School of particle and accelerator

Department of Theoretical Physics

Nuclear parton distribution functions with uncertainties in a general mass variable flavor number scheme

Hamzeh Khanpour

In collaboration with

Maryam Soleymaninia (IPM), Shahin Atashbar Tehrani (IPM),
Hubert Spiesberger (Mainz) and Vadim Guzey (PNPI)

arXiv:2010.00555 [hep-ph]; Phys. Rev. D **104** (2021) 034010

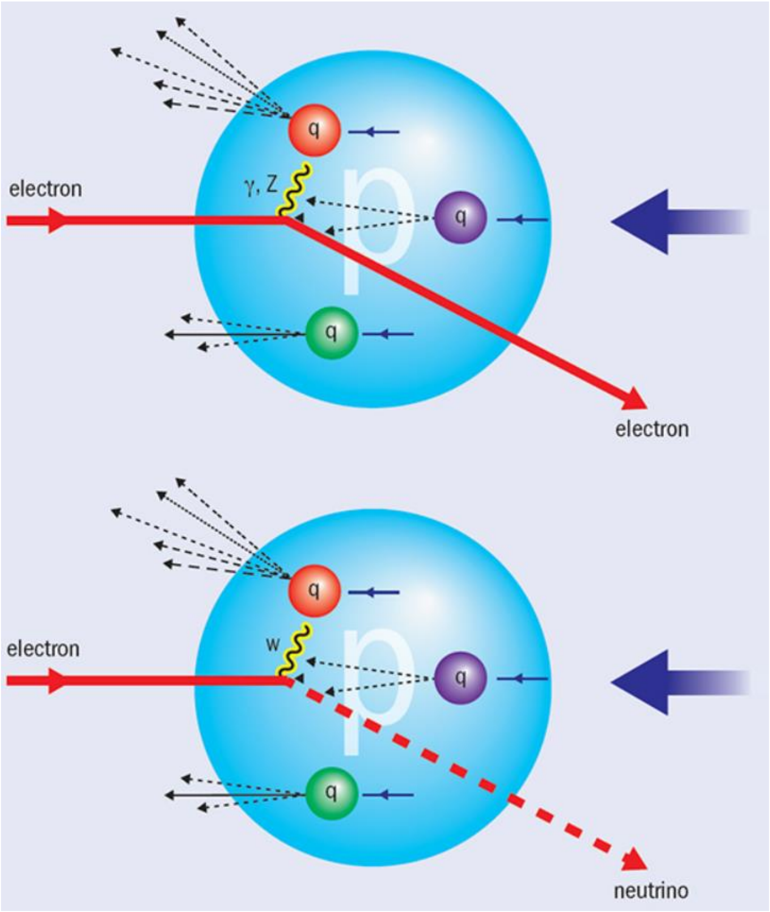
20th Lomonosov Conferences
on Elementary Particle Physics, 23 August 2021, Moscow, **Russia**

Outline

- ❖ Short Introduction
- ❖ Highlights from the nuclear DIS experiments
- ❖ A review on available nuclear PDFs analyses

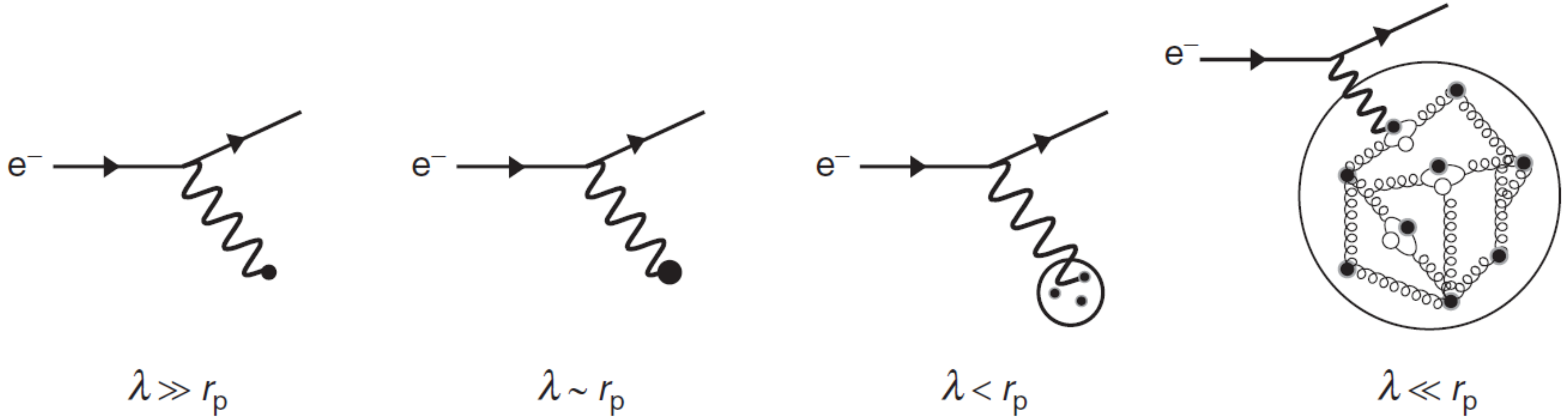
- ❖ Parametrization of KSASG20 nuclear PDFs
- ❖ Experimental data sets used in KSASG20 nuclear PDFs
- ❖ χ^2 analysis method and uncertainty determination

- ❖ KSASG20 results of nuclear PDFs
- ❖ Summary and conclusions



DIS and probing the structure of the proton

- Electron-proton scattering provides a powerful tool for probing the structure of the proton. At low energies, the dominant process is elastic scattering where the proton remains intact. Elastic scattering is described by the interaction of a virtual photon with the proton as a whole, and thus provides a probe of the global properties of the proton, such as its charge radius.
- At high energies, the dominant process is deep inelastic scattering, where the proton breaks up. Here the underlying process is the elastic scattering of the electron from one of the quarks within the proton. Consequently, DIS provides a probe of the momentum distribution of the quarks.



The nature of e⁻p scattering depending on the wavelength of the virtual photon.

$$x = \left(\frac{Q^2}{2p \cdot q}\right) \quad 0 \le x \le 1.$$

$$Q^2 = -q^2$$

Inclusive DIS kinematics

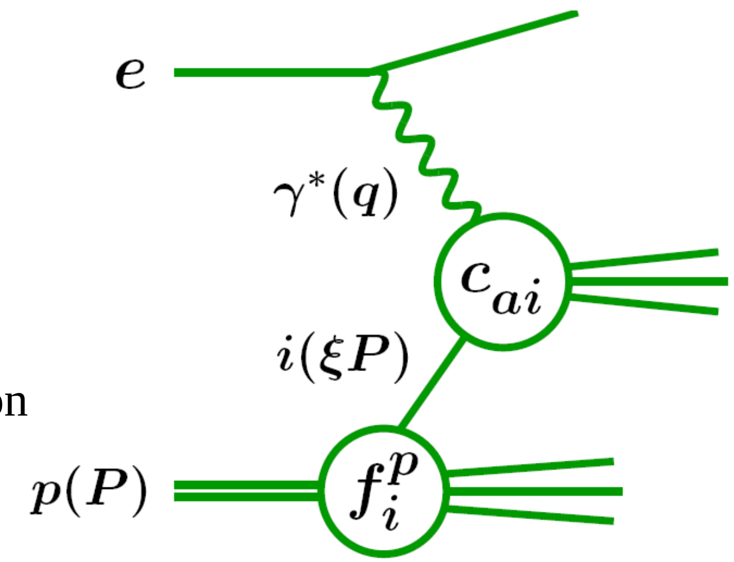
The Quark Parton Model

- The basic idea of the QPM is that in the DIS process, $ep \to eX$, the virtual proton interacts with one of the quark constituents of the proton.
- As a result the *ep* interaction may be written as a sum (of probabilities) of scattering from single free quarks.

$$\frac{d\sigma}{dx dQ^2} = \sum_q \int_0^1 d\xi \; f_q(\xi) \left(\frac{d\hat{\sigma}_{eq}}{dx dQ^2} \right)$$

$f_q(\xi)$ is the probability of finding the quark q in the proton carrying a fraction ξ of its momentum

electron-quark cross section



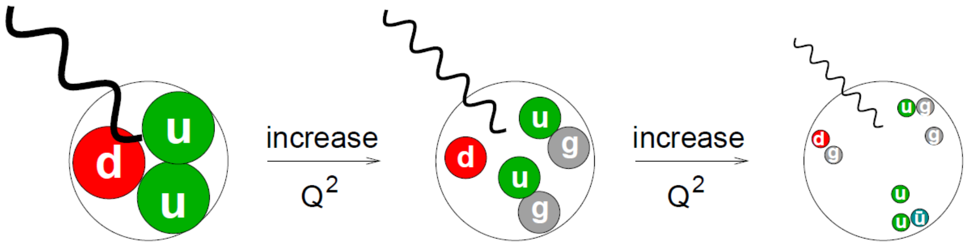
Inclusive DIS structure functions

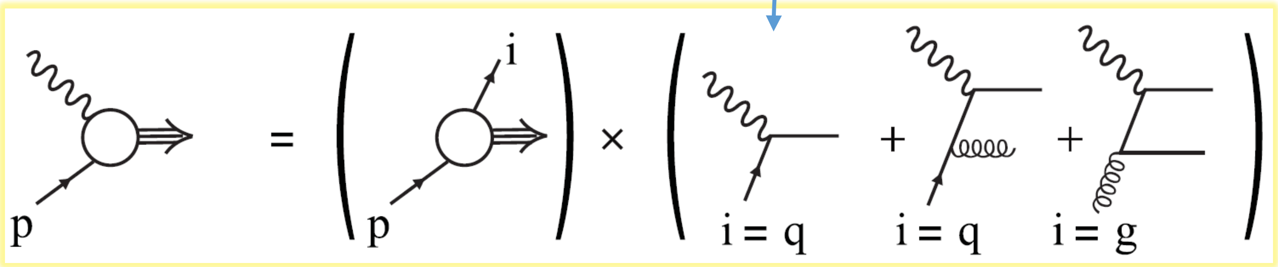
- The structure functions F_a as a physical observables describe the DIS processes.
- For the structure functions F_a, describing the deep inelastic processes $\ell + p \longrightarrow \ell' + X$, the factorization formula has the following form:

$$F_a(x, Q^2) \;=\; \sum_{i=q,\bar{q},g} \int_0^1 \frac{dy}{y} \; f_i(y, Q^2)\; C_{a,i}\left(\frac{x}{y}, \alpha_s(Q^2)\right)$$

Universal parton densities (of the proton). They cannot be calculated in perturbative QCD, but their Q^2 dependence is calculable using the DGLAP evolution equations

Coefficient functions. They are calculable from perturbative QCD as a power series in α_s, but are unique to the particular observable, F_a.





