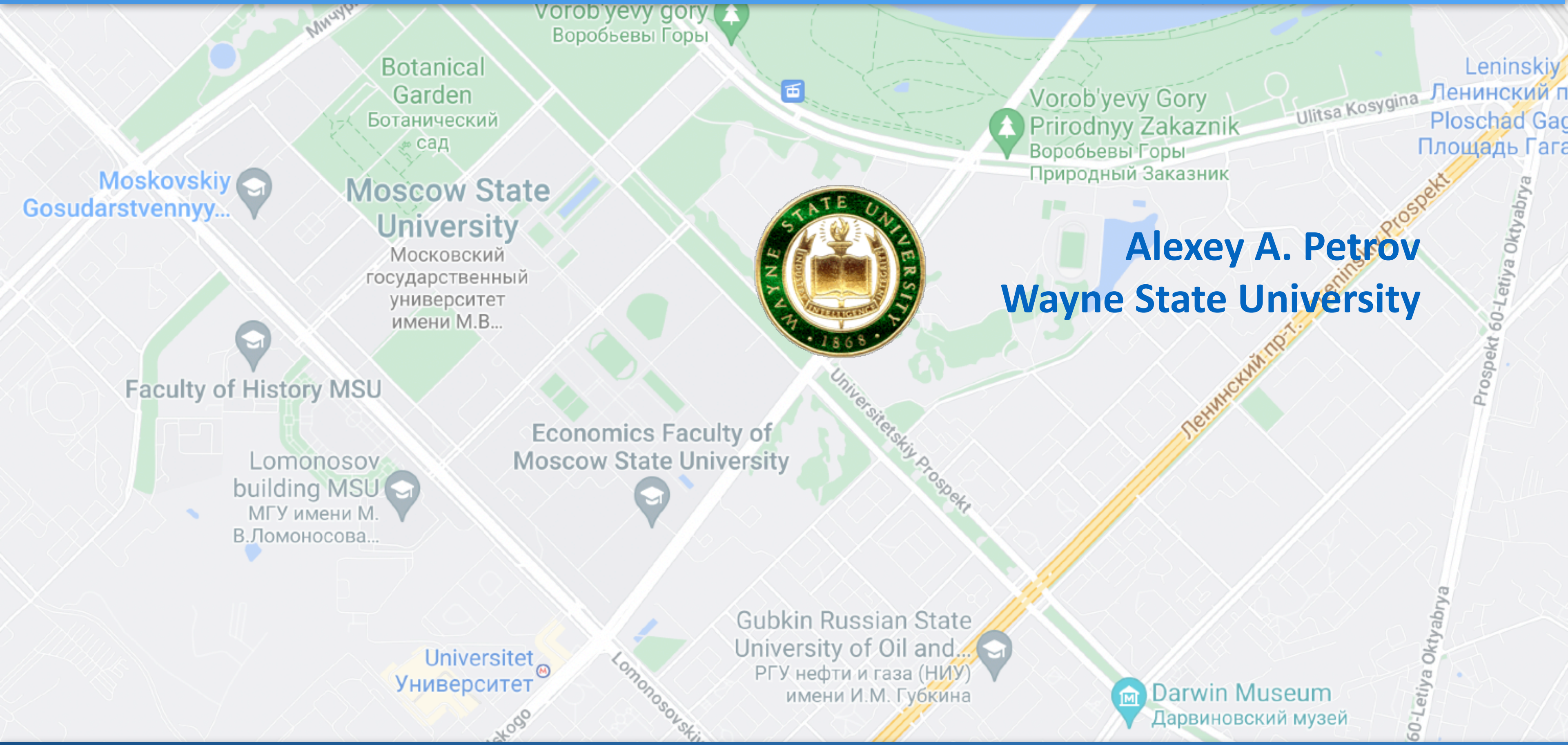
Theoretical developments in the physics of charm quarks



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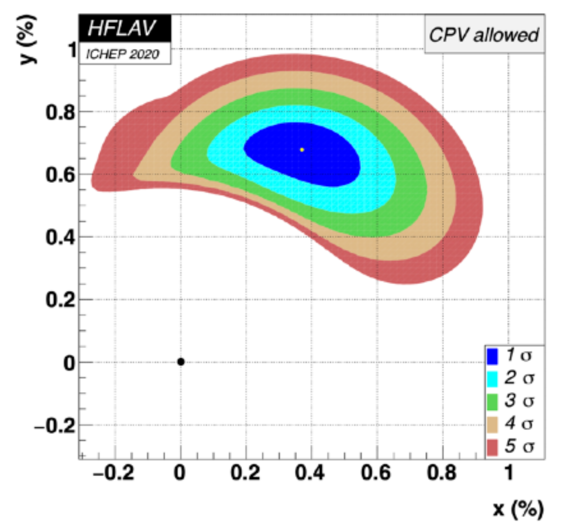
The 20th Lomonosov Conference on Elementary Particle Physics
Moscow State University, August 19-25, 2021

Introduction: charming results

– Charm quark continue to churn out surprises!

- spectroscopy: from X(3872) to X(6900) to pentaquark to T_{cc}^{+}?
  $q\bar{c}c\bar{q}$ $c\bar{c}c\bar{c}$ $c\bar{q}c\bar{q}$
- CP-violation: "anti-superweak" system
  $$\Delta a_{\mathrm{CP}}^{\mathrm{dir}}(KK-\pi\pi) = (-15.4 \pm 2.9)\times 10^{-4}$$
- D-mixing: y > x?
  $$y = 0.68^{+0.06}_{-0.07}$$
  $$x = 0.37 \pm 0.12\%$$



