









# Polarised Study of Diboson Production at NNLO

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#### XII Lomonosov Conference

https://lomcon.ru Moscow State University

Tuesday 24<sup>th</sup> August, 2021

#### Outline of the talk

#### **1. QCD Precision Studies**

Cross section at hadron collider

#### 2. Process

Motivation

Aspects of diboson production

Double-pole approximation

#### 3. Calculation

Setup

NNLO corrections

Loop-induced channel corrections

**Polarisation fractions** 

#### 4. Conclusions

Future plans

**QCD Precision Studies** 

#### Master formula for collinear factorisation:



Figure 1: Parton model description of a hard scattering process (Ellis, Stirling, Webber).



$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \hat{\sigma}_{ij}^{(1)} + \hat{\sigma}_{ij}^{(2)} + \dots$$

The difficulty comes not from UV but IR divergencies.

# Example with quark production (final state radiation):



#### **Kinoshita-Lee-Neuberger theorem**

Regularised virtual poles cancel against real radiation singularities upon inclusive integration of the emission phase space.

Perturbative expansion for  $\sigma_{ab}$  :

$$\begin{split} \hat{\sigma}_{ab}^{(0)} &\sim \int d\Phi_n \left\langle M_n^{(0)} \middle| M_n^{(0)} \right\rangle F_n \qquad \hat{\sigma}_{ab}^{(1)} \sim \int d\Phi_{n+1} \left\langle M_{n+1}^{(0)} \middle| M_{n+1}^{(0)} \right\rangle F_{n+1} + 2 \int d\Phi_n \operatorname{Re} \left\langle M_n^{(0)} \middle| M_n^{(1)} \right\rangle F_n \\ \hat{\sigma}_{ab}^{(2)} &\sim \int d\Phi_{n+2} \left\langle M_{n+2}^{(0)} \middle| M_{n+2}^{(0)} \right\rangle F_{n+2} + 2 \int d\Phi_n \operatorname{Re} \left\langle M_n^{(0)} \middle| M_{n+1}^{(2)} \right\rangle F_n + \int d\Phi_{n+1} \left\langle M_{n+1}^{(1)} \middle| M_{n+1}^{(1)} \right\rangle F_{n+1} \end{split}$$

### **Higher-order corrections**

#### Benefits:



• Significant reduction of scale uncertainty (to 1% at NNLO)

#### Challenges:

- Computational complexity (100-1000x compared to NLO)
- Some schemes tackle only a subset of processes

#### Fact

It took a long time to establish NNLO precision as standard: 2-loop amplitudes for dijet process were available 20 years before cross-section prediction arrived. Theoretical uncertainty is estimated through "7-point scale variation"

$$\frac{1}{2} < \mu_R, \mu_F < 2.$$

#### Common differential observables:

- rapidity (  $y = \frac{1}{2} \ln \left( \frac{E + p_z}{E p_z} \right)$  )
- transverse momentum ( $p_T$ )
- invariant masses
- distance between particles  $dR = \sqrt{y^2 + \Delta \phi^2}$
- event shapes (thrust, N-jettiness, ...): to describe geometry of a group of particles

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#### "Sector-improved residue subtraction scheme"

- Project leader Michal Czakon [1005.0274, 1408.2500]
- **Contributors** Arnd Behring, David Heymes, Alexander Mitov, Andrew Papanastasiou, Mathieu Pellen, Rene Poncelet, A.P.
  - Features ... local numerical cancellation of poles
    - · versatile scheme: any process possible
    - · implemented in C++ library STRIPPER

  - **Processes**  $t\bar{t}$  [1901.05407. 2008.11133].
    - · dijet [1907.12911] .
    - · 3-photon [1911.00479].
    - · V+c [2011.01011],
    - · 2-photon+iet [2105.06940].
    - · 3-iet [2106.05331].

Process

**Longitudinal polarisation** and massiveness of  $W^{\pm}, Z$  bosons is the direct consequence of the Electroweak symmetry breaking mechanism in the Standard Model.