DIS experiments on nuclear targets

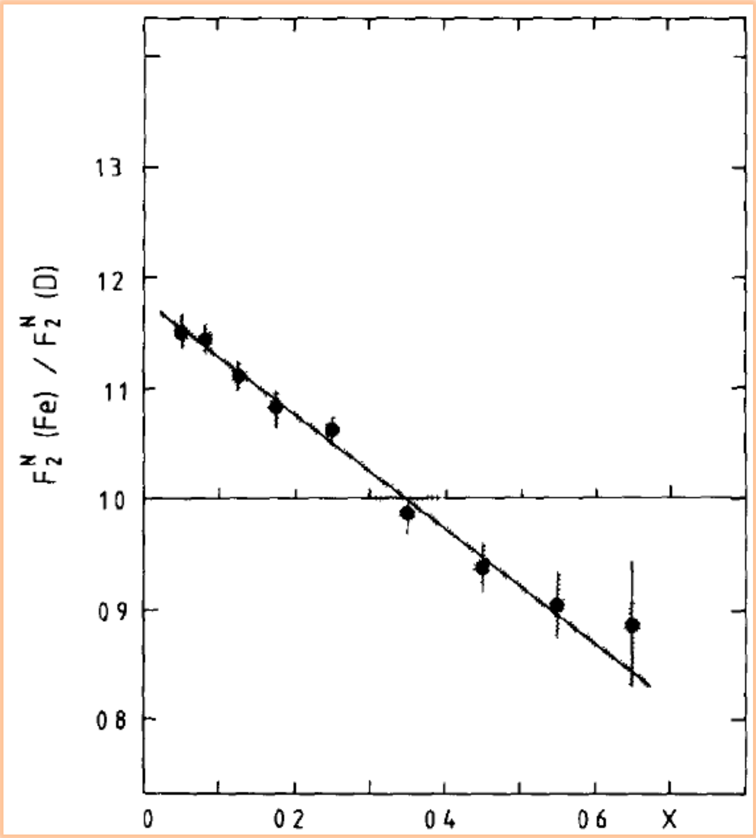
Nuclear EMC effect

CERNCOURIER | Reporting on international high-energy physics

Physics ▾ | Technology ▾ | Community ▾ | In focus | Magazine

SCIENTIFIC PRACTICE | FEATURE

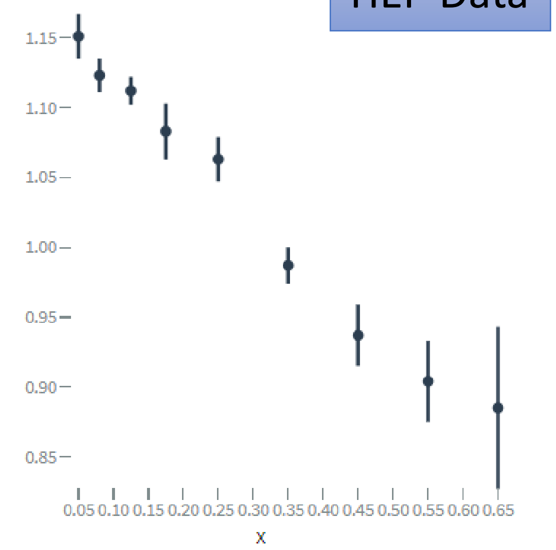
The EMC effect still puzzles after 30 years



RE(Q=DEUT)	MU+ DEUT --> MU+ X
RE(Q=FE)	MU+ FE --> MU+ X
SQRT(S)	0.0 GeV
X	F2(Q=FE)/F2(Q=DEUT)
0.05	1.151 ±0.016
0.08	1.123 ±0.012
0.125	1.112 ±0.01
0.175	1.083 ±0.02
0.25	1.063 ±0.016
0.35	0.987 ±0.013
0.45	0.937 ±0.022
0.55	0.904 ±0.029
0.65	0.885 ±0.058

Visualize

HEP Data



Sum errors Log Scale (X) Log Scale (Y)

Deselect variables or hide different error bars by clicking on them.

This effect was first observed in 1983 at CERN by the European Muon Collaboration, hence the name "EMC effect". The observed *x*-dependence of this ratio is in disagreement with existing theoretical predictions.

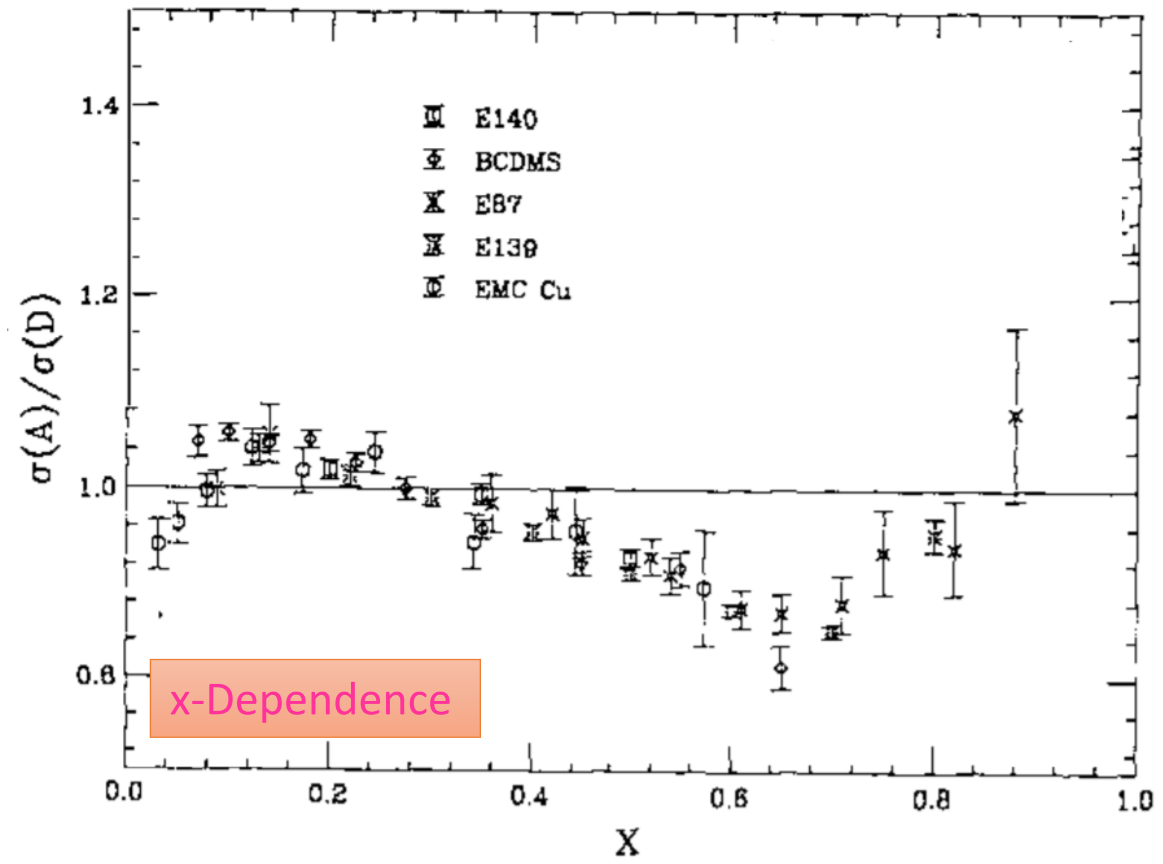
The ratio of the nucleon structure functions F2(Fe)/F2(D) for iron and deuterium
European Muon Collaboration, Published in Phys. Lett. B123 (1983) 275-278, CERN-EP/83-14