– Maybe the first signs of New Physics will come from charm…

Charming CP-violation

- How can CP-violation be observed in charm system?
  - can be observed by comparing CP-conjugated decay rates in various ways, both with and w/out time dependence

$$a_{CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})},$$

  - can manifest itself in charm ΔC=1 transitions (direct CP-violation)

$$\Gamma(D \to f) \neq \Gamma(CP[D] \to CP[f])$$ **dCPV**

  - or in ΔC=2 transitions (indirect CP-violation): mixing $\left|D_{1,2}\right\rangle = p\left|D^0\right\rangle \pm q\left|\overline{D^0}\right\rangle$

$$R_m^2 = |q/p|^2 = \left|\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma}\right|^2 = 1 + A_m \neq 1$$ **CPVmix**

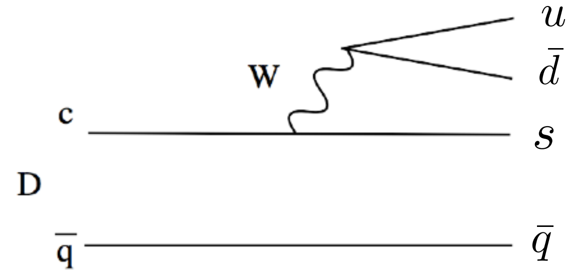
  - or in the interference b/w decays (ΔC=1) and mixing (ΔC=2)

$$\lambda_f = \frac{q}{p}\frac{\overline{A_f}}{A_f} = R_m e^{i(\phi+\delta)}\left|\frac{\overline{A_f}}{A_f}\right|$$ **CPVint**

Introduction: nonleptonic charm decays?

★ Can be classified by SM CKM suppression of tree amplitude ($V_{us} \sim \lambda$)

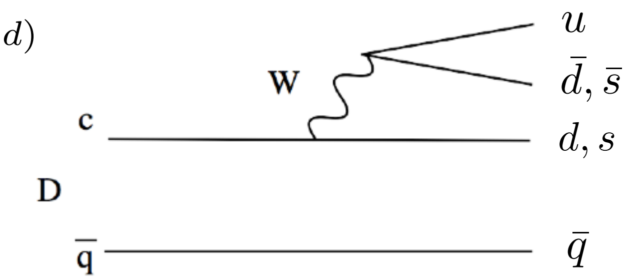
$$V_{cs}V_{ud}^*$$



★ Cabibbo-favored (CF: λ^0) decay

- originates from $c \to s\, u\bar{d}$
- examples: $D^0 \to K^-\pi^+$

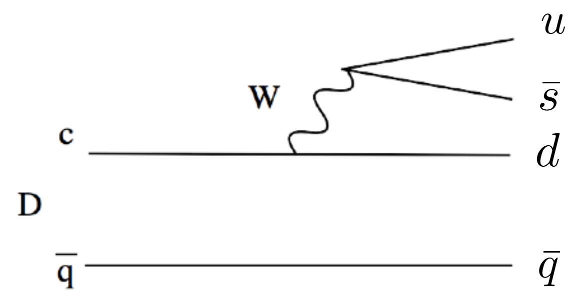
$$V_{cs(d)}V_{us(d)}^*$$



★ Singly Cabibbo-suppressed (SCS: λ^1) decay

- originates from $c \to q\, u\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

$$V_{cd}V_{us}^*$$



★ Doubly Cabibbo-suppressed (DCS: λ^2) decay

- originates from $c \to d\, u\bar{s}$
- examples: $D^0 \to K^+\pi^-$

★ We shall concentrate on SCS decays. Why is that?

Direct CP-violation in charm: realities of life

★ IDEA: consider the DIFFERENCE of decay rate asymmetries: $D \to \pi\pi$ vs $D \to KK$!
For each final state the asymmetry

D^0: no neutrals in the final state!

$$a_f = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})} \Rightarrow a_f = a_f^d + a_f^m + a_f^i$$

direct, mixing, interference

★ A reason: $a^m_{KK} = a^m_{\pi\pi}$ and $a^i_{KK} = a^i_{\pi\pi}$ (for CP-eigenstate final states), so, ideally, mixing asymmetries cancel ($r_f = P_f/A_f$)!

$$a_f^d = 2 r_f \sin\phi_f \sin\delta_f$$

★ ... and the resulting DCPV asymmetry is $\Delta a_{CP} = a^d_{KK} - a^d_{\pi\pi} \approx 2 a^d_{KK}$ (double!)

$$A_{KK} = \frac{G_F}{\sqrt{2}} \lambda \left[(T + E + P_{sd}) + a \lambda^4 e^{-i\gamma} P_{bd} \right]$$

$$A_{\pi\pi} = \frac{G_F}{\sqrt{2}} \lambda \left[(-(T + E) + P_{sd}) + a \lambda^4 e^{-i\gamma} P_{bd} \right]$$

★ ... so it is doubled in the limit of $SU(3)_F$ symmetry

SU(3) is badly broken in D-decays

- Experimental results
  - note that while the new result does constitute an observation of CP-violation in the difference…

$$\Delta a_{CP}^{dir} = a_{CP}(K^- K^+) - a_{CP}(\pi^- \pi^+) = (-0.156 \pm 0.029)\%$$

LHCb 2019

  - … it is not yet so for the individual decay asymmetries

$$\begin{aligned} a_{CP}(K^- K^+) &= (0.04 \pm 0.12\ (\text{stat}) \pm 0.10\ (\text{syst}))\%\,, \\ a_{CP}(\pi^- \pi^+) &= (0.07 \pm 0.14\ (\text{stat}) \pm 0.11\ (\text{syst}))\%\,. \end{aligned}$$

LHCb 2017

- Need confirmation from other experiments (Belle II)
- What does this result mean? New Physics? Standard Model?

Theoretical troubles

ΔA_{CP} within the Standard Model and beyond

Mikael Chala, Alexander Lenz, Aleksey V. Rusov and Jakub Scholtz

Institute for Particle Physics Phenomenology, Durham University, DH1 3LE Durham, United Kingdom

Implications on the first observation of charm CPV at LHCb

Hsiang-nan Li[1*], Cai-Dian Lü[2†], Fu-Sheng Yu[3‡]

[1]*Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China*

The Emergence of the $\Delta U = 0$ Rule in Charm Physics

Yuval Grossman[*] and Stefan Schacht[†]

Department of Physics, LEPP, Cornell University, Ithaca, NY 14853, USA

Revisiting CP violation in $D \to PP$ and VP decays

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Calculating CP-asymmetries?

