \left[ (U^\dagger \Theta)_{iJ} \gamma^\mu P_L - (U^\dagger \Theta)_{iJ}^* \gamma^\mu P_R \right] N_J Z_\mu$$



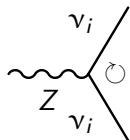
# HNL decays: Feynman rules for Majorana fermions

Width calculation for Majorana fermions.



A Feynman diagram showing a Majorana fermion  $N_l$  (represented by a solid line) decaying into a  $Z$  boson (represented by a wavy line) and a neutrino  $\nu_j$  (represented by a solid line). A small circle with a dot is attached to the vertex.

$$-i \frac{g}{2 \cos \theta_W} \gamma^\mu \left[ (U^\dagger \Theta)_{jl}^* P_L - (U^\dagger \Theta)_{jl} P_R \right]$$



A Feynman diagram showing a Majorana fermion  $\nu_i$  (represented by a solid line) decaying into a  $Z$  boson (represented by a wavy line) and another Majorana fermion  $\nu_i$  (represented by a solid line). A small circle with a dot is attached to the vertex.

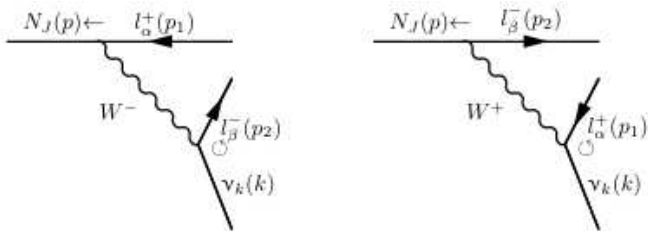
$$-i \frac{g}{2 \cos \theta_W} \gamma^\mu (P_L - P_R) = i \frac{g}{2 \cos \theta_W} \gamma^\mu \gamma^5$$

Таблица: Sample Feynman rules for Majorana fermions implemented in LanHEP/CompHEP

$$\Gamma_{N_{2,3}} = \Gamma(\rightarrow h^\pm + l^\mp) + \Gamma(\rightarrow h^0 + \nu) + \Gamma(\rightarrow l^+ l^- \nu), \quad \tau_{N_{2,3}} = \Gamma_{N_{2,3}}^{-1}$$

# Majorana case interferences and the "Dirac limit": example

$$N \rightarrow l_{\alpha}^{+} l_{\beta}^{-} \nu$$



Decay width

$$\Gamma(N_J \rightarrow \sum_{k=1}^3 \nu_k l_{\alpha}^{+} l_{\beta}^{-}) = \frac{G_F^2 M_J^5}{192\pi^3} \left( |\Theta_{\alpha J}|^2 + |\Theta_{\beta J}|^2 - \frac{4}{M_J} \sum_{k=1}^3 m_k \text{Re}\{\Theta_{\alpha J} \Theta_{\beta J}^* U_{\beta k}^* U_{\alpha k}\} \right)$$

$e^{Im(\omega)} = 1100$  at  $\omega = 7$ . If  $\alpha = \beta$  third interfering diagram with intermediate  $Z$  appears.



# Partial widths of three-particle decays

- $\Gamma(N_I \rightarrow \sum_i \nu_i, \nu_j, \nu_j) = \frac{G_F^2 M_I^5}{192\pi^3} \sum_{\alpha=e,\mu,\tau} |\Theta_{\alpha I}|^2$
- $\Gamma(N_I \rightarrow \sum_{i=1,2,3} \nu_i l_{\alpha}^+ l_{\alpha}^-) = \frac{G_F^2 M_I^5}{96\pi^3} \left( \left[ (C_1^2 + C_2^2) \sum_{\beta} |\Theta_{\beta I}|^2 + (1 - 2C_1) |\Theta_{\alpha I}|^2 \right] \mathcal{F}_1(r) + \left[ (2C_1 C_2) \sum_{\beta} |\Theta_{\beta I}|^2 - 2C_2^2 |\Theta_{\alpha I}|^2 \right] \mathcal{F}_2(r) \right)$ , где  $C_1 = s_W^2 - \frac{1}{2}$ ,  $C_2 = s_W^2$ ,  $r_{\alpha} = \frac{m_{\alpha}^2}{M_I^2}$ ,

$$\mathcal{F}_1(r) = (1 - 14r - 2r^2 - 12r^3) \sqrt{1 - 4r} + 12r^2(1 - r^2) \ln\left(\frac{1 - 3r + (1 - r)\sqrt{1 - 4r}}{r(1 - \sqrt{1 - 4r})}\right),$$

$$\mathcal{F}_2(r) = (2r + 10r^2 - 12r^3) \sqrt{1 - 4r} - (6r^2 - 12r^3 + 12r^4) \ln\left(\frac{1 - 3r + (1 - r)\sqrt{1 - 4r}}{r(1 - \sqrt{1 - 4r})}\right).$$