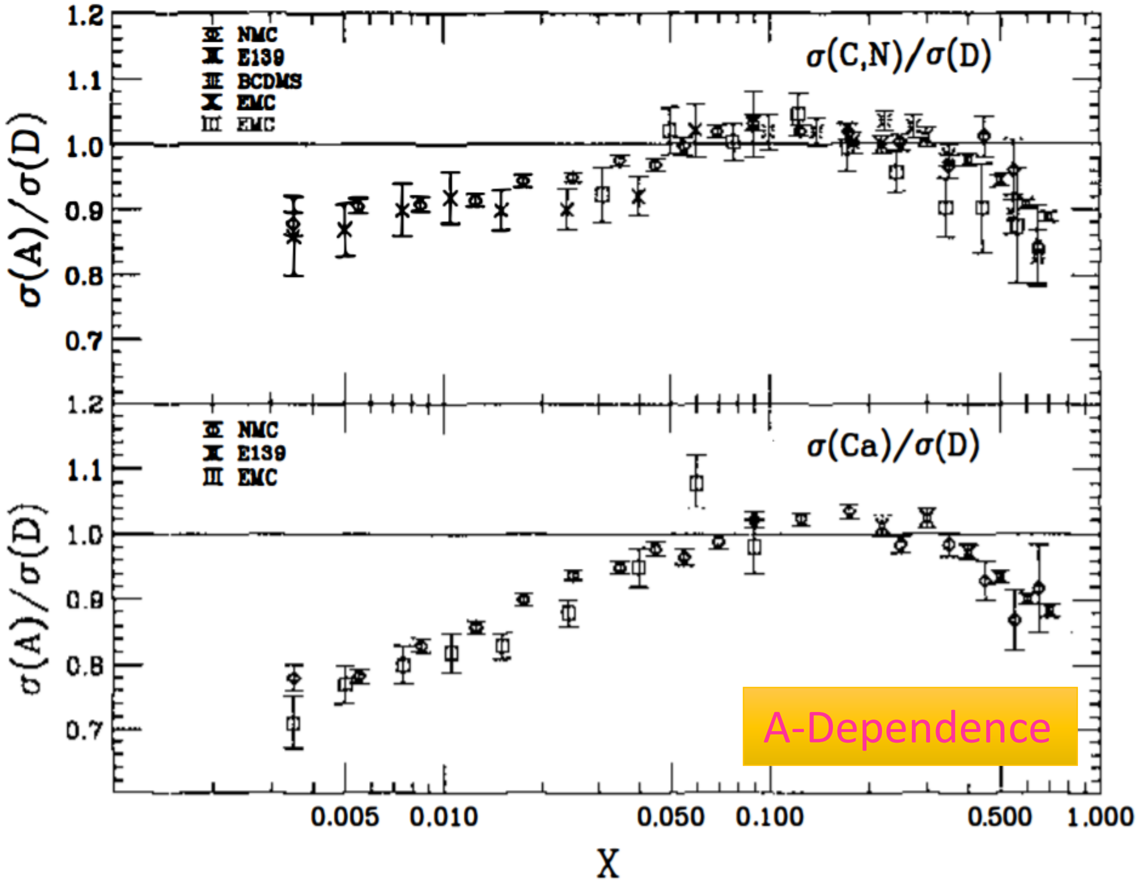
Experimental efforts

Laboratory/ collaboration	Beam	Energy (GeV)	Measurement	Target	Reference	Year
SLAC E139	e	8-24.5	σ	D, ^{4}He, Be, C, Ca, Fe, Ag, Au	(2, 124)	1994, 1984
SLAC NE-11		1.5–9.8	νW_2, R $x \sim 1$	Al	(125)	1992
SLAC E140		3.75–19.5	R_a-R_b, σ_a/σ_b	D, Fe, Au	(3, 9)	1988
SLAC		8–24.5	σ	H, D	(126, 127)	1992, 1990
CERN NMC	■	90,280	σ	H, D	(128)	1992
		90,280	ΔR	D to H, Ca to C	(129)	1992
		90	σ_A/σ_{Li}	^{6}Li, ^{12}C, ^{40}Ca	(130)	1992
		200,280	$\sigma_{J/\psi}$	Sn/C	(131)	1991
		200	σ_A/σ_D	D, ^{4}He, C, Ca	(132)	1991
		90–280	σ_D/σ_H	D, H	(133)	1991
		90–280	σ_D/σ_H	D, H	(134)	1994
CERN BCDMS	μ	200	$\sigma_C(x \sim 1)$	C	(135)	1994
		120,200,280	σ_D, σ_H	H, D	(136)	1990
		200	σ_A/σ_D	Fe, D	(4)	1987
		120,200,280	σ_C, R	C	(137)	1987
		280	σ_A/σ_D	D, N, Fe	(138)	1985
CERN EMC	μ	100–280	σ_A/σ_D	Cu, D	(5)	1993
		280	σ_A/σ_D	D, C, Ca	(139)	1988
		100–280	σ_A/σ_D	D, C, Cu, Sn	(6)	1988
		280	σ_A/σ_D	H, D, Fe	(140)	1987
		120–280	σ_A	Fe	(141)	1986
		100–280	σ_A/σ_D	D, Fe	(1)	1983
FNAL E665	■	490	Final State	D, Xe	(142)	1995
		490	Final State	D, Xe	(143)	1994
		470	σ_D/σ_H	H, D	(144)	1995
		490	σ_A/σ_D	D, Xe	(145)	1992
		490	σ_A/σ_D	D, Xe	(146)	1992

Annu. Rev. Nucl. Part. Sci. 1995.45:337-390

Nuclear EMC Effects



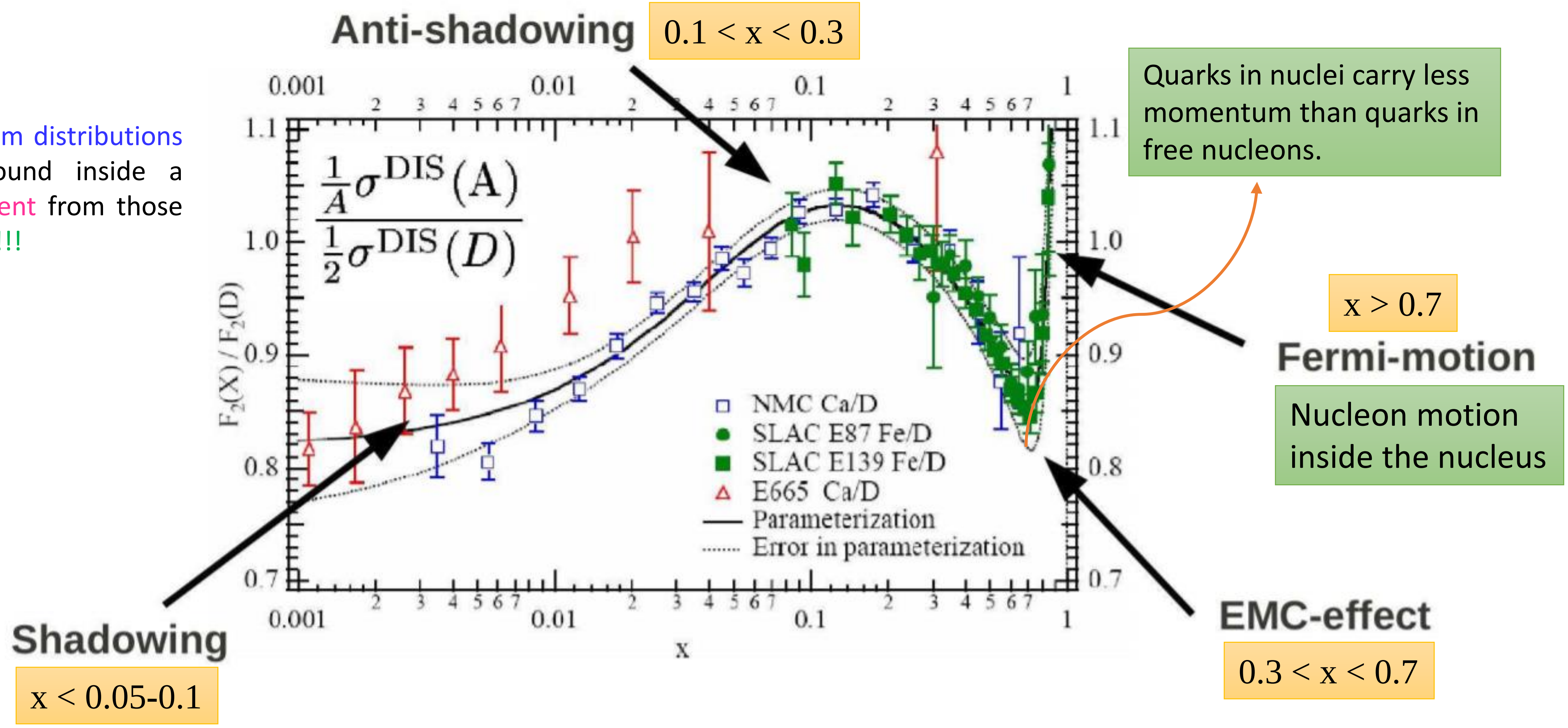


Ratios of the deep inelastic cross section on nuclear several targets.

Annu. Rev. Nucl. Part. Sci. 1995.45:337-390

Typical nuclear effects seen in the DIS measurement

Quark momentum distributions in nucleons bound inside a nuclei are different from those of free nucleons!!!

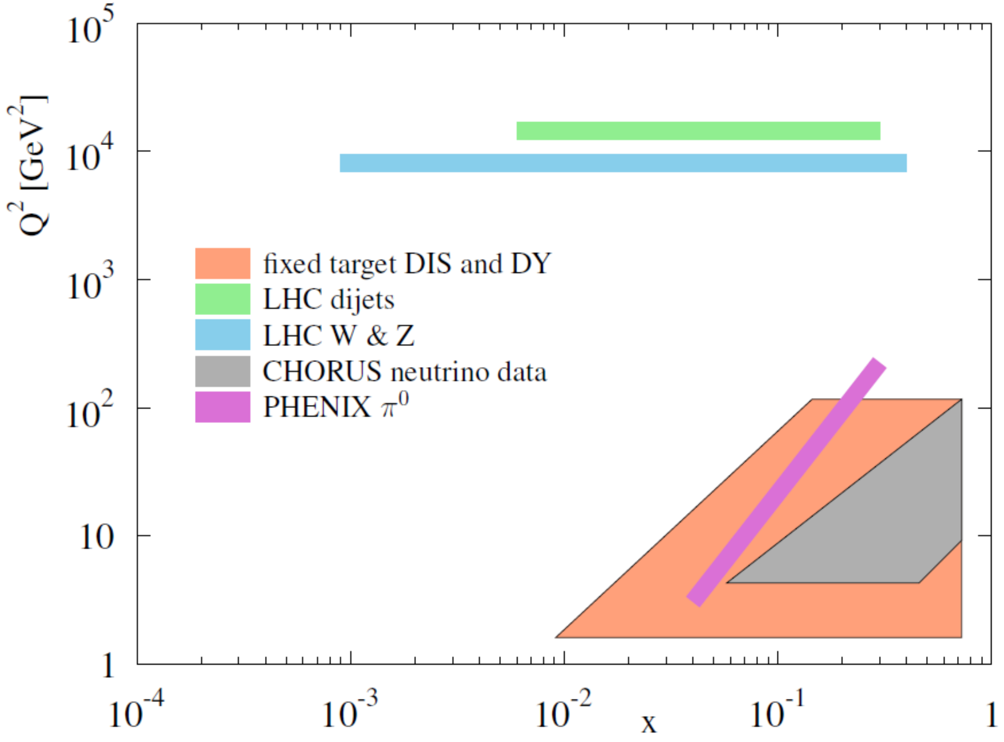


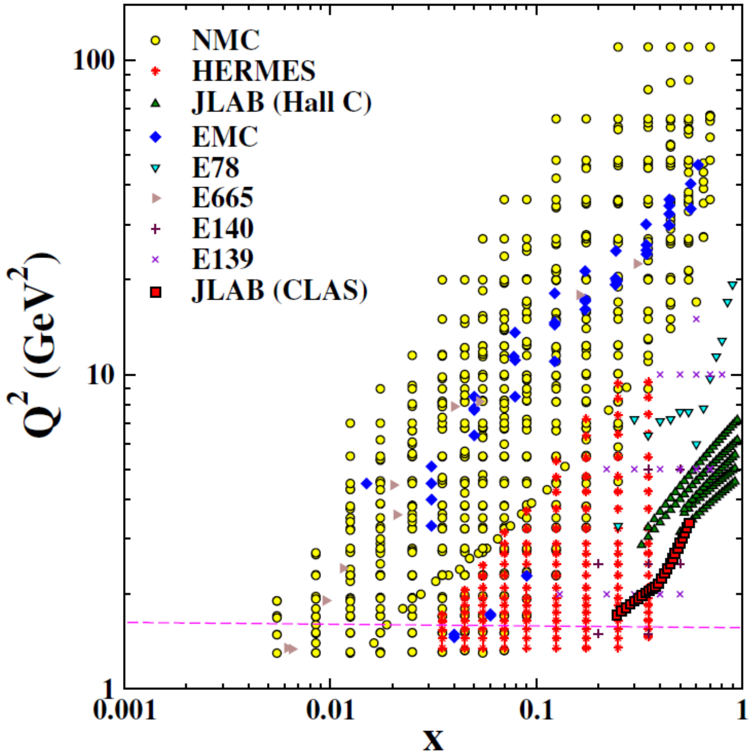
Rev. Mod. Phys. 89 (2017) 4, 045002

Physics Reports 240 (1994) 301-393

Charged-lepton scattering of the nuclei and nuclear PDFs

- ❖ These data do not have the power to constrain all of the nuclear PDFs components.
- ❖ Parton densities are less precisely known for nuclei than for nucleons.
- ❖ The LHC data opens a previously unexplored kinematic region.





Nominal x and Q^2 coverage of nuclear DIS.

JLAB (CLAS), Nature 566, no.7744, 354-358 (2019) [arXiv:2004.12065 [nucl-ex]].