- Effective Hamiltonian for singly Cabibbo-suppressed (SCS) decays
  - drop all "penguin" operators (Q_i for $i \geq 3$) as C_i are small, $\lambda_q = V_{uq} V^*_{cq}$,

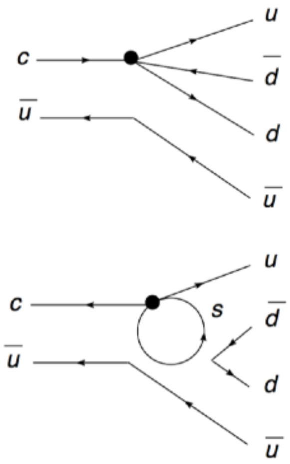
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) - \lambda_b \sum_{i=3,\dots,6,8g} C_i Q_i \right]$$

$$Q_1^q = (\bar{u}\Gamma_\mu q)(\bar{q}\Gamma^\mu c)\,, \qquad Q_2^q = (\bar{q}\Gamma_\mu q)(\bar{u}\Gamma^\mu c)$$

  - recall that $\sum_{q=d,s,b} \lambda_q = 0$ or $\lambda_d = -(\lambda_s + \lambda_b)$ and $\mathcal{O}^q \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i Q_i^q\,, \quad \text{with} \;\; q = d, s.$

without QCD

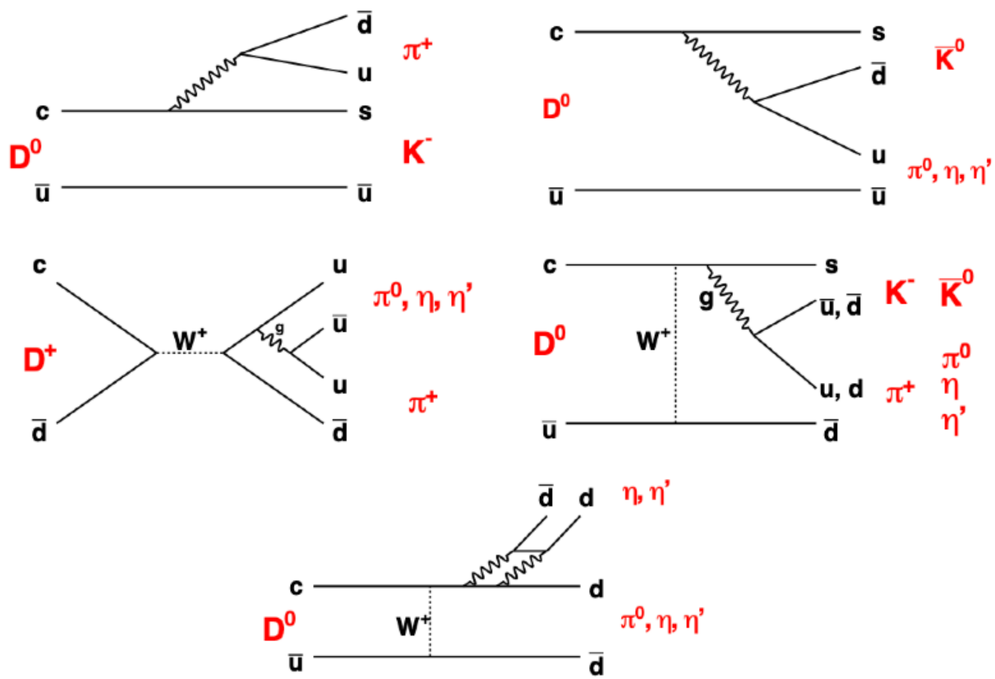
with QCD



CP-asymmetry: topological flavor flow

- Topological flavor-flow diagrams could be used to deal with hadronic uncertainties

Recent: B. Bhattacharya, A. Datta, AAP, J. Waite, 2107.13564



T and C

A and E

SE

- Fit many decay modes, assume SM weak phase!

