

# Polarization effects in the search for dark vector boson in $e^+ e^-$ colliders

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Fei-Fan Lee, GLL and Vo Quang Nhat, arXiv:2008.07769 [hep-ph];  
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# Introduction and motivations

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- Growing interests in searching for DM related phenomenon with high statistics and high precision measurements.
- Such phenomenon has to do hidden sector\*, assumed to interact with the visible sector through a messenger particle.
- A popular proposal for such a messenger is the so-called dark photon\*\*, which mixes with  $U(1)_Y$  in SM.

\*B. Holdom, Phys. Lett. 166B, 196 (1986); P. Galison and A. Manohar, Phys. Lett. 136B, 279 (1984)

\*\*J. Alexander *et al.*, arXiv:1608.08632 [hep-ph]

# Introduction and motivations

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- Such a mixing induces EM couplings between dark photon and SM fermions, which generate rich phenomenology.
- The search for light boson with the reaction  $e^+e^- \rightarrow A' + \gamma$  has been proposed\*.
- Many new proposals to search for dark photons with the above process—see the list next page
- These proposals are based upon either fixed target or electron-positron collider

\*C. Boehm and P. Fayet, Nucl. Phys. B 683, 219 (2004); N. Borodatchenkova, D. Choudhury and M. Drees, Phys. Rev. Lett. 96, 141802 (2006); P. Fayet, Phys. Rev. D 75, 115017 (2007).

# Introduction and motivations

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- **V. Kozuharov [PADME Collaboration], Nuovo Cim. C 40, no. 5, 192 (2017)**
- **T. Araki *et al.*, Phys. Rev. D 95, no. 5, 055006 (2017)**
- **B. Wojtsekhowski *et al.*, JINST 13, no. 02, P02021 (2018)**
- **I. Alikhanov and E. A. Paschos, Phys. Rev. D 97, no. 11, 115004 (2018)**
- **L. Marsicano *et al.*, Phys. Rev. D 98, no. 1, 015031 (2018)**
- **J. Jiang *et al.*, Eur. Phys. J. C 78, no. 6, 456 (2018)**
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# Introduction and motivations

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- **The dark photon interaction with EM current is given by**
$$\mathcal{L}_{\text{int}} = \varepsilon_\gamma e J_{\text{em}}^\mu A'_\mu \quad \varepsilon_\gamma \equiv \varepsilon \text{ in}$$
$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} \frac{\varepsilon}{\cos \theta_W} B^{\mu\nu} A'_{\mu\nu} - \frac{1}{4} A'_{\mu\nu} A'^{\mu\nu}$$
- **The neutral current interaction is suppressed in the limit  $M_{A'} \ll M_Z$**
- **The detection of  $A'$  determines the mixing parameter and the mass of the dark photon.**
- **On the other hand, there could be other mixing between dark boson and SM gauge bosons, such as\***

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} M_Z^2 Z_\mu^0 Z^{0\mu} - \delta m^2 Z_\mu^0 A'^\mu + \frac{1}{2} M_{A'}^2 A'_\mu A'^\mu$$

\*H. Davoudiasl, H. S. Lee and W. J. Marciano, Phys. Rev. D 85, 115019 (2012)

# Introduction and motivations

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- With the above mass mixing, an independent neutral current coupling between dark boson and SM fermions is induced:

$$\mathcal{L}_{\text{int}} = \varepsilon_Z \frac{g}{\cos \theta_W} J_{\text{NC}}^\mu A'_\mu \text{ with } \varepsilon_Z \equiv \delta m^2 / M_Z^2$$

- Considering both mixings, the interaction between dark boson (renamed as  $Z_d$  from now on) and SM fermions becomes

$$e\varepsilon \bar{f}(g_{f,V}\gamma_\mu + g_{f,A}\gamma_\mu\gamma_5)f Z_d^\mu$$

- In the search for  $Z_d$  with  $e^+e^- \rightarrow Z_d + \text{gamma}$ , can one determine the relative strength of vector and axial-vector couplings?
- The key is on the polarization of  $Z_d$