- $\Gamma(N_I \rightarrow \sum_{i=1,2,3} \nu_i l_{\alpha}^+ l_{\beta}^-) = \frac{G_F^2 M_I^5}{192\pi^3} (|\Theta_{\alpha I}|^2 + |\Theta_{\beta I}|^2) \mathcal{G}(r_{\alpha}, r_{\beta})$ ,

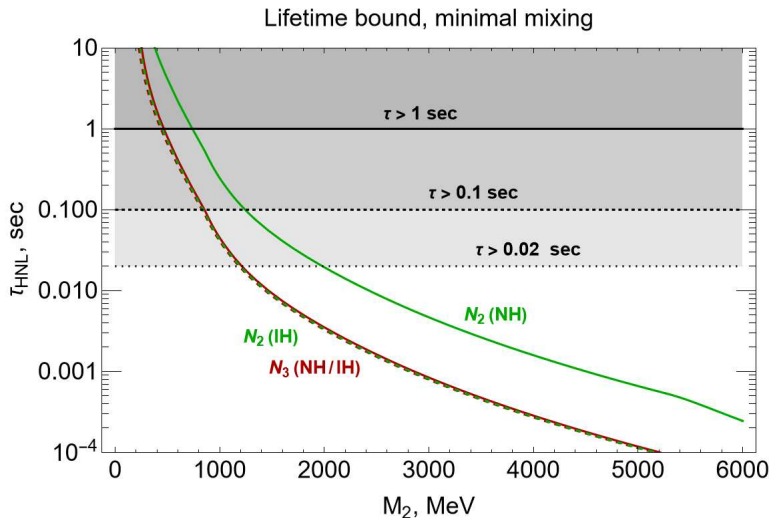
$$\mathcal{G}(x, y) = (1 - 7x - 7x^2 + x^3 + 12xy - 7y - 7y^2 + y^3 - 7x^2y - 7xy^2)R +$$

$$+ 12(y^2 + x^2y^2 - 2x^2) \ln\left(\frac{1 + x - y + R}{2}\right) + 12x^2(1 - y^2) \ln\left(\frac{1}{x}\right) +$$

$$+ 12y^2(1 - x^2) \ln\left(\frac{1 - x - y + R}{1 - x + y - R}\right), \quad R = \lambda^{1/2}(1, x, y)$$