Deeply inelastic scattering (DIS) on nuclear targets

$$\sigma_{\mathrm{DIS}}^{\ell+A} = \sum_i \underbrace{f_i^A(\mu_{\mathrm{fact}}^2)}_{\substack{\text{nuclear PDFs, obey} \\ \text{the usual DGLAP}}} \otimes \underbrace{\hat{\sigma}_{\mathrm{DIS}}^{\ell+i}(\mu_{\mathrm{fact}}^2, \mu_{\mathrm{ren}}^2)}_{\substack{\text{usual pQCD} \\ \text{coefficient functions}}} + \mathcal{O}(1/Q^n)$$

- The partonic coefficient functions $\hat{\sigma}$ and the DGLAP evolution of f_i^A are the same as in the case of free proton scattering. The goal is to carry out very similar program as in the case of free proton analyses.

Standard definition of nuclear PDFs

$$f_i^{\mathrm{p}/A}(x,Q^2) \equiv R_i^A(x,Q^2)\; f_i^{\mathrm{p}}(x,Q^2)$$

Nuclear modifications

Free proton baseline

$$d\sigma = \sum_{i,j} f_i^{\mathrm{p}}(Q_f^2) \otimes d\sigma_{ij}(Q_f^2, Q_r^2) \otimes f_j^{\mathrm{Pb}}(Q_f^2)$$

$$f_i^{\mathrm{p}/A}(x,Q^2) \equiv R_i^A(x,Q^2)\; f_i^{\mathrm{p}}(x,Q^2)$$

- Nuclear modification at low and large x poorly constrained by present data.

Recent determination of nuclear PDFs in 2019/2021

- **nNNPDF** [Eur. Phys. J. C 79, no. 6, 471 (2019), [arXiv:1904.00018 [hep-ph]].

- **TUJU19** [Phys. Rev. D 100, no. 9, 096015 (2019), [arXiv:1908.03355 [hep-ph]].

- **KSASG20** [Phys. Rev. D 104 (2021) 3, 034010; arXiv:2010.00555 [hep-ph]]

- **nCTEQ15HIX** [Phys. Rev. D 103 (2021) 11, 114015, arXiv:2012.11566 [hep-ph]].

KSASG20 nuclear PDFs

KSAG20 nuclear PDFs parameterization

❑ We work within the conventional approach which defines the nuclear PDFs for a bound proton in a nucleus with atomic mass number A with respect to those for a free proton through a multiplicative nuclear modification factor

$$x f_i^{p/A}(x, Q_0^2; A, Z) = \mathcal{W}_i(x, A, Z) \times x f_i^{p}(x, Q_0^2)$$

$$x f_i^{p,\mathrm{CT18}}(x, Q_0^2)$$

T. J. Hou et al., "New CTEQ global analysis of quantum chromodynamics with high-precision data from the LHC," Phys. Rev. D 103 (2021) 014013.

$$w_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^{\alpha}}\right) \times \frac{a_i(A, Z) + b_i(A)x + c_i(A)x^2 + d_i(A)x^3}{(1-x)^{\beta_i}}$$

- In order to give further and enough flexibility to our modification factor, we have considered the fit parameters to be A-dependent:

$$
\begin{aligned}
a_i(A) &= a_i + \left(1 - \frac{1}{A^{\epsilon_1}}\right) \qquad (i = \bar{q}),\\
b_i(A) &= b_i + \left(1 - \frac{1}{A^{\epsilon_2}}\right) \qquad (i = v, \bar{q}, g),\\
c_i(A) &= c_i + \left(1 - \frac{1}{A^{\epsilon_3}}\right) \qquad (i = v, \bar{q}, g),\\
d_i(A) &= d_i + \left(1 - \frac{1}{A^{\epsilon_4}}\right) \qquad (i = v, \bar{q}).
\end{aligned}
$$

- Combining the weight function with PDFs and assuming the isospin symmetry, will yield us nuclear PDFs as in what follows:

$$u_v^{(A,Z)}(x, Q_0^2) = w_{u_v}(x, A, Z)\frac{Zu_v(x, Q_0^2) + Nd_v(x, Q_0^2)}{A},$$

$$d_v^{(A,Z)}(x, Q_0^2) = w_{d_v}(x, A, Z)\frac{Zd_v(x, Q_0^2) + Nu_v(x, Q_0^2)}{A},$$

$$\bar{u}^{(A,Z)}(x, Q_0^2) = w_{\bar{q}}(x, A, Z)\frac{Z\bar{u}(x, Q_0^2) + N\bar{d}(x, Q_0^2)}{A},$$

$$\bar{d}^{(A,Z)}(x, Q_0^2) = w_{\bar{q}}(x, A, Z)\frac{Z\bar{d}(x, Q_0^2) + N\bar{u}(x, Q_0^2)}{A},$$

$$s^{(A,Z)}(x, Q_0^2) = \bar{s}^{(A,Z)}(x, Q_0^2) = w_{\bar{q}}(x, A, Z)s(x, Q_0^2),$$

$$g^{(A,Z)}(x, Q_0^2) = w_g(x, A, Z)g(x, Q_0^2).$$

Total list of experimental data in KSASG20

Total list of experimental data in KSASG20

Nucleus	Experiment	Number of data points	χ^2_{NLO}	χ^2_{NNLO}
F_2^A/F_2^C				
Be/C	NMC-96	15	9.11	11.72
Al/C	NMC-96	15	5.53	5.54
Ca/C	NMC-96	20	14.85	13.82
	NMC-96	15	7.80	7.42
Fe/C	NMC-96	15	9.04	8.50
Sn/C	NMC-96	144	135.86	150.24
	NMC-96	15	21.51	27.21
Pb/C	NMC-96	15	11.49	14.14
Total		**254**		
F_2^A/F_2^{Li}				
C/Li	NMC-95	20	16.95	17.39
Ca/Li	NMC-95	20	23.49	25.34
Total		**40**		

Nuclear DIS

Nucleus	Experiment	Number of data points	χ^2_{NLO}	χ^2_{NNLO}
D	NMC-96	126	201.90	88.50
D	BCDMS	53	58.52	64.86
D	BCDMS	155	245.89	216.84
D	HERMES	39	10.28	5.57
D/p	NMC-96	156	148.98	152.71
Total		**529**		

The measurements of the deuteron structure function and deuteron-proton ratio.

Nucleus	Experiment	Number of data points	χ^2_{NLO}	χ^2_{NNLO}
F_2^A/F_2^D				
He/D	SLAC-E139	18	21.86	21.86
	NMC-95	16	9.91	9.84
Li/D	NMC-95	15	12.16	12.92
Li/D (Q^2dep.)	NMC-95	153	163.87	168.86
Be/D	SLAC-E139	17	41.68	38.40
C/D	EMC-88	9	8.97	9.13
	EMC-90	2	0.13	0.05
	SLAC-E139	7	14.56	14.05
	NMC-95	15	7.78	7.15
	FNAL-E665	4	3.81	3.50
C/D (Q^2dep.)	NMC-95	164	144.90	146.02
N/D	BCDMS-85	9	10.20	12.10
	HERMES-03	92	55.72	65.12
Al/D	SLAC-E49	18	31.39	30.18
	SLAC-E139	17	7.23	6.64
Ca/D	EMC-90	2	1.96	1.78
	NMC-95	15	30.91	39.48
	SLAC-E139	7	4.28	4.02
	FNAL-E665	4	5.39	5.99
Fe/D	SLAC-E87	14	7.18	7.61
	SLAC-E139	23	27.58	25.92
	SLAC-E140	6	10.69	10.93
	BCDMS-87	10	16.60	15.74
Cu/D	EMC-93	19	12.15	12.59
Kr/D	HERMES-03	84	73.67	88.16
Ag/D	SLAC-E139	7	11.12	14.47
Sn/D	EMC-88	8	16.85	18.72
Xe/D	FNAL-E665-92	4	3.24	2.84
Au/D	SLAC-E139	18	31.85	34.77
Pb/D	FNAL-E665-95	4	9.01	8.64
Total		**781**		

Nuclear DIS

Nucleus	Experiment	Number of data points	χ^2_{NLO}	χ^2_{NNLO}
ν Pb	CHORUS	532	459.71	569.92
$\bar{\nu}$ Pb	CHORUS	532	552.67	549.52
ν Fe	CDHSW	698	790.06	733.92
$\bar{\nu}$ Fe	CDHSW	696	695.79	679.00
Total		**2458**		

The charged-current (anti)neutrino-nucleus DIS data.

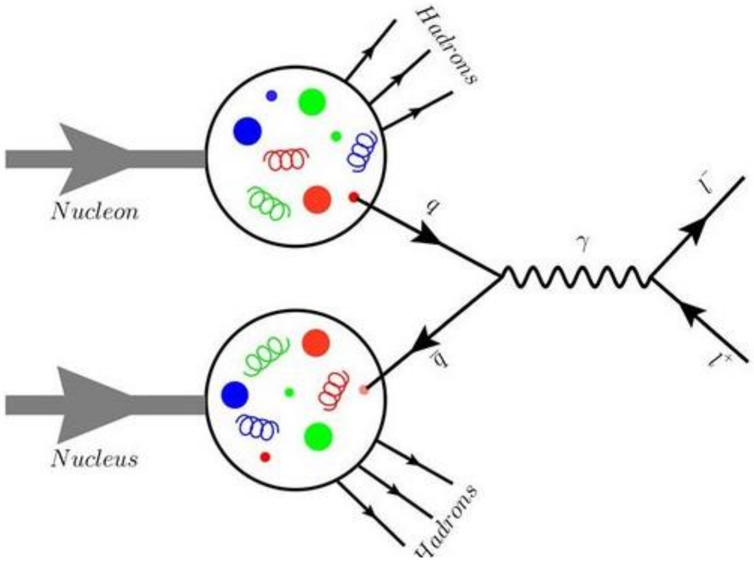
The structure functions for neutrino and antineutrino scattering

$$F_2^{\nu A} \simeq (d^A + s^A + b^A + \bar{u}^A + \bar{c}^A)$$
$$F_3^{\nu A} \simeq (d^A + s^A + b^A - \bar{u}^A - \bar{c}^A)$$

$$F_2^{\bar{\nu} A} \simeq (u^A + c^A + \bar{d}^A + \bar{s}^A + \bar{b}^A)$$
$$F_3^{\bar{\nu} A} \simeq (u^A + c^A - \bar{d}^A - \bar{s}^A - \bar{b}^A)$$

Nucleus	Experiment	Number of data points	χ^2_{NLO}	χ^2_{NNLO}
$\sigma^A_{\text{DY}}/\sigma^{A'}_{\text{DY}}$				
Fe/Be	FNAL-E866/NuSea	28	28.38	28.13
W/Be	FNAL-E866/NuSea	28	37.17	32.09
C/D	FNAL-E772-90	9	30.12	33.83
Ca/D	FNAL-E772-90	9	4.35	6.13
Fe/D	FNAL-E772-90	9	25.98	29.31
W/D	FNAL-E772-90	9	14.04	14.44
Total		**92**		