# Outline

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- **Heuristic derivation of  $Z_d$ -fermion interactions**
- **Ward-Takahashi identity and the polarization of  $Z_d$  in  $e^+e^- \rightarrow Z_d + \text{gamma}$**
- **Differential cross section of  $e^+e^- \rightarrow Z_d + \text{gamma}$  for each polarization of  $Z_d$  and the decay distribution of  $Z_d \rightarrow l^+ l^-$**
- **Searching for  $Z_d$  by  $e^+e^- \rightarrow Z_d + \text{gamma}$  and  $Z_d$  decaying to muon pairs in BaBar and Belle II**
- **Summary**

# Heuristic derivation of $Z_d$ -fermion interactions

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The mixing terms give two point functions

$$i\Pi_{AZ_d}^{\mu\nu} = i\varepsilon k^2 g^{\mu\nu},$$

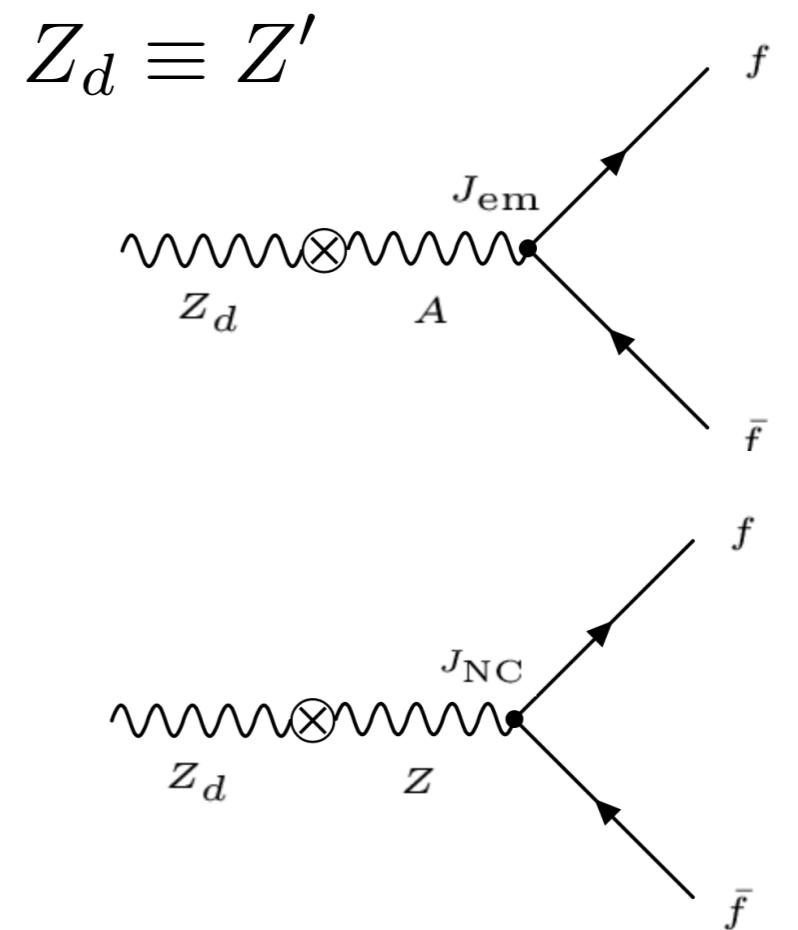
$$i\Pi_{ZZ_d}^{\mu\nu} = -i(\varepsilon \tan \theta_W k^2 + \delta m^2)g^{\mu\nu},$$

The *EM* interactions of dark boson

$$ie J_{\text{em}}^\alpha \frac{-ig_{\alpha\mu}}{k^2} i\varepsilon k^2 g^{\mu\nu} Z_{d\nu} = ie\varepsilon J_{\text{em}}^\nu Z_{d\nu}.$$