CP-asymmetry: topological flavor flow

- All SCS decays can be written in terms of the set of flavor flow diagrams

	Mode	Representation
D^0	$\pi^+\pi^-$	$\lambda_d(0.96T+E_d)+\lambda_p(P_p+PE_p+PA_p)$
	$\pi^0\pi^0$	$\frac{1}{\sqrt{2}}\lambda_d(-0.78C+E_d)+\frac{1}{\sqrt{2}}\lambda_p(P_p+PE_p+PA_p)$
	$\pi^0\eta$	$-\lambda_d(E_d)\cos\phi-\frac{1}{\sqrt{2}}\lambda_s(1.28C)\sin\phi+\lambda_p(P_p+PE_p)\cos\phi$
	$\pi^0\eta'$	$-\lambda_d(E_d)\sin\phi+\frac{1}{\sqrt{2}}\lambda_s(1.28C)\cos\phi+\lambda_p(P_p+PE_p)\sin\phi$
	$\eta\eta$	$\frac{1}{\sqrt{2}}\lambda_d(0.78C+E_d)\cos^2\phi+\lambda_s(-\frac{1}{2}1.08C\sin 2\phi+\sqrt{2}\,E_s\sin^2\phi)+\frac{1}{\sqrt{2}}\lambda_p(P_p+PE_p+PA_p)\cos^2\phi$
	$\eta\eta'$	$\frac{1}{2}\lambda_d(0.78C+E_d)\sin 2\phi+\lambda_s(\frac{1}{\sqrt{2}}1.08C\cos 2\phi-E_s\sin 2\phi)\ +\frac{1}{2}\lambda_p(P_p+PE_p+PA_p)\sin 2\phi$
	K^+K^-	$\lambda_s(1.27T+E_s)+\lambda_p(P_p+PE_p+PA_p)$
	$K^0\overline{K}^0$	$\lambda_d(E_d)+\lambda_s(E_s)+2\lambda_p(PA_p)$
D^+	$\pi^+\pi^0$	$\frac{1}{\sqrt{2}}\lambda_d(0.97T+0.78C)$
	$\pi^+\eta$	$\frac{1}{\sqrt{2}}\lambda_d(0.82T+0.93C+1.19A)\cos\phi-\lambda_s(1.28C)\sin\phi+\sqrt{2}\lambda_p(P_p+PE_p)\cos\phi$
	$\pi^+\eta'$	$\frac{1}{\sqrt{2}}\lambda_d(0.82T+0.93C+1.61A)\sin\phi+\lambda_s(1.28C)\cos\phi+\sqrt{2}\lambda_p(P_p+PE_p)\sin\phi$
	$K^+\overline{K}^0$	$\lambda_d(0.85A)+\lambda_s(1.28T)+\lambda_p(P_p+PE_p)$
D_s^+	π^+K^0	$\lambda_d(1.00T)+\lambda_s(0.84A)+\lambda_p(P_p+PE_p)$
	π^0K^+	$\frac{1}{\sqrt{2}}[-\lambda_d(0.81C)+\lambda_s(0.84A)+\lambda_p(P_p+PE_p)]$
	$K^+\eta$	$\frac{1}{\sqrt{2}}\lambda_p[0.92C\delta_{pd}+1.14A\delta_{ps}+P_p+PE_p]\cos\phi-\lambda_p[(1.31T+1.27C+1.14A)\delta_{ps}+P_p+PE_p]\sin\phi$
	$K^+\eta'$	$\frac{1}{\sqrt{2}}\lambda_p[0.92C\delta_{pd}+1.14A\delta_{ps}+P_p+PE_p]\sin\phi+\lambda_p[(1.31T+1.27C+1.14A)\delta_{ps}+P_p+PE_p]\cos\phi$

H.-Y. Cheng, C.W. Chiang
Phys.Rev.D 100 (2019) 9, 093002

CP-asymmetry: topological flavor flow

- Fits to experimental data in SCS and CF results in

Decay Mode	$\mathcal{B}_{\rm SU(3)}$	$\mathcal{B}_{\rm SU(3)-breaking}$	$\mathcal{B}_{\rm expt}$
$D^0 \to \pi^+\pi^-$	2.28 ± 0.02	1.47 ± 0.02	1.455 ± 0.024
$D^0 \to \pi^0\pi^0$	1.50 ± 0.03	0.82 ± 0.02	0.826 ± 0.025
$D^0 \to \pi^0\eta$	0.83 ± 0.02	0.92 ± 0.02	0.63 ± 0.06
$D^0 \to \pi^0\eta'$	0.75 ± 0.02	1.36 ± 0.03	0.92 ± 0.10
$D^0 \to \eta\eta$	1.52 ± 0.03	1.82 ± 0.04	2.11 ± 0.19
	1.52 ± 0.03	2.11 ± 0.04	
$D^0 \to \eta\eta'$	1.28 ± 0.05	0.69 ± 0.03	1.01 ± 0.19
	1.28 ± 0.05	1.63 ± 0.08	
$D^0 \to K^+K^-$	1.91 ± 0.02	4.03 ± 0.03	4.08 ± 0.06
	1.91 ± 0.02	4.05 ± 0.05	
$D^0 \to K_SK_S$	0	0.141 ± 0.007	0.141 ± 0.005
	0	0.141 ± 0.007	
$D^+ \to \pi^+\pi^0$	0.89 ± 0.02	0.93 ± 0.02	1.247 ± 0.033
$D^+ \to \pi^+\eta$	1.90 ± 0.16	4.08 ± 0.16	3.77 ± 0.09
$D^+ \to \pi^+\eta'$	4.21 ± 0.12	4.69 ± 0.08	4.97 ± 0.19
$D^+ \to K^+K_S$	2.29 ± 0.09	4.25 ± 0.10	3.04 ± 0.09
$D_s^+ \to \pi^+K_S$	1.20 ± 0.04	1.27 ± 0.04	1.22 ± 0.06
$D_s^+ \to \pi^0K^+$	0.86 ± 0.04	0.56 ± 0.02	0.63 ± 0.21
$D_s^+ \to K^+\eta$	0.91 ± 0.03	0.86 ± 0.03	1.77 ± 0.35
$D_s^+ \to K^+\eta'$	1.23 ± 0.06	1.49 ± 0.08	1.8 ± 0.6

H.-Y. Cheng, C.W. Chiang
Phys.Rev.D 100 (2019) 9, 093002
but see also:
B. Bhattacharya, A. Datta, AAP,
J. Waite, 2107.13564

- Individual asymmetries:

$$a_{CP}^{\rm dir}(\pi^+\pi^-) = (0.80 \pm 0.22) \times 10^{-3},$$

$$a_{CP}^{\rm dir}(K^+K^-) = \begin{cases} (-0.33 \pm 0.14) \times 10^{-3} & \text{Solution I,} \\ (-0.44 \pm 0.12) \times 10^{-3} & \text{Solution II.} \end{cases}$$

- Asymmetry differences

$$\Delta a_{CP}^{\rm dir} = \begin{cases} (-1.14 \pm 0.26) \times 10^{-3} & \text{Solution I,} \\ (-1.25 \pm 0.25) \times 10^{-3} & \text{Solution II.} \end{cases}$$

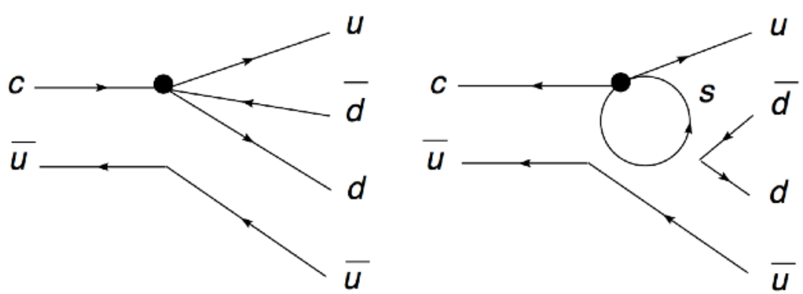
Consistent with Standard Model?