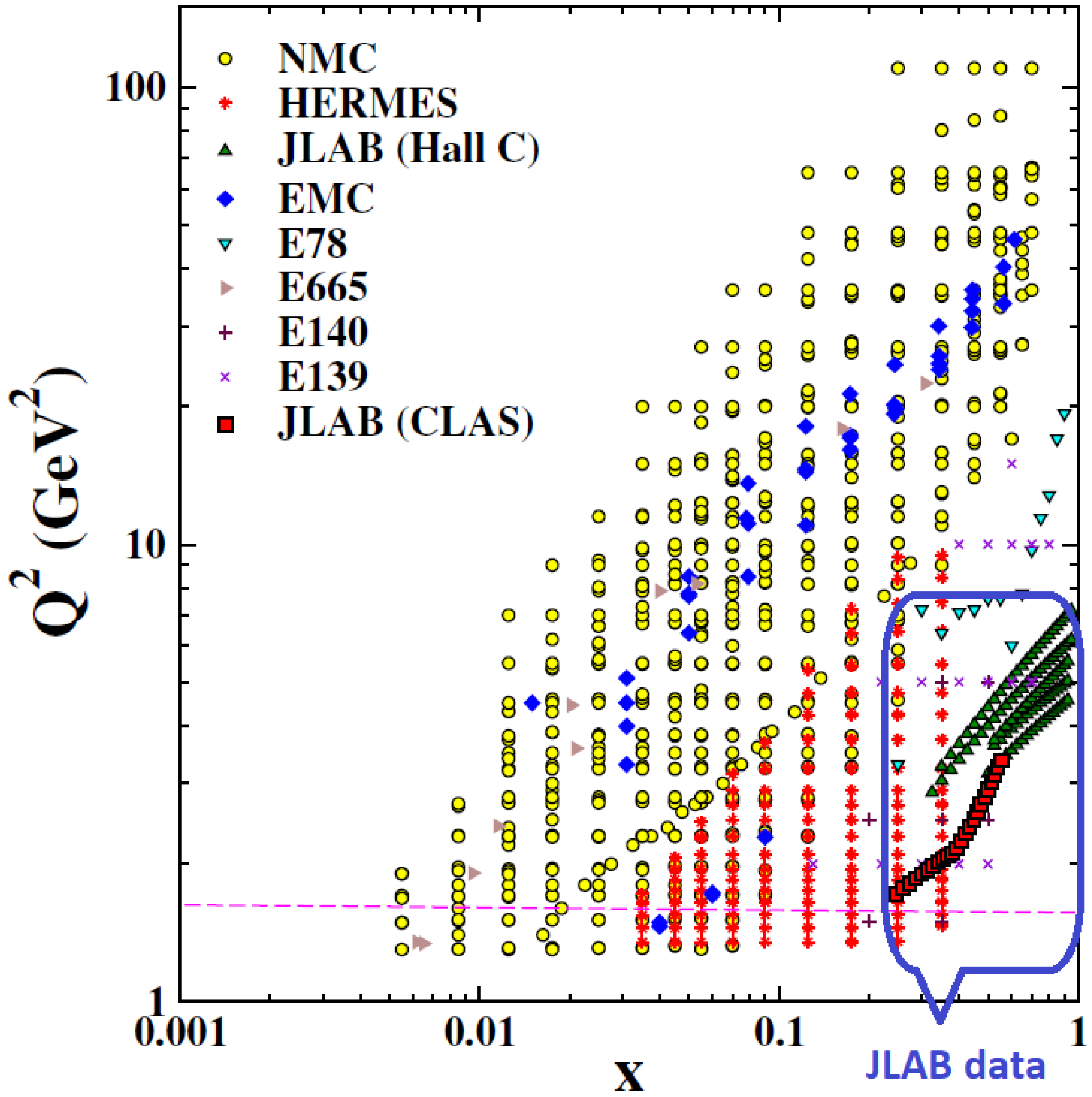
The Drell-Yan cross section ratios measured by FNAL Collaboration at JLAB experiment.



$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9M^2} \sum_i e_i^2 [q_i^1(x_1)\bar{q}_i^2(x_2) + \bar{q}_i^1(x_1) q_i^2(x_2)]$$

Nucleus	Experiment	Number of data points	χ^2_{NLO}	χ^2_{NNLO}
F_2^A/F_2^D				
C/D	JLAB Hall C	103	158.88	154.51
Pb/D	JLAB CLAS	24	9.98	13.38
Fe/D	JLAB CLAS	24	26.19	26.97
Al/D	JLAB CLAS	24	16.39	15.23
C/D	JLAB CLAS	24	14.19	14.66
Total		**199**		

The neutral-current charged lepton DIS experimental datasets from Jefferson Lab **Hall C** and **CLAS** measured by JLAB during the 6 GeV electron beam operation.



JLAB (CLAS), Nature 566, no.7744, 354-358 (2019) [arXiv:2004.12065 [nucl-ex]].

Target Mass Corrections

- **TMC** effects are most pronounced at large-*x* regions and moderate-to-small values of photon virtuality Q^2,

Nachtmann variable $\xi = \frac{2x}{1+\tau}$

$\eta = \frac{M_p^2}{Q^2}$

$$\boxed{F_2^{\mathrm{TMC}}(x,Q) = \frac{x^2}{\xi^2\tau^3}F_2^0(\xi,Q) + \frac{6\eta x^3}{\tau^2}\int_\xi^1 dy F_2(y,Q)/y^2}$$

The target-mass corrected structure functions

$\tau = 1 + 4\eta x^2$

Higher Twist Corrections

- The inclusion of the Higher Twist (HT) corrections in which particular important at high x and low Q values

$$F_2^A(x,Q) \to F_2^{(\mathrm{LT},\mathrm{A})}(x,Q)[1 + C_{\mathrm{HT}}(x,A)/Q^2]$$

$$C_{\mathrm{HT}}(x,A) = H_0\ x^{H_1}(1 + H_2 x)A^{1/3}$$

- ✓ For our NLO analysis: **CJ15**: Phys. Rev. D 93, no.11, 114017 (2016), [arXiv:1602.03154 [hep-ph]]
- ✓ For our NNLO analysis: M. Goharipour and S. Rostami, Phys. Rev. D 101, no.7, 074015 (2020), [arXiv:2004.03403 [hep-ph]].

χ^2 minimization

- Global QCD extractions of nuclear PDFs are implemented around an effective χ^2 function that quantifies the goodness of the fit to the data for a given set of theoretical parameters, which determine the nuclear PDFs at some input scale Q_0^2.

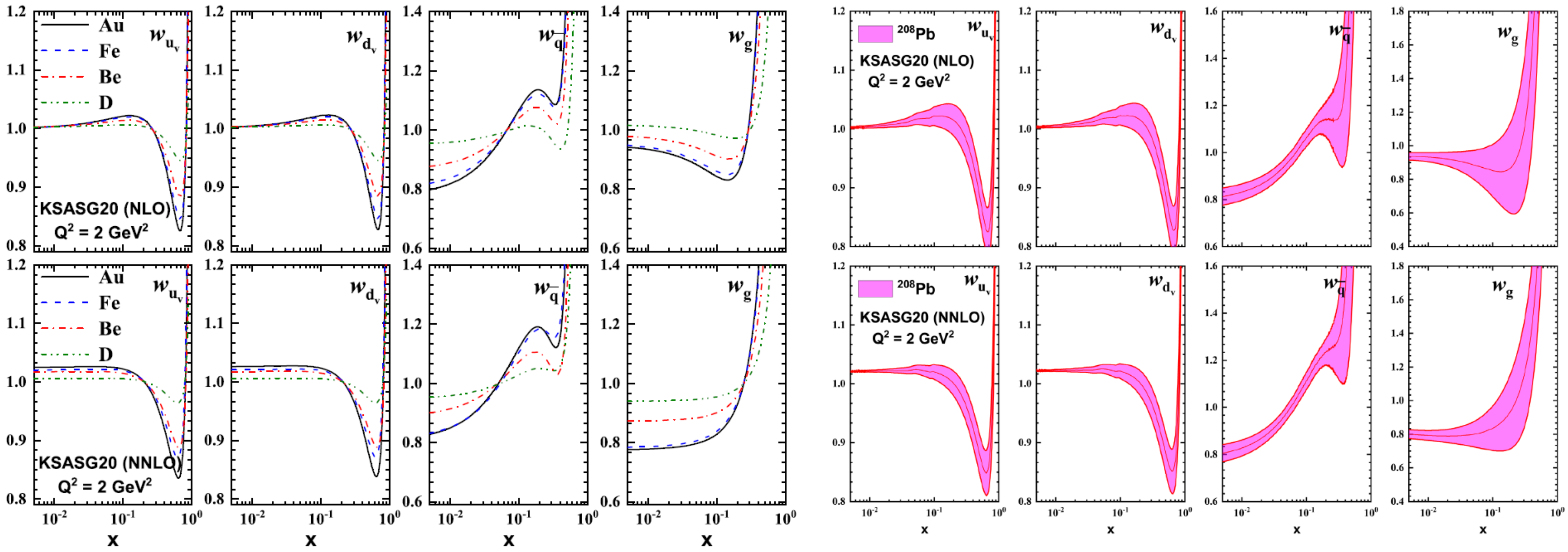
$$\chi_n^2(\{p_i\}) = \left(\frac{1-\mathcal{N}_n}{\Delta\mathcal{N}_n}\right)^2 + \sum_{j=1}^{N_n^{\text{data}}} \left(\frac{(\mathcal{N}_n \text{Data}_j - \text{Theory}_j(\{p_i\})}{\mathcal{N}_n \delta\text{Data}_j}\right)^2,$$

- The χ^2 function is minimized by the CERN program library MINUIT.