<u>Features</u> of  $pp \rightarrow W^+ W^- \rightarrow e^+ \nu_e \mu^- \overline{\nu}_{\mu}$ :

- Leptonic decay channel is a clean experimental signature
- Largest  $\sigma$  among diboson processes
- Luminosities of Run 2/3 will allow for precise measurement

#### Relevant theoretical papers:

- Seminal papers on W-boson polarisation [Bern et al. 1103.5445]
   [Stirling et al. 1204.6427]
- Polarised diboson production at NLO QCD [Denner et al. 2006.14867, 2010.07149]
- Double pole approximation (DPA)
   [Billoni et al. 1310.1564]
   [Ballestrero et al. 1710.09339, 1907.04722]
- Off-shell W<sup>+</sup> W<sup>-</sup> production up to NNLO QCD + EW NLO [Caola et al. 1511.08617]
   [Grazzini et al. 1605.02716, 1912.00068]
   [Lombardi et al. 2103.12077]

#### Polarised diboson production: technical aspects



- 1. On-shell amplitudes:
  - → polarisation is defined for on-shell bosons;
  - → non-resonant background effects due to missing SR amplitudes;
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- 2. Interference between polarisations:
  - $\rightarrow$  caused by **cross terms**  $\mathcal{A}^*_{\lambda} \mathcal{A}_{\lambda'}$  (polarisation information lost after leptonic decay);
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- 3. Loop-induced channel:
  - → significant effects and **dominating** scale uncertainty at  $\mathcal{O}(\alpha_s^2)$ .



- A selected on-shell projection defined on-shell sub-amplitudes (we choose to preserve leptonic angles in the decay frames, and boson angles in the diboson frame).
- Cross-term amplitude contributions coming from  $A_{\alpha}A_{\tilde{\alpha}}$  terms create **interferences** for cross sections.

# Calculation

**Process:**  $pp \rightarrow W^+ W^- \rightarrow e^+ \nu_e \mu^- \overline{\nu}_{\mu}$  @ 13 TeV.

**Details:** Massive *b*-quarks scheme (*Nf*=4),  $G_{\mu}$ -scheme, complex-mass scheme.

**PDF sets:** NNPDF31\_[n]nlo\_as\_0118 (5-flavour).

Scales: fixed central scale  $\mu_R = \mu_F = M_W$ , 7-point variation scheme with  $1/2 \le \mu_R/\mu_F \le 2$ .

**Cuts:** ATLAS-inspired<sup>1</sup> fiducial setup:

$p_{Tmiss}>20{ m GeV}$	to avoid D-Y background
$M_{e^+\mu^-} > 55{ m GeV}$	to avoid Higgs background
perfect b-quark jet veto	to avoid $t\overline{t}$ background
$p_{T,l} > 27  { m GeV},   y_l  < 2.5$	detector cuts
jet veto: $\left \eta_{j} ight  < 4.5$ , $p_{T,j} > 35{ m GeV}$	to reduce QCD corrections

Finally, an **implicit cut** on  $M_{W^+,W^-} > 2M_W$  comes from double resonant parts in DPA, NWA.

<sup>&</sup>lt;sup>1</sup>[Aaboud et al. 1902.05759]









#### Features:

Polarisation interference
 Non-resonant background



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(2)

3 "Monte-Carlo true" polarisation distributions



# 

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 $\overline{2}$ 

1

62

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#### Summary:

- → NNLO effects are **2-3%** of  $\sigma_{tot}$  for all setups except  $W_L^+ W_L^-$  where it is **9%**.
- → Scale uncertainty is reduced by a factor of 3 w.r.t NLO.



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0 100 200 300



100 200 300

 $p_T(e^+)$ 

Ó.

100 200 300

NNLO

- NNLO+LI

Ô.

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#### Summary:

- → Corrections of 6-9% to  $\sigma_{tot}$ .
- → Overall scale uncertainty increased by a factor of 2.
- → Correction profile does not follow NNLO K-factor.

In our fiducial setup, interferences are small (2 - 3%), allowing for polarisation fraction extraction:

$$f_i = \frac{\sigma_i}{\sigma_{tot}}.$$

- +  $W_L^+ W_L^-$  setup is significantly affected by NNLO corrections, others are stable;
- Loop-induced channel affects  $W_T^+ W_T^-$  setup the most.