Theoretical troubles

★ But these asymmetries are notoriously difficult to compute

★ In the Standard Model

- need to estimate size of penguin/penguin contractions vs. tree



- unknown penguin contributions
  - SU(3) analysis: some ME are enhanced?

    Golden & Grinstein PLB 222 (1989) 501; Pirtshalava & Uttayarat 1112.5451
  - could expect large $1/m_c$ corrections (E/PE/PA/…)

    Isidori et al PLB 711 (2012) 46; Brod et al 1111.5000
  - flavor-flow diagrams

    Broad et al 1203.6659; Bhattacharya et al PRD 85 (2012) 054014; Cheng & Chiang 1205.0580; 1909.03063; Gronau, Rosner

★ General comments on SU(3)/flavor flow — type analyses

- fit both SM and (possible) NP parts of the amplitudes: can one claim SM-only?
- many parameters: can one claim $O(10^{-4})$ precision if rates are known to $O(10^{-2})$?

★ Need direct calculations of amplitudes/CPV-asymmetries

- QCD sum rule calculations of Δa_{CP} — Khodjamirian, AAP
- SU(3) breaking analyses of $D \to PV, VV$
- constant (but slow) lattice QCD progress in $D \to \pi\pi, \pi\pi\pi$ — Hansen, Sharpe

- Recipe for calculation of CPV asymmetry
  - prepare decay amplitudes (and using $\lambda_d = -(\lambda_s + \lambda_b)$)

$$A(D^0 \to \pi^-\pi^+) = \lambda_d \langle \pi^-\pi^+|\mathcal{O}^d|D^0\rangle + \lambda_s \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$$
$$A(D^0 \to K^-K^+) = \lambda_s \langle K^-K^+|\mathcal{O}^s|D^0\rangle + \lambda_d \langle K^-K^+|\mathcal{O}^d|D^0\rangle$$

  - add and subtract $\lambda_b \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$, put in a new form

$$A(D^0 \to \pi^-\pi^+) = -\lambda_s \mathcal{A}_{\pi\pi} \left[1 + \frac{\lambda_b}{\lambda_s}\left(1 + r_\pi \exp(i\delta_\pi)\right)\right]$$
$$A(D^0 \to K^-K^+) = \lambda_s \mathcal{A}_{KK} \left[1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K)\right]$$

  - define things we cannot compute (extract from branching ratios)

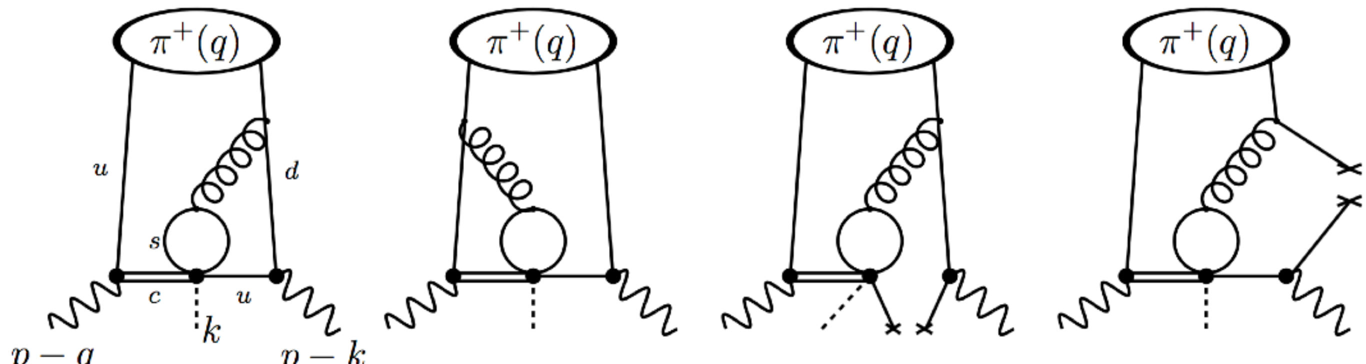
$$\mathcal{A}_{\pi\pi} = \langle \pi^-\pi^+|\mathcal{O}^d|D^0\rangle - \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$$
$$\mathcal{A}_{KK} = \langle K^-K^+|\mathcal{O}^s|D^0\rangle - \langle K^-K^+|\mathcal{O}^d|D^0\rangle$$

  - … and things we can $\mathcal{P}^s_{\pi\pi} = \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$, $\mathcal{P}^d_{KK} = \langle K^-K^+|\mathcal{O}^d|D^0\rangle$

$$r_\pi = \left|\frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right|, \quad r_K = \left|\frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}}\right|$$

dCPV: calculating matrix elements

- Evaluate (leading) diagrams contributing to the correlation function
  - calculate OPE in terms of known LC DAs — Khodjamirian, AAP: PLB774 (2017) 235



(a) (b) (c) (d)

  - extract $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ amplitudes from measured branch. fractions

$$|\mathcal{A}_{\pi\pi}| \simeq \lambda_s^{-1} \left|A(D \to \pi^-\pi^+)\right| = (2.10 \pm 0.02) \times 10^{-6}\ \text{GeV},$$

$$|\mathcal{A}_{KK}| \simeq \lambda_s^{-1} \left|A(D \to K^-K^+)\right| = (3.80 \pm 0.03) \times 10^{-6}\ \text{GeV}.$$

LCSR: predictions

- As a result… $$\langle \pi^+\pi^-|\widetilde{\mathcal{Q}}_2^s|D^0\rangle = (9.50 \pm 1.13)\times 10^{-3}\exp[i(-97.5^o \pm 11.6)]\,\mathrm{GeV}^3$$ $$\langle K^+K^-|\widetilde{\mathcal{Q}}_2^d|D^0\rangle = (13.9 \pm 2.70)\times 10^{-3}\exp[i(-71.6^o \pm 29.5)]\,\mathrm{GeV}^3$$

- Thus, $$r_\pi = \frac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \frac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015$$

  and with $$\Delta a^{dir}_{CP} = -2 r_b \sin\gamma (r_K \sin\delta_K + r_\pi \sin\delta_\pi)$$

- Phases of $r_{\pi\pi(KK)}$ are given by the phases of $\mathcal{P}^{s(d)}_{\pi\pi(KK)}$?