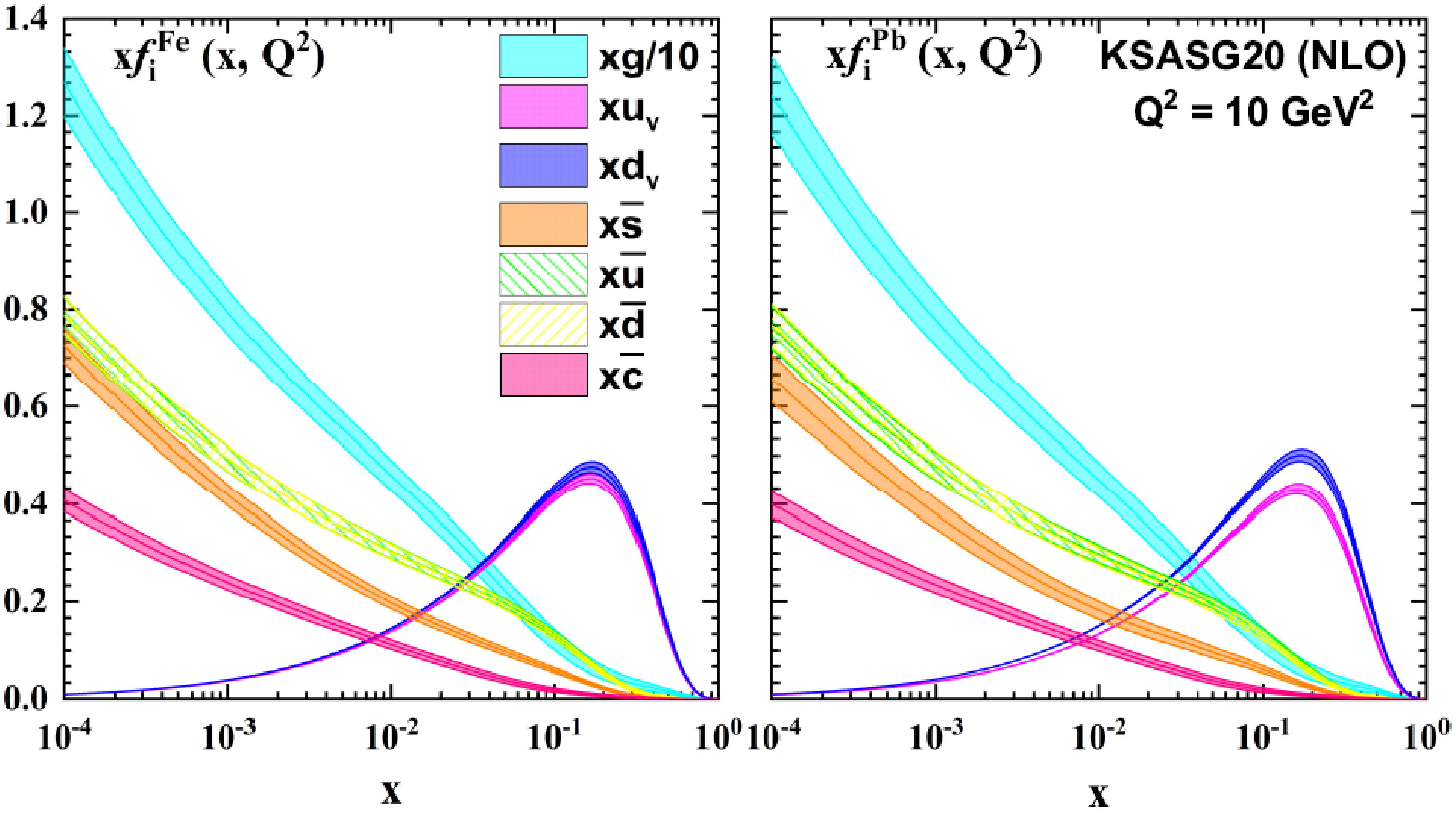
F. James and M. Roos, Comput. Phys. Commun. 10, 343 (1975).

- Nuclear PDFs uncertainties: Standard Hessian method

KSASG20 nuclear PDFs and uncertainties

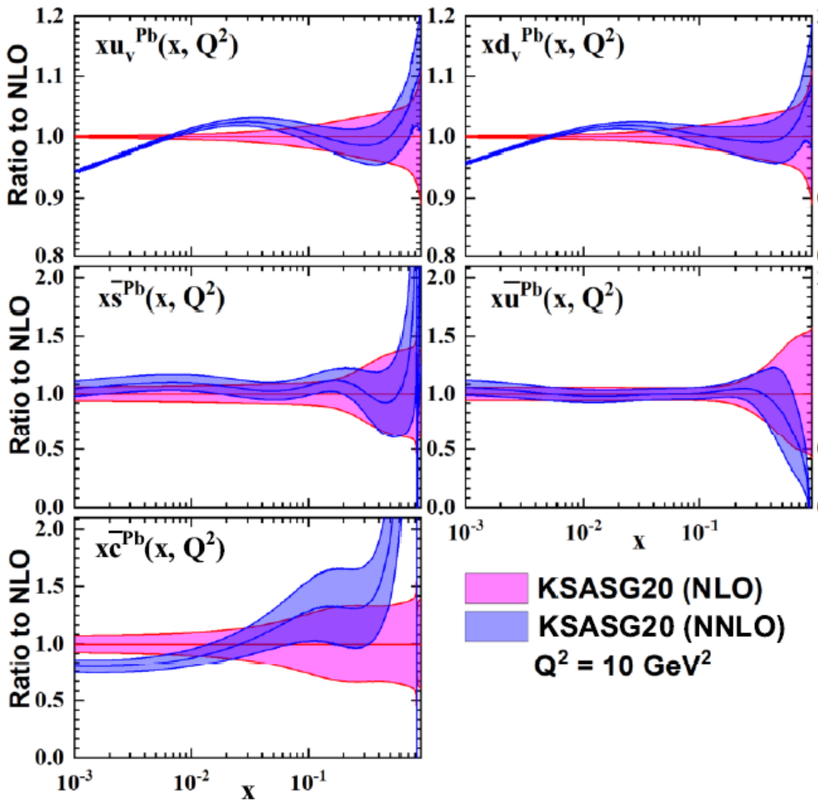


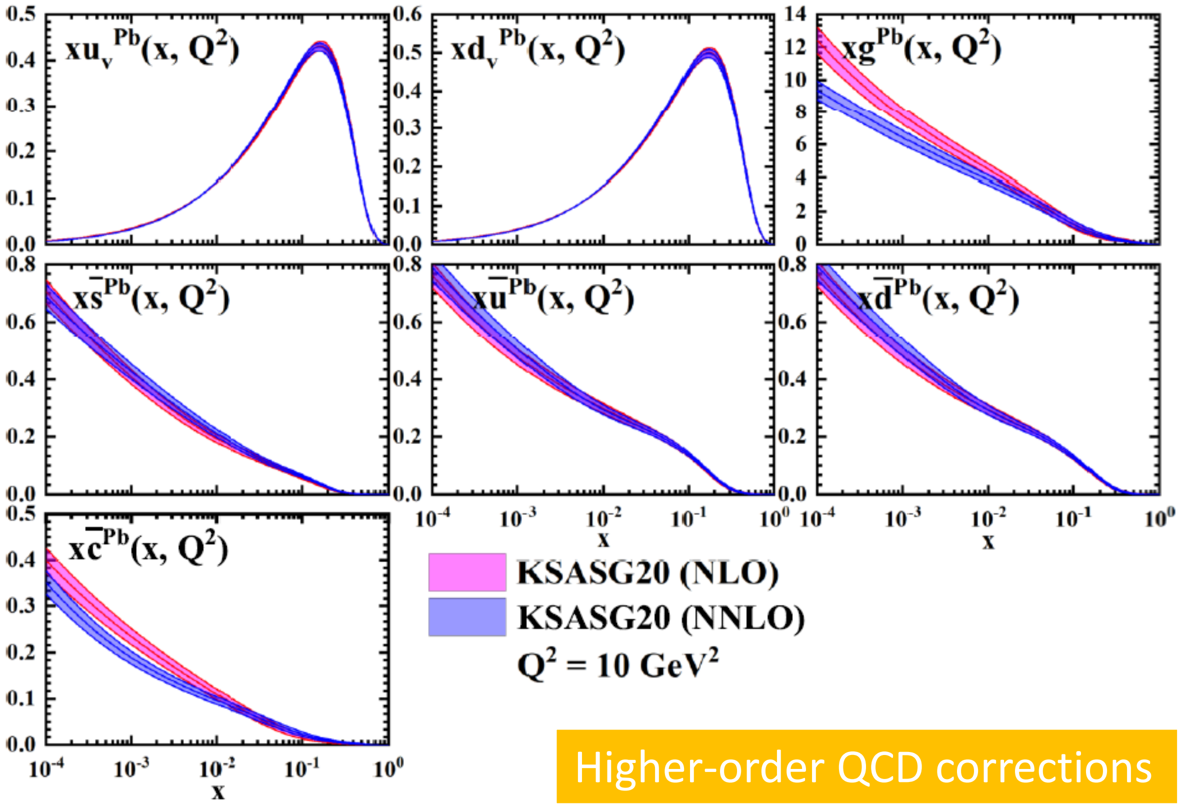
The nuclear modification factors defined as ratios of proton PDFs bound in deuterium (D), Beryllium (Be), iron (Fe) and gold (Au) to the corresponding free-proton PDFs of CT18 at **NLO** (top row) and **NNLO** (bottom row) accuracy in pQCD and at the initial scale Q^2 = 2.0 GeV.



KSASG20 nuclear PDFs and their uncertainties at $Q^2 = 10\ GeV^2$ for iron (left) and lead (right).