The *Neutral-Current* interactions of dark boson

$$\begin{aligned} & \frac{ig}{\cos \theta_W} J_{\text{NC}}^\alpha \frac{-i}{k^2 - M_Z^2} \left( g_{\alpha\mu} - \frac{k_\alpha k_\mu}{M_Z^2} \right) \cdot (-i)(\varepsilon \tan \theta_W k^2 + \delta m^2) g^{\mu\nu} Z_{d\nu} \\ &= \frac{-ig}{\cos \theta_W} J_{\text{NC}}^\nu Z_{d\nu} \frac{(\varepsilon \tan \theta_W M_{Z_d}^2 + \delta m^2)}{(M_{Z_d}^2 - M_Z^2)}. \end{aligned}$$



# Heuristic derivation of $Z_d$ -fermion interactions

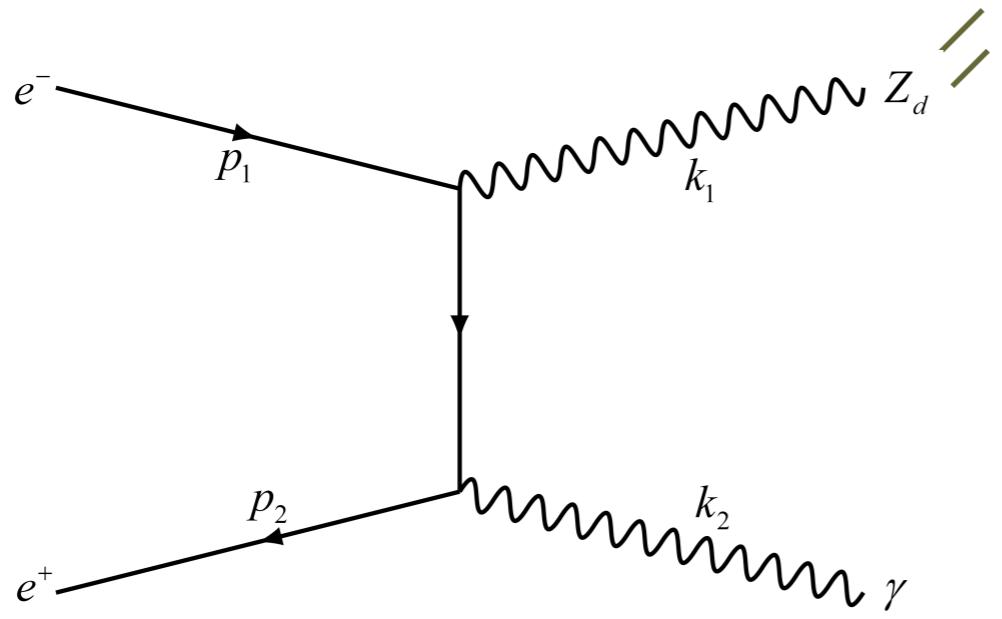
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In the limit  $M_{Z_d} \ll M_Z$

$$\mathcal{L}_{\text{int}} = \left( \varepsilon_\gamma e J_{\text{em}}^\mu + \varepsilon_Z \frac{g}{\cos \theta_w} J_{\text{NC}}^\mu \right) Z_{d\mu},$$