No:

$$\left|a^{dir}_{CP}(\pi^-\pi^+)\right| < 0.012 \pm 0.001\%,$$
$$\left|a^{dir}_{CP}(K^-K^+)\right| < 0.009 \pm 0.002\%,$$
$$\left|\Delta a^{dir}_{CP}\right| < 0.020 \pm 0.003\%\,.$$

Yes:

$$a^{dir}_{CP}(\pi^-\pi^+) = -0.011 \pm 0.001\%,$$
$$a^{dir}_{CP}(K^-K^+) = 0.009 \pm 0.002\%.$$
$$\Delta a^{dir}_{CP} = 0.020 \pm 0.003\%\,.$$

Khodjamirian, AAP: PLB774 (2017) 235

- Again, experiment: $$\Delta a^{dir}_{CP} = (-0.156 \pm 0.029)\%$$

Charming mixing

★ How can one tell that a process is dominated by long-distance or short-distance?

★ To start thing off, mass and lifetime differences of mass eigenstates...

$$x_D = \frac{M_2 - M_1}{\Gamma_D}, \; y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

★ ...can be calculated as real and imaginary parts of a correlation function

$$y_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \,{\rm Im}\, \langle \overline{D^0}| \, i \int {\rm d}^4x \, T\left\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\right\}|D^0\rangle$$

bi-local time-ordered product

$$x_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \,{\rm Re}\, \left[2\langle \overline{D^0}|H^{|\Delta C|=2}\,|D^0\rangle + \langle \overline{D^0}| \, i \int {\rm d}^4x \, T\left\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\right\}|D^0\rangle\right]$$

local operator
(b-quark, NP): small?

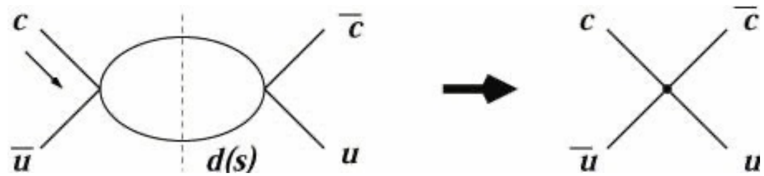
bi-local time-ordered product

★ ... or can be written in terms of hadronic degrees of freedom...

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|\overline{D}^0\rangle + \langle \overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle\right]$$

Mixing: short-distance computation

★ SD calculation: expand the operator product in $1/m_c$, e.g.



★ Note that $1/m_c$ is not small, while factors of m_s make the result small

- keep $V_{ub} \neq 0$, so the leading SU(3)-breaking contribution is suppressed by $\lambda_b^2 \sim \lambda^{10}$
- ... but it is tiny, so look for SU(3)-breaking effects that come from mass insertions and quark condensates

$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right) + 2\lambda_s\lambda_b \left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) - \lambda_b^2 \, \Gamma_{12}^{dd}$$

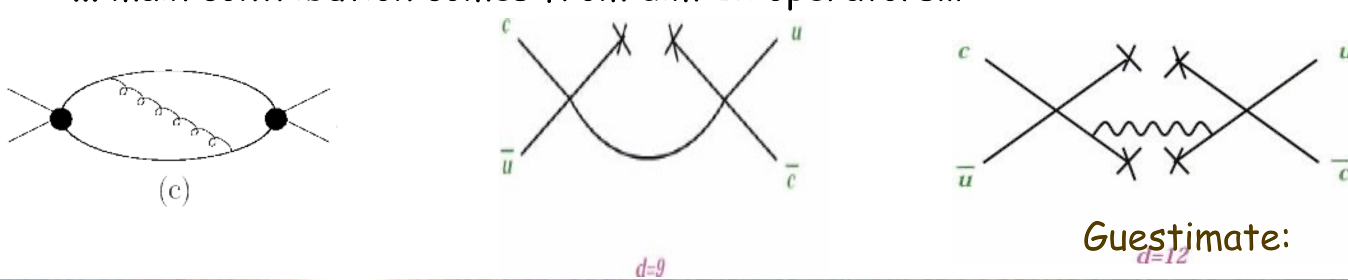
LO:	$O(m_s^4)$	$O(m_s^2)$	$O(1)$
NLO:	$O(m_s^3)$	$O(m_s^1)$	$O(1)$

H. Georgi, …
I. Bigi, N. Uraltsev

M. Bobrowski et al
JHEP 1003 (2010) 009

E. Golowich and A.A.P.
Phys. Lett. B625 (2005) 53

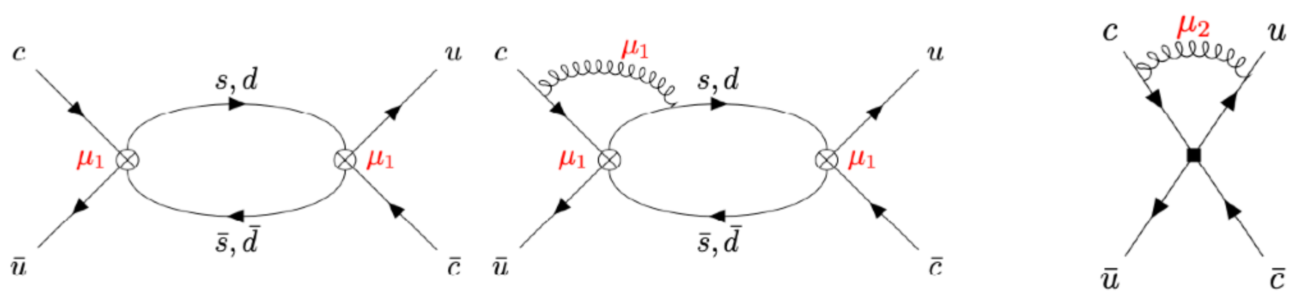
- ... main contribution comes from dim-12 operators!!!