# Conclusions

We studied W-boson polarisation in the fully leptonic channel of  $pp \to W^+ W^-$  process. Polarisations were separated on the amplitude level in the framework of DPA.

- NNLO corrections bring scale uncertainty down to 1% and are well-behaved.
- QCD corrections are polarisation dependent and are particularly strong for the doubly-longitudinal setup.
- Loop-induced channel has a 6 9% effect on the results and increases scale uncertainty by a factor of 2.

# Future plans:

- Calculate NLO corrections to LI channel in diboson production.
- Study polarised W + j process at NNLO.



From polarised study of W + j at NNLO [in preparation]

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Backup

- STRIPPER: general purpose framework for fixed-order calculations up to NNLO QCD. [Czakon et al. 1907.12911, 1408.2500]
- AvH: tree-level amplitudes.
   [Bury, van Hameren 1503.08612]
- OPENLOOPS: 1-loop amplitudes (privately modified for polarised study). [Buccioni et al. 1907.13071, 1710.11452] [Cascioli et al. 1111.5206]
- RECOLA: checks in 1-loop amplitudes (private version used by authors of the NLO study). [Actis et al. 1211.6316, 1605.01090]
- ◊ VVAMP: 2-loop amplitudes for q q channel. [Gehrmann et al. 1503.04812]
- LHAPDF: particle distribution functions framework. [Buckley et al. 1412.7420]

#### Brief look at competing schemes:

- Slicing method:
  - resummation formulae are used to approximate divergent phase-space regions within a small cutoff;
  - q<sub>T</sub> subtraction (MATRIX code) and N-jettiness use corresponding observables as cut-off variables;
  - · used for diboson and boson+jet production;
- Subtraction method:
  - Fully differential subtraction with numerically integrated subtraction terms;
  - Antenna subtraction first used as NLO scheme, promoted to NNLO;
  - CoLoRFulNNLO developed for colourless initial states;
  - Local analytic sector subtraction with analytic counterterms;
  - used for e.g. 2-jet, V+j,  $t\overline{t}\text{, }e^+e^- \rightarrow 3\text{j}.$

#### "Sector-improved residue subtraction scheme"

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 $\mathbb{P}_{\mu}$ 

Double pole approximation:

$$\begin{split} \mathcal{A}_{\lambda} &= \mathcal{P}_{\mu} \frac{\varepsilon_{\lambda}^{\mu} \varepsilon_{\lambda}^{\nu*}}{k^2 - M_V^2 + i M_V \Gamma_V} \mathcal{D}_{\nu} \\ \mathcal{M} &= \sum_{\lambda} |A_{\lambda}|^2 + \sum_{\lambda \neq \lambda'} A_{\lambda}^* A_{\lambda'} \end{split}$$

Polarisation only defined for physical vectors (present physical vectors), so define on-shell projection (OSP) for kinematics and evaluate  $\mathcal{P}, \mathcal{D}$  at this point. Narrow width approximation:

$$\mathcal{M}_{\mathrm{pp}\to\mathrm{e}^+\nu\mathrm{e}\mu^-\bar\nu\mu}\sim\sum_{h,h'\in\Lambda}\mathcal{M}_{\mathrm{pp}\to\mathrm{W}^+\mathrm{W}^-}^{h,h'}\Gamma^h_{\mathrm{W}^+\to\mathrm{e}^+\nu\mathrm{e}}\Gamma^{h'}_{\mathrm{W}^-\to\mu^-\bar\nu\mu}.$$

Precision of a method that uses on-shell amplitudes is of  $\mathcal{O}(\Gamma_W/M_W)$  for inclusive computation.

Beware of interference terms in the unpolarised case:

$$\left|M\right|^{2} = \sum_{\lambda} \left|M_{\lambda}\right|^{2} + \sum_{\lambda \neq \lambda'} M_{\lambda}^{*} M_{\lambda'}$$

Analytic result for polarised massive vector boson decay in its CM frame:

$$\frac{1}{\sigma} \frac{d^2 \sigma}{d \cos \theta^* d \phi^*} = \frac{3}{16\pi} \Big[ (1 + \cos^2 \theta^*) + A_0 \frac{1}{2} (1 - 3\cos^2 \theta) + A_1 \sin(2\theta^*) \cos \phi^* + A_2 \frac{1}{2} \sin^2 \theta^* \cos(2\phi^*) \\ + A_3 \sin \theta^* \cos \phi^* + A_4 \cos \theta^* + A_5 \sin^2 \theta^* \sin(2\phi^*) + A_6 \sin(2\theta^*) \sin \phi^* + A_7 \sin \theta^* \sin \phi^* \Big]$$