$$\varepsilon_Z \equiv \delta m^2 / m_Z^2.$$

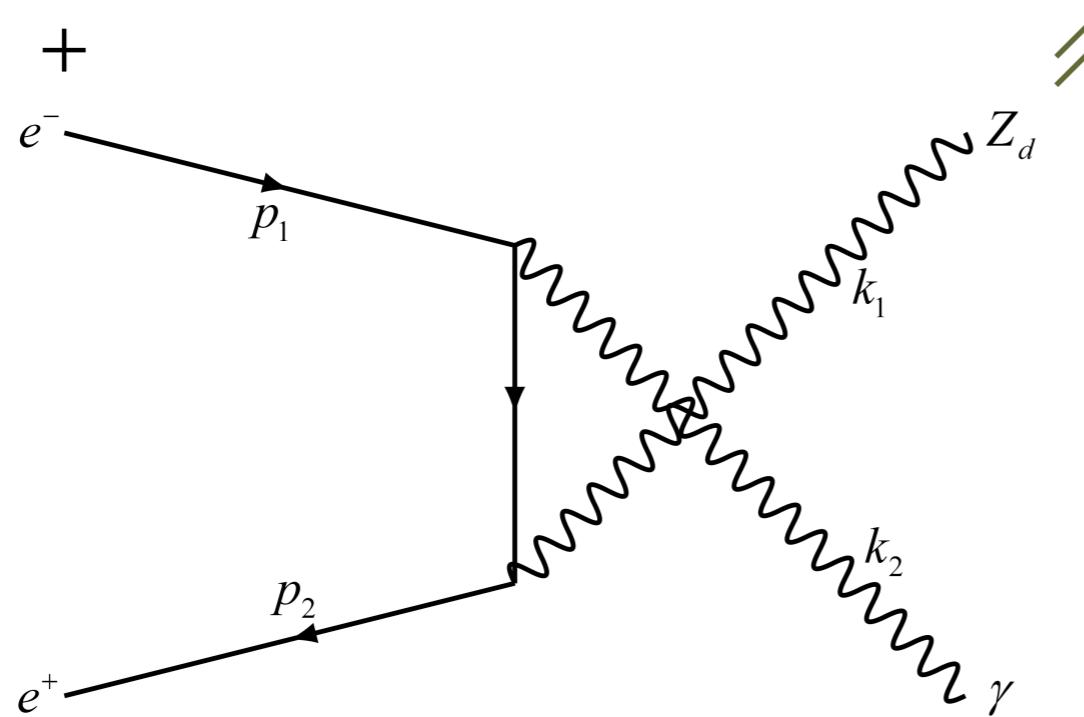
# Ward-Takahashi identity and $Z_d$ polarization