Guestimate: $x \sim y \sim 10^{-3}$?

Scale-setting in charm mixing?

★ SD calculation: non-universal perturbative scales?



(a) (b)

★ Recall: $$\Gamma_{12} = \sum_{q_1 q_2 = ss, sd, dd} \Gamma_3^{q_1 q_2}(\mu_1^{q_1 q_2}, \mu_2^{q_1 q_2}) \langle Q \rangle (\mu_2^{q_1 q_2}) \frac{1}{m_c^3} + \ldots$$

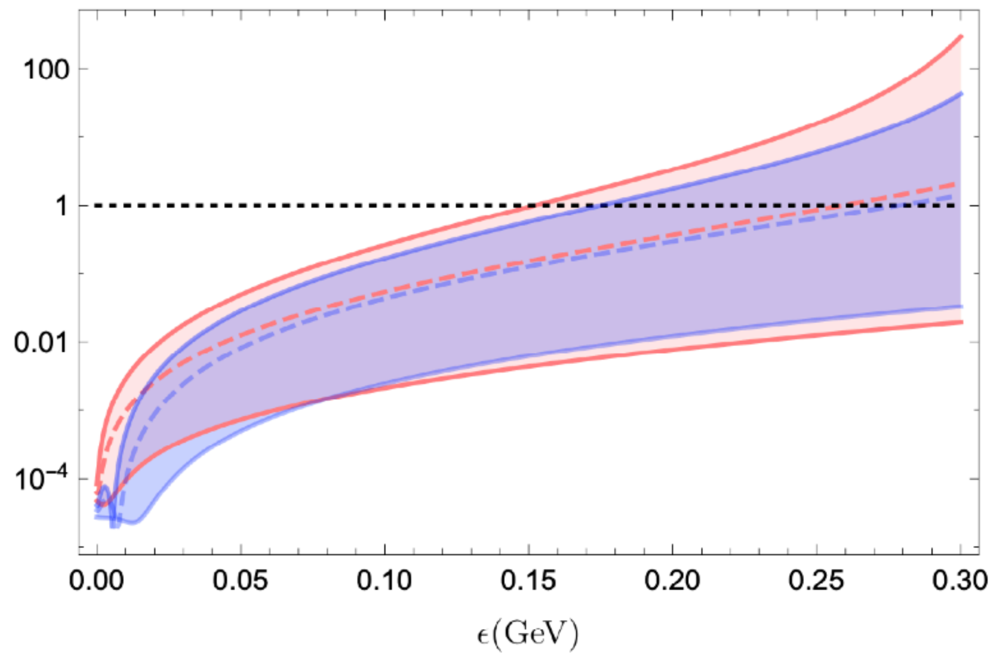
★ Why should the contributions to dd-, sd-, and ss- be evaluated at the same scale?

$$\begin{aligned}\Gamma_{12} &= -\left(\lambda_s^2\, \Gamma_{12}^{ss} + 2\, \lambda_s \lambda_d\, \Gamma_{12}^{sd} + \lambda_d^2\, \Gamma_{12}^{dd}\right) \\ &= -\lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right) \\ &\quad + 2\lambda_s \lambda_b \left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) - \lambda_b^2 \Gamma_{12}^{dd}.\end{aligned}$$

Lenz, Piscopo, Vlahos
arXiv:2007.03022

- Scale uncertainty: variation of scale from M/2 to 2M
  - Try varying scales in dd-, sd-, and ss- independently
  - Try phase-space modulated: $\mu_1^{ss} = \mu - 2\epsilon$ $\mu_1^{sd} = \mu$ $\mu_1^{sd} = \mu - \epsilon$

$$\Omega = \frac{2\,|\Gamma_{12}|^{\rm SM}}{0.028\,{\rm ps}^{-1}}$$



  - Consistent with the Standard Model result! NLO?

Things to take home

- Philosophy: does exclusive approach to mixing constitute a prediction?
- Computation of charm mixing amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
- Charm quark is neither heavy nor light enough for a clean application of well-established techniques
  - "heavy-quark-expansion" techniques miss threshold effects
  - "heavy-quark" techniques give numerically leading contribution that is parametrically suppressed by $1/m^6$
  - "hadronic" techniques need to sum over large number of intermediate states, AND cannot use current experimental data on D-decays
  - "hadronic" techniques currently neglect some sources of SU(3) breaking
  - "quark-level" computation needs to be revisited!

Things to take home

- Theory/Experiment relation:

Theory ✗ **Experiment** ✗ Not a very interesting case…	**Theory** ✗ **Experiment** ✓ SM wins again?
Theory ✓ **Experiment** ✗ SM wins again!	**Theory** ✓ **Experiment** ✓ New Physics!

- Observation of CP-violation in the current round of experiments could have provided a "smoking gun" signals for New Physics
  - But latest LHCb observation seem to be broadly consistent (?) with SM

$$\Delta a_{CP}^{dir} = (-0.156 \pm 0.029)\%$$ LHCB-PAPER-2019-006

  - Maybe if we only have a reliable calculation of the SM effects…

$$\left|\Delta a_{CP}^{dir}\right| < 0.020 \pm 0.003\%$$ Khodjamirian, AAP: PLB774 (2017) 235

$$|\Delta A_{CP}| \le (2.0 \pm 1.0) \times 10^{-4}$$ Chala, Lenz, Rusov, Scholtz: JHEP 1907 (2019) 161


EXTRA!