(coefficients depend on the choice of a coordinate system, e.g "helicity", "Collins-Soper", etc) In case of inclusive phase space azimuthal angle can be integrated out:

$$\frac{1}{\sigma} \frac{d\sigma}{d(\cos \theta^*)} = \frac{3}{8} (1 \mp \cos \theta^*)^2 f_- + \frac{3}{8} (1 \pm \cos \theta^*)^2 f_+ + \frac{3}{4} \sin^2 \theta^* f_L$$

where

$$f_{\pm} = \frac{1}{4}(2 - A_0 \pm A_4), \qquad f_0 = \frac{1}{2}A_0.$$

or through angular measurements:

$$f_{\pm} = -\frac{1}{2} \pm \langle \cos \theta^* \rangle + \frac{5}{2} \langle \cos^2 \theta^* \rangle, \qquad f_0 = 2 - 5 \langle \cos^2 \theta^* \rangle.$$

	$\frac{\sigma_{NLO}}{\sigma_{LO}}$	$\frac{\sigma_{NNLO}}{\sigma_{NLO}}$	NNLO+LI [fb]	$\frac{\sigma_{NNLO+LI}}{\sigma_{NLO}}$
unpol. (dpa)	1.095	1.023	$232.7(4)^{+1.4\%}_{-1.1\%}$	1.061
unpol. (nwa)	1.097	1.025	$241.0(6)^{+1.5\%}_{-1.1\%}$	1.060

# DPA vs NWA: positron emission angle



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- Matching description of normalised bulk-defined distributions between DPA & NWA.

#### DPA vs NWA: leading lepton transverse momentum



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#### DPA vs NWA: azimuthal angle between leptons



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- Close K-factors for all QCD corrections.
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- Differences come from the bulk region.
- Interesting interplay in azimuthal angle.
- ⇒ Overall **good description** of differential distributions **by NWA**.

#### Top-quark loop contribution in LI channel: off-shell setup



- Main effect in the tail:
  - $\rightarrow$  value increase by up to 8%
  - ightarrow scale variation band increased by 30%
- Effect on total cross section:
  - ightarrow cross section increased by 0.6%
  - $\rightarrow$  scale uncertainty increased by 7%

# Top-quark loop contribution in LI channel: bulk observable



No significant effect on bulk observables from top-quark loop.

#### Effects of the PDF set



**Figure 5:** Comparison between calculations with nf = 4 and nf = 5 PDF sets. Uncertainty bands correspond to **factorisation** scale uncertainty.

- the discrepancy falls withing this band at NLO;
- total cross-section effect: < 0.6% (largest for LL).

#### Distribution poorly suited for polarisation study



Single resonant contributions are dominating at high  $p_{T,miss}$ .



Figure 6: Dominating single resonant contribution at high  $p_T$  [Biedermann et al. 1605.03419]

# Extra figures: $M_{e^+,\mu^-}$



Extra figures:  $\cos \theta_{e^+,\mu^-}$ 



#### **Extra figures:** $y_e^+$



#### Extra figures: $e^+$ vs $\mu^-$



Positron transverse momentum

 $\log_{10} \left| \frac{\sigma_{p_T}(\mathbf{e}^+) - \sigma_{p_T}(\mu)}{\sigma_{p_T}(\mathbf{e}^+) + \sigma_{p_T}(\mu)} \right|$ 

1.00

# Extra figures: $\phi_{e^+,\mu}^-$



LI channel has large overall shift in TT and unpolarised setups. Interesting shape in LL setup.

#### Effects of LI channel: positron transverse momentum



- Sizeable increase of scale uncertainty in the tail, particularly in longitudinal setups
- Corrections are polarisation-dependent:
  - ightarrow in the bulk for transverse setups
  - $\rightarrow\,$  in the tail for longitudinal setups
  - ightarrow in both places for LL setup