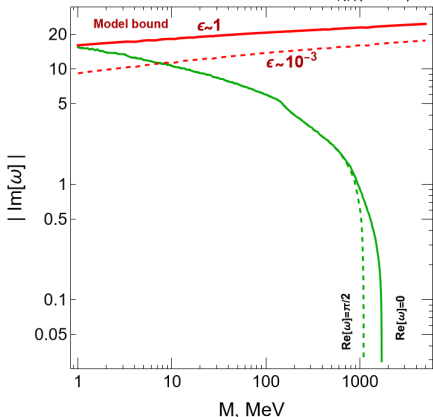


# BBN restriction in the "minimal mixing" scenario

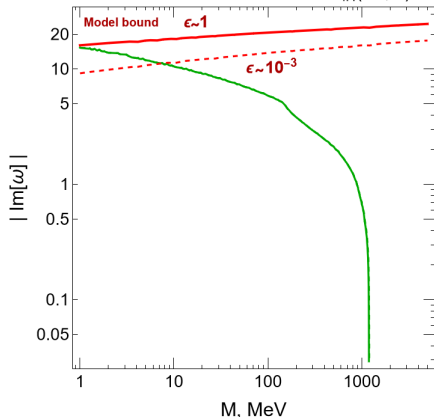


# BBN restrictions for $\omega$ parameter

Lifetime bound for case  $\Omega = \Omega_{\text{NH}}(\pm 1, \omega)$

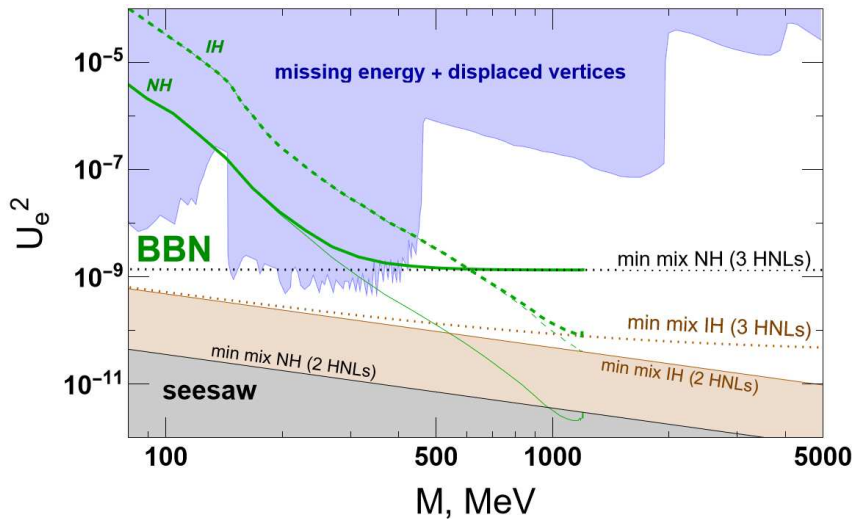


Lifetime bound for case  $\Omega = \Omega_{\text{IH}}(\pm 1, \omega)$

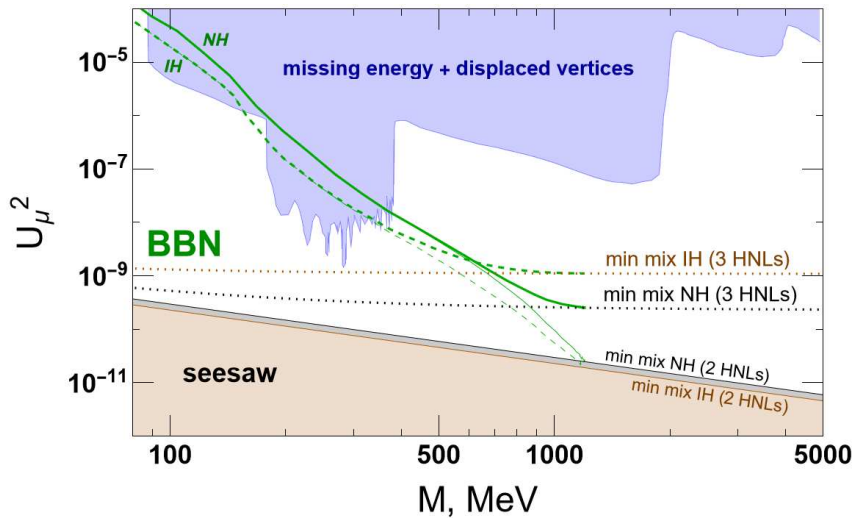


$$\Omega^{-1} = \Omega^T + \frac{1}{3} \hat{M}^{-1} (\Omega^{-1})^* \hat{m}, \quad (\text{up to } \mathcal{O}(M_D \theta) \text{ terms})$$

# Exclusion contours for $U_e^2$ mixing variable



# Exclusion contours for $U_{\mu}^2$ mixing variable



# Parameter of lepton universality violation in the meson decays

The value of lepton universality violation (**LUV**) in meson decays  $M = \pi^+, K^+$  is defined as

$$\Delta r_M = \frac{R_M}{R_M^{SM}} - 1, \quad \text{where} \quad R_M = \frac{\Gamma(M \rightarrow e\nu) + \Gamma(M \rightarrow eN)}{\Gamma(M \rightarrow \mu\nu) + \Gamma(M \rightarrow \mu N)}$$

to  $R_M^{SM}$  only active (SM) neutrino contribute

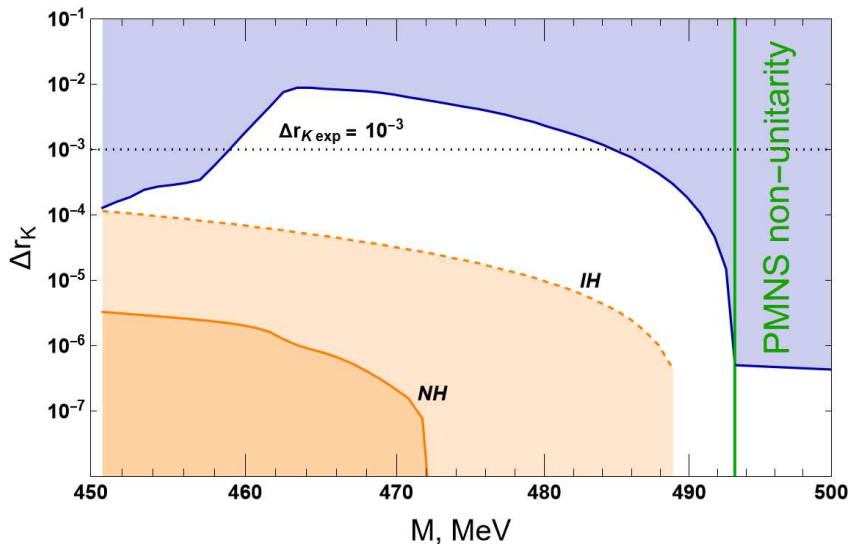
Unitarity condition of the  $6 \times 6$  matrix  $\mathcal{U}$

