# Emergent Planck mass and dark energy from affine gravity

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# Affine gravity

Types of gravity, based on independent variables:

- Metric:
  *GR*
  $$g_{\mu\nu}\,, \quad \Gamma^{\lambda}_{\mu\nu} = \left\{ {}^{\lambda}_{\mu\nu} \right\}\,.$$
- Metric-affine:
  *Palatini approach*
  $$g_{\mu\nu}\,, \quad \Gamma^{\lambda}_{\mu\nu}\,.$$
- Affine:
  *Einstein, Eddington, Schrodinger, ...*
  $$\cancel{g_{\mu\nu}}\,, \quad \Gamma^{\lambda}_{\mu\nu}\,.$$

# Affine gravity

Lagrangian of the affine gravity:

$$\mathcal{L} = \sqrt{-\det\left(R_{(\mu\nu)}(\Gamma)\right)}$$

Affine gravity is equivalent to GR with $\Lambda \neq 0$:

$$\sqrt{-g}(M_{pl}^2 R + 2\Lambda) \quad \longleftrightarrow \quad 2\frac{M_{pl}^4}{\Lambda}\sqrt{-\det R_{\mu\nu}}$$

# Affine gravity

Metric is the momentum, canonically conjugated to the connection:

$$\mathfrak{g}_{\lambda}^{\mu\nu\rho} = \frac{\delta\mathcal{L}}{\delta\partial_\rho\Gamma^{\lambda}_{\mu\nu}} \,, \qquad \mathfrak{g}^{\mu\nu} = \frac{\delta\mathcal{L}}{\delta R_{(\mu\nu)}} \,.$$

Lagrange formulation: $\mathcal{L}\,,\quad d\mathcal{L} = J_{\lambda}^{\mu\nu} d\Gamma^{\lambda}_{\mu\nu} + \mathfrak{g}^{\mu\nu} dR_{(\mu\nu)}$

Hamiltonian formulation: $L = \mathcal{L} - \mathfrak{g}^{\mu\nu} R_{(\mu\nu)}\,,$

$$dL = J_{\lambda}^{\mu\nu} d\Gamma^{\lambda}_{\mu\nu} - R_{(\mu\nu)} d\mathfrak{g}^{\mu\nu} \,, \qquad R_{(\mu\nu)} = -\frac{\delta L}{\delta\mathfrak{g}^{\mu\nu}}$$

# Pros and cons of affine gravity

- Natural from geometrical viewpoint
- Allows for a wide range of modifications
- Unified description of SM (fermions) and gravity

# Scalar–affine gravity

$$\varphi: \qquad \varphi \to J\varphi\,, \quad \nabla_\mu \varphi = \partial_\mu \varphi - \Gamma^\sigma_{\mu\sigma}\varphi \quad (w=1)\,.$$

Lagrangian: $\mathcal{L} = \sqrt{-\det L_{\mu\nu}} + \varphi\,, \qquad L_{\mu\nu} = R_{(\mu\nu)} - \frac{3}{2}\frac{\nabla_\mu\varphi\nabla_\nu\varphi}{\varphi^2}$

Model is scale invariant (no dimensionfull parameters)

Spontaneous symmetry breaking:

$$-\nabla_\lambda\left(\mathfrak{g}^{\mu\nu}\frac{\delta L_{\mu\nu}}{\delta\nabla_\lambda\varphi}\right) + \mathfrak{g}^{\mu\nu}\frac{\delta L_{\mu\nu}}{\delta\varphi} + 1 = 0$$

$$\Downarrow$$

$$\varphi = \varphi_0^4 e^\pi\,, \quad \mathfrak{g}^{\mu\nu} = -\varphi_0^2\sqrt{-g}\,g^{\mu\nu}\,, \quad \varphi_0 \neq 0\,, \quad g_{\mu\nu} \neq 0\,.$$

# Scalar–affine gravity

Vacuum solution:

$$g_{\mu\nu} = \mathrm{diag}\left(-1, a^2(t), a^2(t), a^2(t)\right)$$

$$\Gamma^0_{ij} = a^2(H + 3v)\gamma_{ij}\,, \quad \Gamma^i_{0j} = (H + v)\delta^i_j\,, \quad \Gamma^0_{00} = -v\,, \quad H \equiv \frac{a'}{a}\,,$$

$$6\left(3Hv + v'\right) = -\varkappa\varphi_0^2\,.$$

Master equation:

$$\left(\frac{H}{M_{pl}}\right)^2 = \frac{\varkappa + 4}{6}\,.$$

# Conclusion

Scalar–affine gravity:

- realizes the no–scale scenario
- yields the emergent Plank mass,
- scalar $\varphi$ acting as dark energy
- is phenomenologically viable (Newton's law and Linearized limit)