Experimental analysis from LHCb

★ Since we are comparing rates for D^0 and anti-D^0: need to tag the flavor at production

$$D^{*+} \to D^0 \pi_s^+$$

"D*-trick" -- tag the charge of the slow pion (or muon for D's produced in B-decays)

★ The difference Δa_{CP} is also preferable experimentally, as

$$a_f^{\rm raw} = a_f^{CP} + a_f^{\rm detect,\ D} + a_D^{\rm detect,\ \pi_s} + a_{D*}^{\rm prod}$$

- a_f^{CP}: physics
- $a_f^{\rm detect,\ D}$: detection asymmetry of D^0
- $a_D^{\rm detect,\ \pi_s}$: detection asymmetry of soft pion
- $a_{D*}^{\rm prod}$: production asymmetry of D^{*+}

★ D* production asymmetry and soft pion asymmetries are the same for KK and ππ final states-- they cancel in Δa_{CP}!

★ Integrate over time,

$$a_{CP,\ f} = \int_0^\infty a_{CP}(f;t) D(t) dt = a_f^d + \frac{\langle t \rangle}{\tau} a_f^{ind}$$

$D(t)$: distribution of proper decay time

★ Viola! Report observation!

dCPV: calculating matrix elements

Khodjamirian, NPB 605 (2001) 558

- Use modified light-cone QCD Sum Rule (LCSR) method
  - start with the correlation function ($j_5^{(D)} = im_c\bar{c}\gamma_5 u$ and $j_{\alpha 5}^{(\pi)} = \bar{d}\gamma_\alpha\gamma_5 u$)

$$F_\alpha(p,q,k) = i^2 \int d^4x e^{-i(p-q)x} \int d^4y e^{i(p-k)y} \langle 0|\, T\left\{ j_{\alpha 5}^{(\pi)}(y) \mathcal{Q}_1^s(0) j_5^{(D)}(x) \right\} |\pi^+(q)\rangle$$
$$= (p-k)_\alpha F((p-k)^2, (p-q)^2, P^2) + \dots,$$

  - use dispersion relation in (p-k) and (p-q), perform Borel transform, extract matrix element:

Khodjamirian, Mannel, Melic, PLB571 (2003) 75

$$\langle \pi^-(-q)\pi^+(p)|\mathcal{Q}_1^s|D^0(p-q)\rangle = \frac{-i}{\pi^2 f_\pi f_D m_D^2} \int_0^{s_0^\pi} ds e^{-s/M_1^2} \int_{m_c^2}^{s_0^D} ds' e^{(m_D^2 - s')/M_2^2} \mathrm{Im}_{s'} \mathrm{Im}_s F(s, s', m_D^2)$$

  - perform LC expansion of F(s, s' m_D^2) to get $\mathcal{P}_{\pi\pi}^s$
  - note that $C_1 \mathcal{Q}_1^s + C_2 \mathcal{Q}_2^s = 2C_1 \widetilde{\mathcal{Q}}_2^s + \left(\frac{C_1}{3} + C_2\right) \mathcal{Q}_2^s$ with $\widetilde{\mathcal{Q}}_2^s = \left(\bar{s}\Gamma_\mu \frac{\lambda^a}{2} s\right)\left(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} c\right)$

thus $\mathcal{P}_{\pi\pi}^s = \frac{2G_F}{\sqrt{2}} C_1 \langle \pi^+\pi^- | \widetilde{\mathcal{Q}}_2^s | D^0 \rangle$

Error budget: parameter uncertainties

Parameter values and references	Parameter rescaled to $\mu = 1.5$ GeV
$\alpha_s(m_Z) = 0.1181 \pm 0.0011$ [6]	0.351
$\bar{m}_c(\bar{m}_c) = 1.27 \pm 0.03$ GeV [6]	1.19 GeV
$\bar{m}_s(2\,\mathrm{GeV}) = 96^{+8}_{-4}\,\mathrm{MeV}$ [6]	105 MeV
$\langle \bar{q}q \rangle(2\,\mathrm{GeV}) = (-276^{+12}_{-10}\,\mathrm{MeV})^3$ [6]	$(-268\,\mathrm{MeV})^3$
$\langle \bar{s}s \rangle = (0.8 \pm 0.3)\langle \bar{q}q \rangle$ [21]	$(-249\ \mathrm{MeV})^3$
$a_2^\pi(1\,\mathrm{GeV}) = 0.17 \pm 0.08$ [22]	0.14
$a_4^\pi(1\,\mathrm{GeV}) = 0.06 \pm 0.10$ [22]	0.045
$\mu_\pi(2\,\mathrm{GeV}) = 2.48 \pm 0.30\,\mathrm{GeV}$ [6]	2.26 GeV
$f_{3\pi}(1\,\mathrm{GeV}) = 0.0045 \pm 0.015\ \mathrm{GeV}^2$ [19]	$0.0036\,\mathrm{GeV}^2$
$\omega_{3\pi}(1\,\mathrm{GeV}) = -1.5 \pm 0.7$ [19]	-1.1
$a_1^K(1\,\mathrm{GeV}) = 0.10 \pm 0.04$ [23]	0.09
$a_2^K(1\,\mathrm{GeV}) = 0.25 \pm 0.15$ [19]	0.21
$\mu_K(2\,\mathrm{GeV}) = 2.47^{+0.19}_{-0.10}$ GeV [6]	2.25
$f_{3K} = f_{3\pi}$	$0.0036\,\mathrm{GeV}^2$
$\omega_{3K}(1\,\mathrm{GeV}) = -1.2 \pm 0.7$ [19]	-0.99
$\lambda_{3K}(1\,\mathrm{GeV}) = 1.6 \pm 0.4$ [19]	1.5