NLO/NNLO comparison

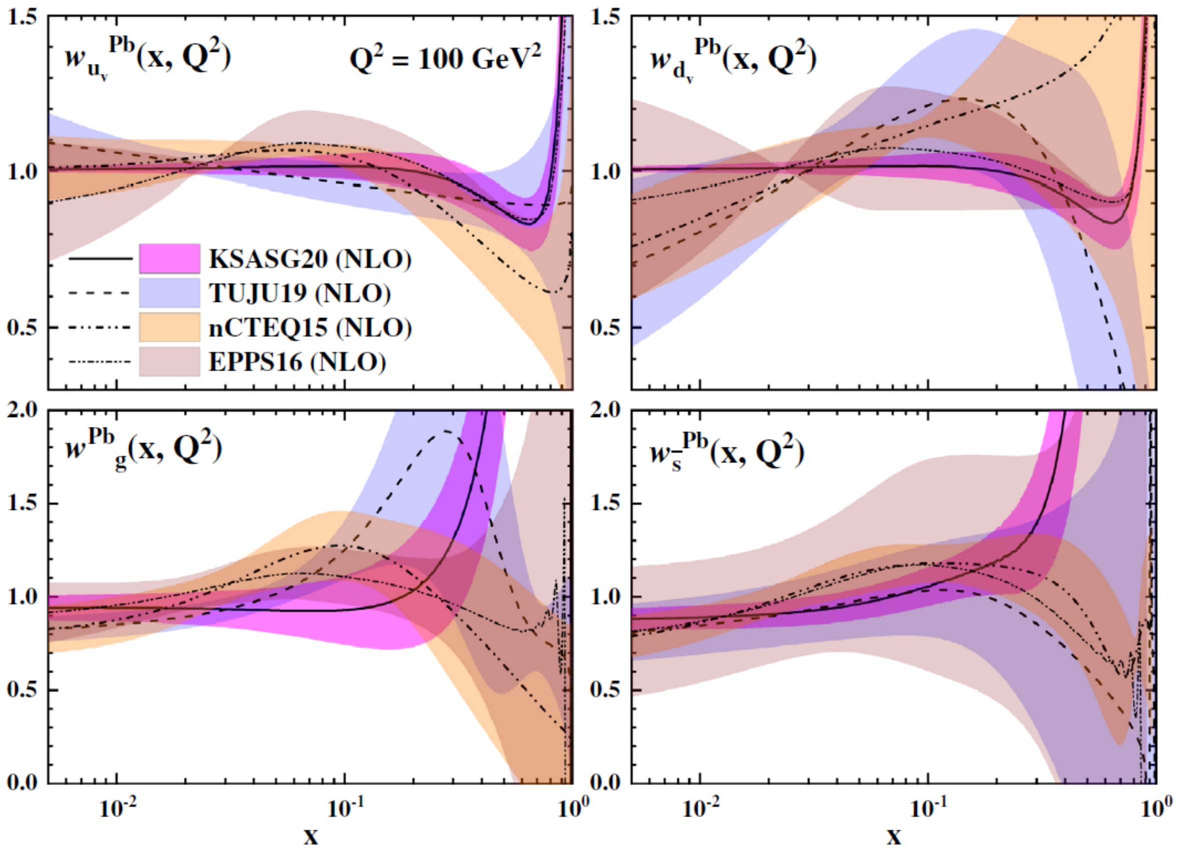




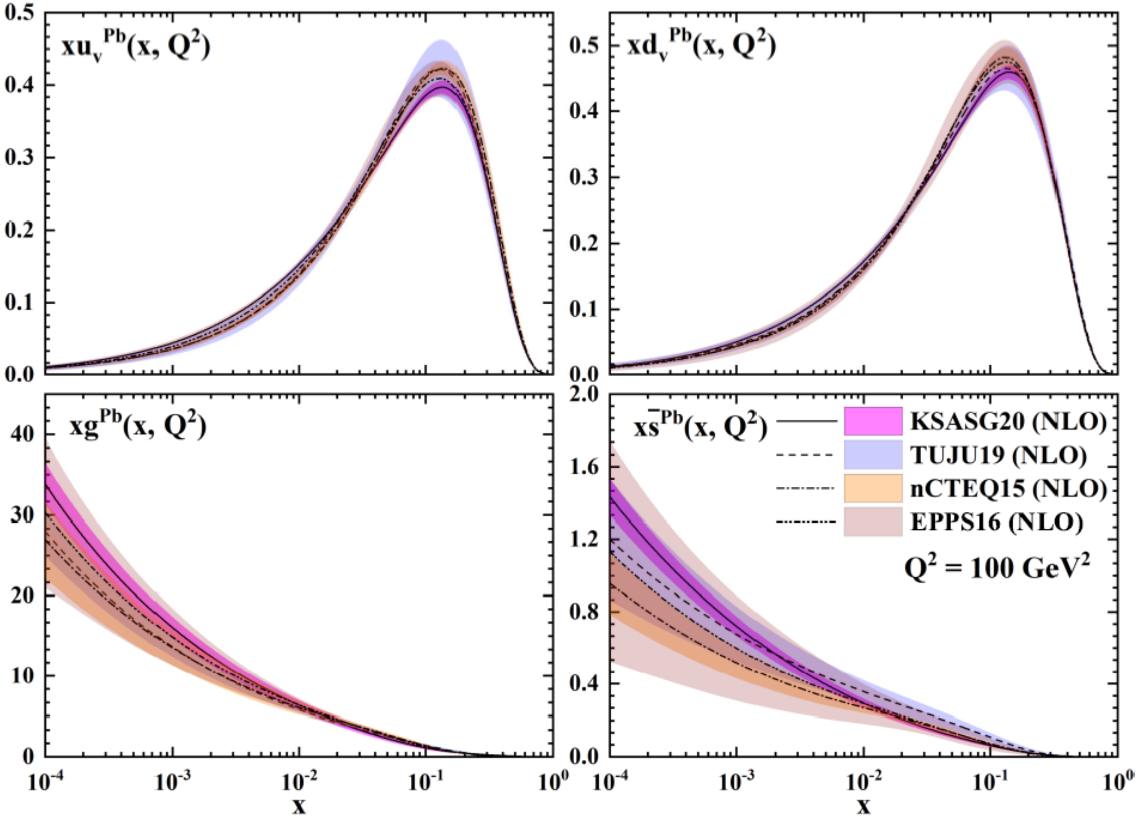
Higher-order QCD corrections

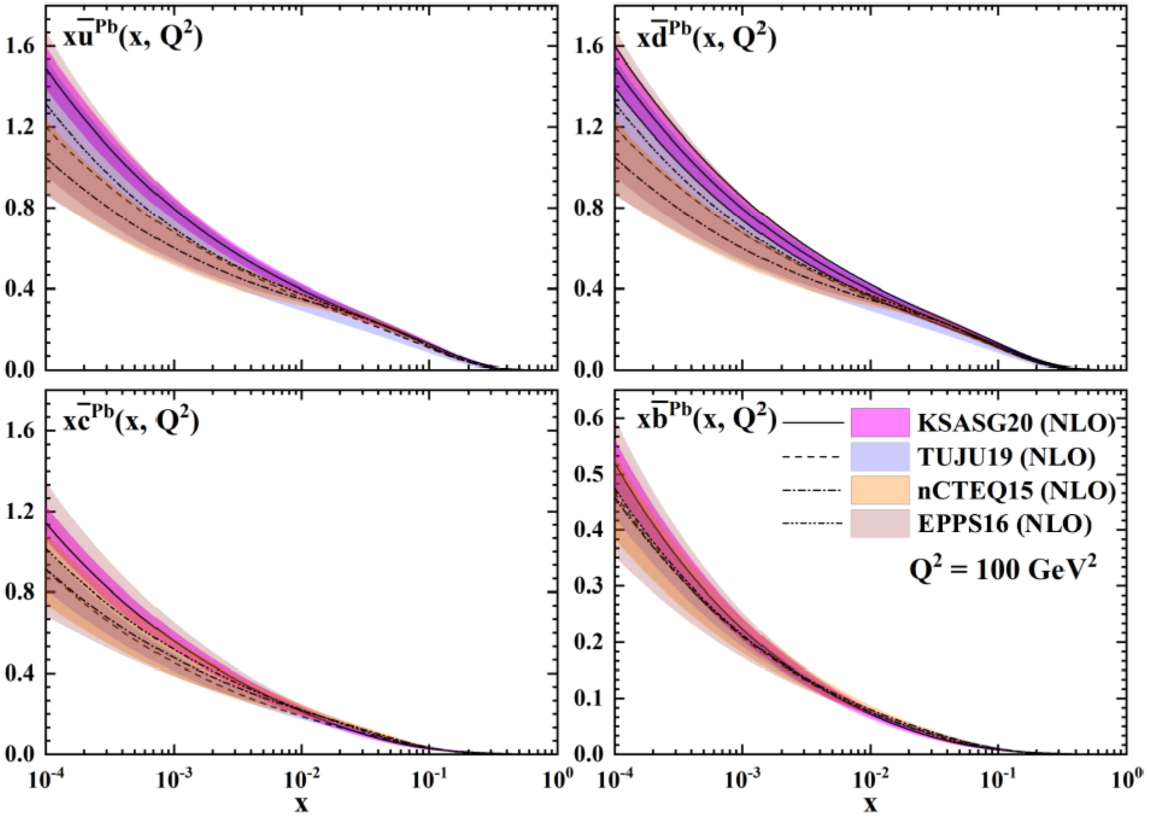
Our bound proton PDFs at $Q^2 = 10$ GeV^2 for lead at NLO and NNLO accuracy.

Comparison with other groups



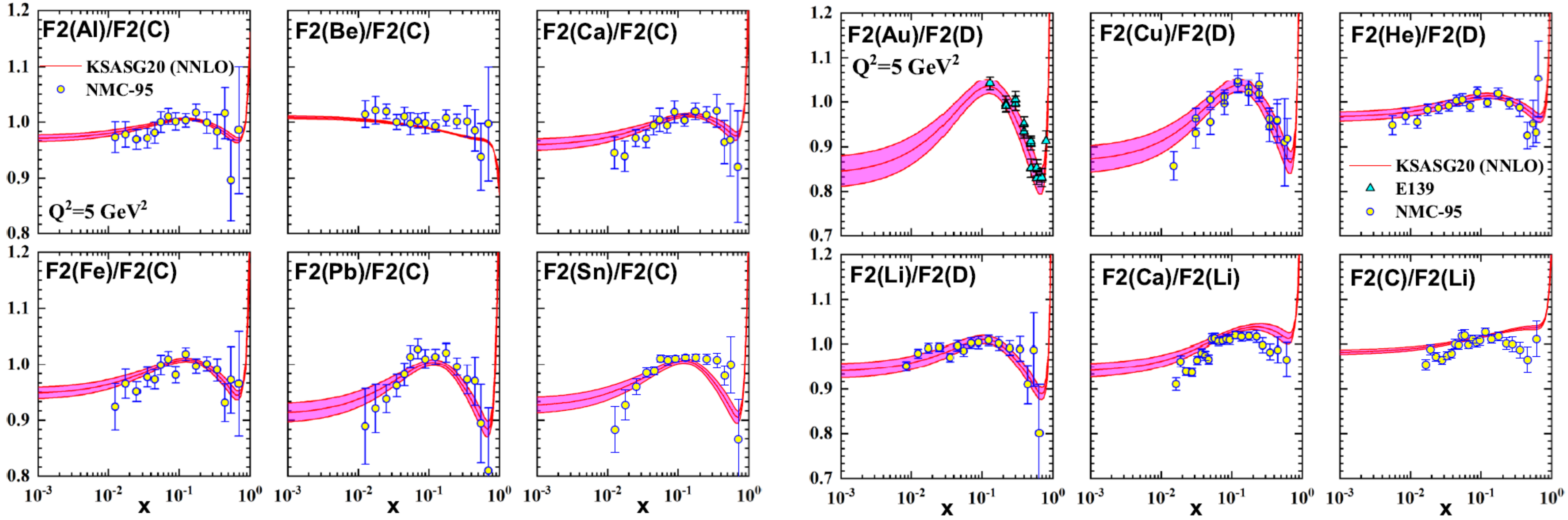
The nuclear parton distribution functions KSASG20 in lead at NLO accuracy compared to the nuclear PDF sets nCTEQ15, EPPS16 and TUJU19 shown at a higher scale $Q^2 = 10$ GeV^2.