$$\sum_{i=1}^3 |U_{\alpha i}|^2 + \sum_{l=1}^3 |\Theta_{\alpha l}|^2 = 1$$

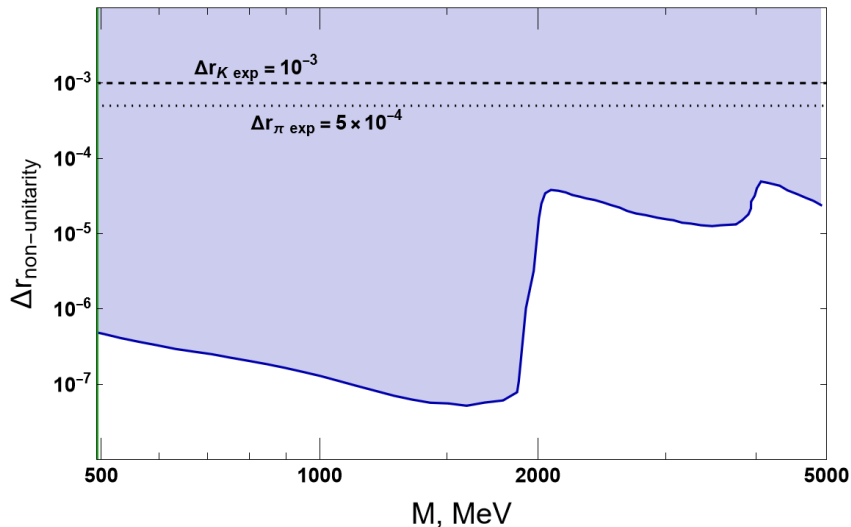
$$\Delta r_M = \frac{1 + \sum_l |\Theta_{el}|^2 (G_{el}^M - 1)}{1 + \sum_l |\Theta_{\mu l}|^2 (G_{\mu l}^M - 1)} - 1$$

$$G_{\alpha l}^M = \begin{cases} \frac{\lambda^{1/2}(1, r_l, r_\alpha) [r_l + r_\alpha - (r_\alpha - r_l)^2]}{r_\alpha(1 - r_\alpha)^2}, & M_l < m_M - m_\alpha \\ 0, & M_l > m_M - m_\alpha \end{cases}$$

# Lepton universality violation parameter in $K$ decays



# Lepton universality violation parameter in the mass region kinematically closed



# Main results (1)

- Limitations on the lifetime and particle density of a light sterile neutrino  $N_1$  of dark matter show that its mass is 0.4 - 40 keV (in the region of mixings experimentally accessible), exponential mixing is favored for enhancements in the  $N_2 - N_3$  sector where HNL signals can be amplified by the exponential multiplier  $e^{Im(\omega)} \sim 1000$  at  $\omega = i \times 6-7$ .
- Consideration of the sector  $N_2 - N_3$  for  $\nu$ MSM-like models showed a significant dependence on the mixing component with a light sterile neutrino of dark matter. The model taking into account all three generations significantly raises the lower seesaw bound for mixing parameters at masses  $M_{2,3} > 0.5$  GeV.
- In a model with three generations of HNL with the mass of a light active neutrino  $m_{1(3)} \sim 10^{-5}$  eV and  $M_1 \simeq 5$  keV, BBN the boundary of the mass of HNL  $M > 407$  MeV (NH), instead of and  $M > 340$  MeV for a model with two HNLs. Thus, taking into account the permissible non-zero values of the mass of a light active neutrino significantly shifts the BBN constraints.
- For LUV, a "window" was found in kaon decays in which the experimental value of  $\Delta r_K = (4 \pm 4) \times 10^{-3}$  is exceeded.










## Main results (2)

- Careful calculation of the lifetime in conjunction with the accelerator data gives the following acceptable parameter ranges:

- 134 MeV  $< M < 144$  MeV for NH – small "window"  $U_e^2$ :  
 $1.5 \cdot 10^{-7} < U_e^2 < 2.7 \cdot 10^{-7}$  (Requires a more accurate analysis of experimental data);
- 155 MeV  $< M < 177$  MeV for IH:  $1, 2 \cdot 10^{-6} < U_\mu^2 < 3, 5 \cdot 10^{-7}$ .
- For heavier HNLs that do not fall under the aforementioned ranges, the following boundaries appear from BBN:  
 $M > 407$  MeV for  $U_e^2$  with NH,  $M > 452$  MeV for  $U_e^2$  with IH;  
 $M > 370$  MeV for  $U_\mu^2$  with NH,  $M > 340$  MeV for  $U_\mu^2$  with IH.
- For «minimal mixing» the BBN-bound gives an estimate of the minimum HNL mass:  $M > 1.2$  GeV for IH and  $M > 2$  GeV for NH. For these masses, LUV is determined only by the deviation from unitarity and is an unobservable value of  $\mathcal{O}(10^{-11})$ .

For HNL signals at collider energies see the talk by A. Drutskoi.



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