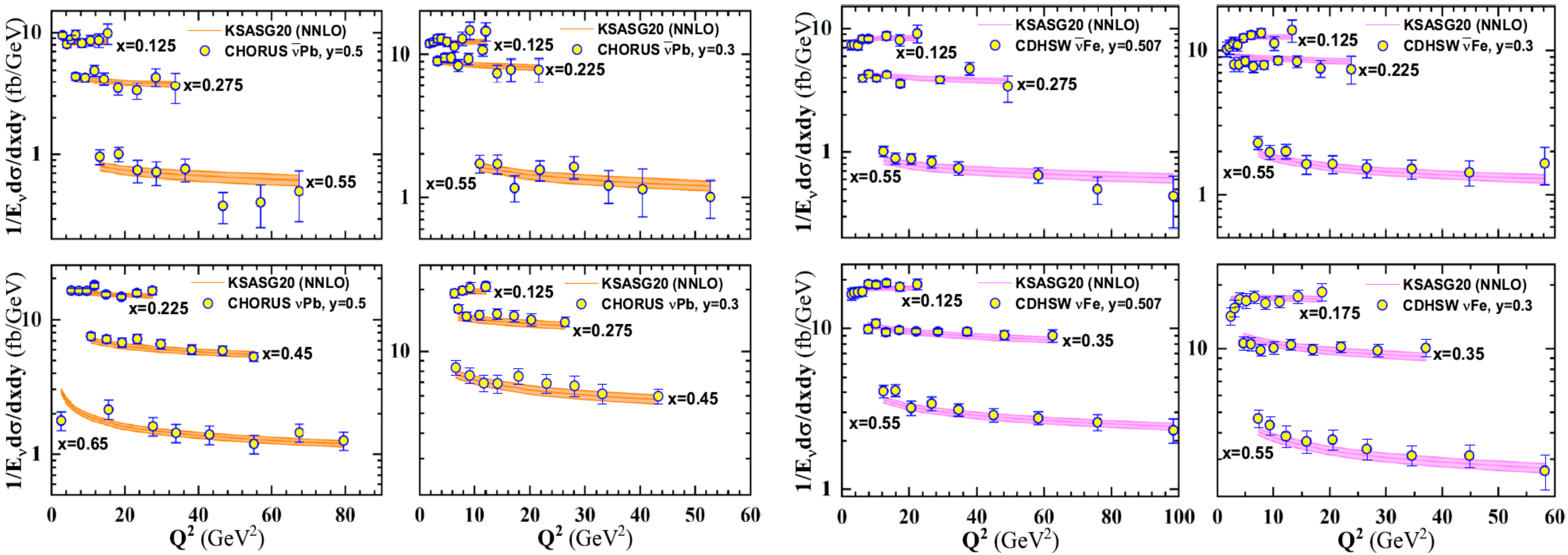


Our bound proton PDFs at the scale $Q^2 = 100$ GeV^2 for lead at NLO accuracy. The most recent results from nCTEQ15, EPPS16 and TUJU19 are shown for comparison.

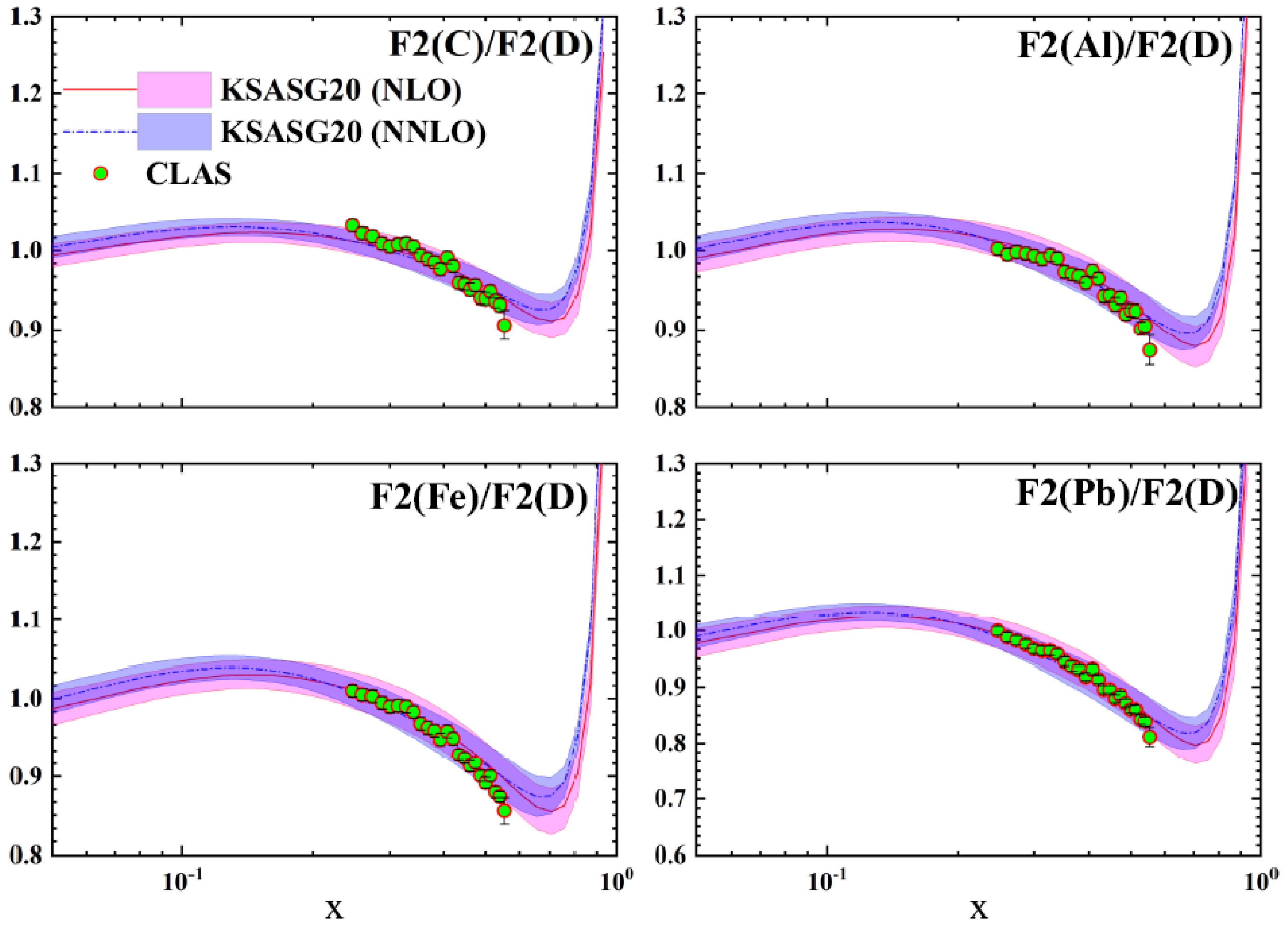
Fit quality and comparison of data and theory



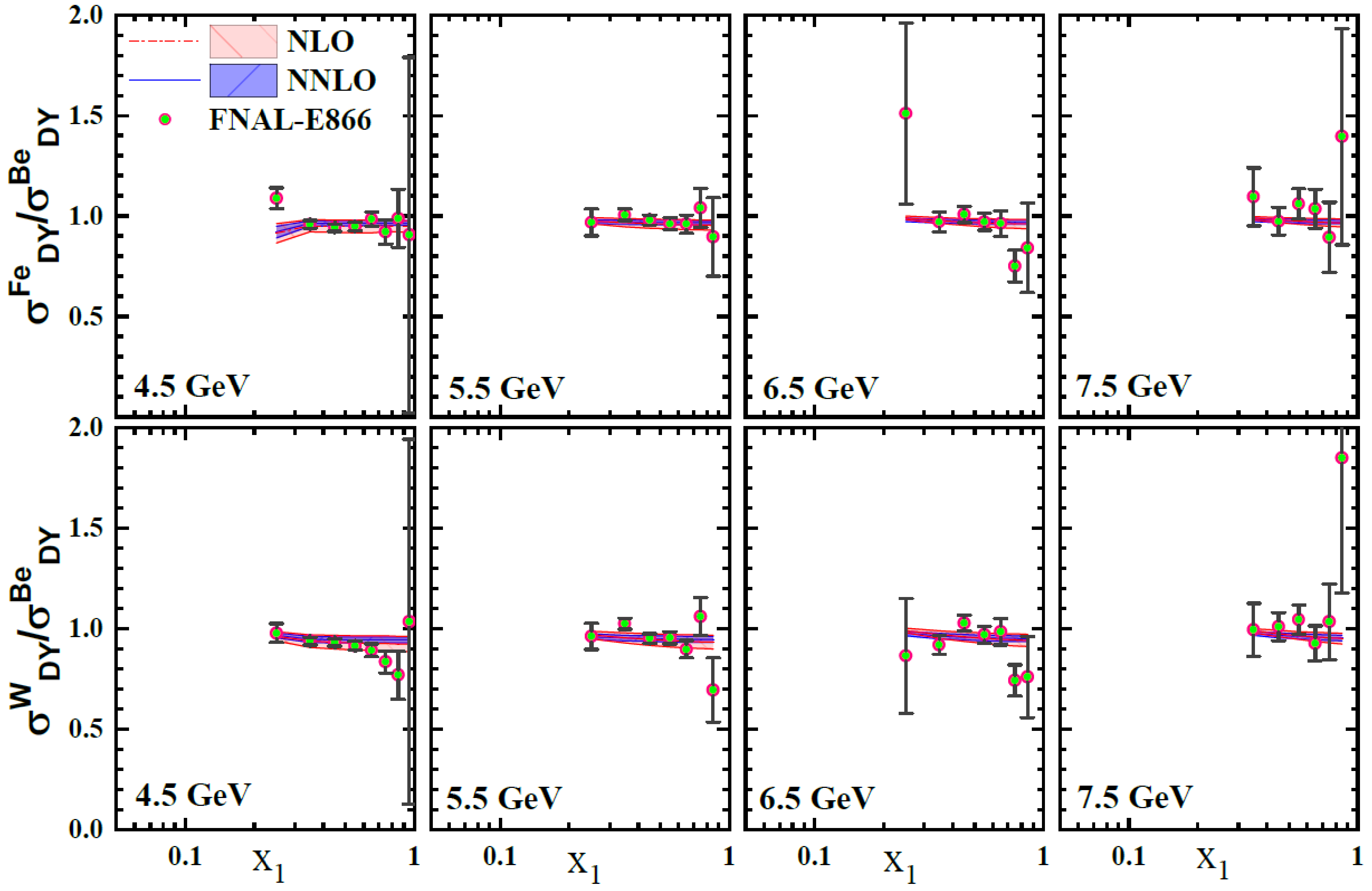
Comparison of our NNLO theory predictions for the ratio F2A/F2C and F2A/F2D as a function of x with some selected nuclear DIS data.



The neutrino(antineutrino) lead DIS data compared with the KSASG20 NNLO theory predictions.



Comparison of our NLO and NNLO theory predictions for the structure functions ratios F2A/F2D with the most recent dara from Jefferson Lab CLAS as a function of x with some selected nuclear targets.



Comparison of the KSASG20 NLO theory predictions for the Drell-Yan cross section ratios

Summary and Conclusions

- We have given a very short introduction to nuclear PDFs and reviewed some available global nuclear PDFs analyses.
- We have presented the **KSASG20** NLO and NNLO nuclear PDFs and their uncertainties.
- We have introduced several improvements aimed to obtained a well-established nuclear PDFs.
- Consistent QCD fit for wide variety of nuclear targets and wide kinematic range have been achieved.
- We look forward to more high-precision data from LHC proton-lead ($p - pb$) collisions to better determination of nuclear gluon PDF (especially at small-x)
- Future HERA-like measurements for many nuclear targets covering very small range of Bjorken x (FCC-he & LHeC).
- Plans are being made to test this hypothesis with the **12 GeV** upgrade of Jefferson Lab by including a more complete set of light nuclei.


THANK
YOU