$$\epsilon^\mu(k_1) = (|\vec{k}_1|, E_{Z_d} \hat{k}_1)/m_{Z_d}$$

$$= k_1^\mu/m_{Z_d} + \mathcal{O}(m_{Z_d}/E_{Z_d})$$

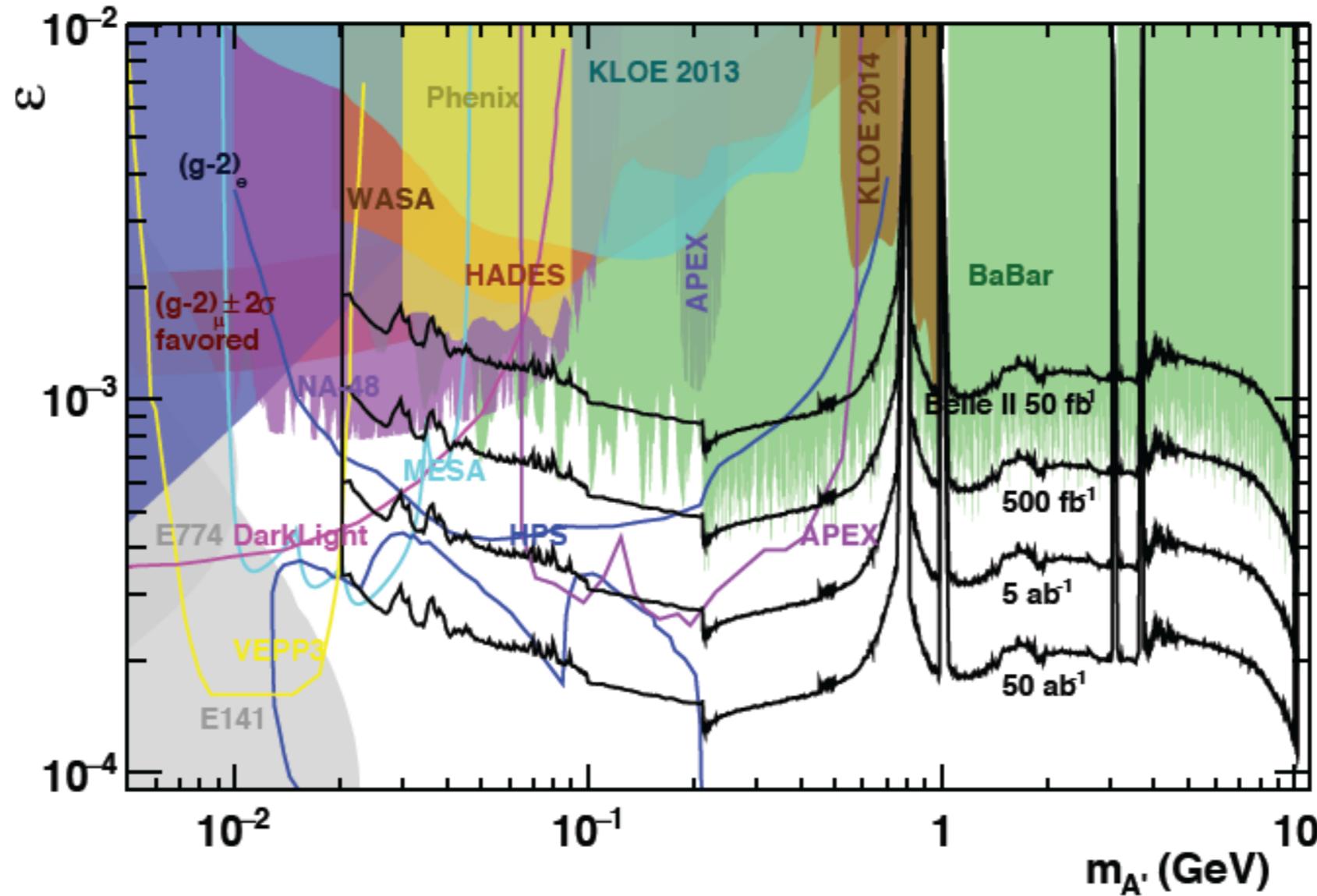
$Z_d$  is expected to be transversely polarized for dark boson mass much less than CM energy



$$= 0 \text{ for } m_e \rightarrow 0$$

# BaBar search result and Belle II sensitivity to $A' \rightarrow e^+e^-, \mu^+\mu^-, hh$

The Belle II physics book, arXiv:1808.10567



Take  $\varepsilon = 7 \times 10^{-4}$ ;  $m_{Z_d} / \sqrt{s} = 0.1, 0.3, \text{ and } 0.8$ .

J. P. Lees et al. [BaBar Collaboration], Phys. Rev. Lett. 113, no. 20, 201801 (2014)

# Polarized amplitudes

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$\theta$  the direction of  $Z_d$  with respect to e- direction in CM frame

$$|\bar{\mathcal{M}}|_+^2 = \frac{8\pi^2\alpha^2\varepsilon^2}{(t - m_e^2)(u - m_e^2)} \left[ (1 + \cos^2 \theta)(s^2 + m_{Z_d}^4) + \rho \cos \theta (s - m_{Z_d}^2)^2 \right],$$
$$|\bar{\mathcal{M}}|_-^2 = \frac{8\pi^2\alpha^2\varepsilon^2}{(t - m_e^2)(u - m_e^2)} \left[ (1 + \cos^2 \theta)(s^2 + m_{Z_d}^4) - \rho \cos \theta (s - m_{Z_d}^2)^2 \right],$$
$$|\bar{\mathcal{M}}|_\parallel^2 = \frac{8\pi^2\alpha^2\varepsilon^2}{(t - m_e^2)(u - m_e^2)} (4m_{Z_d}^2 s \sin^2 \theta),$$

where  $\rho = 4g_{f,V}g_{f,A}$ .  $g_{f,V}^2 + g_{f,A}^2 = 1$

**$m_e$  is neglected except in the denominator**

# Polarized differential cross sections

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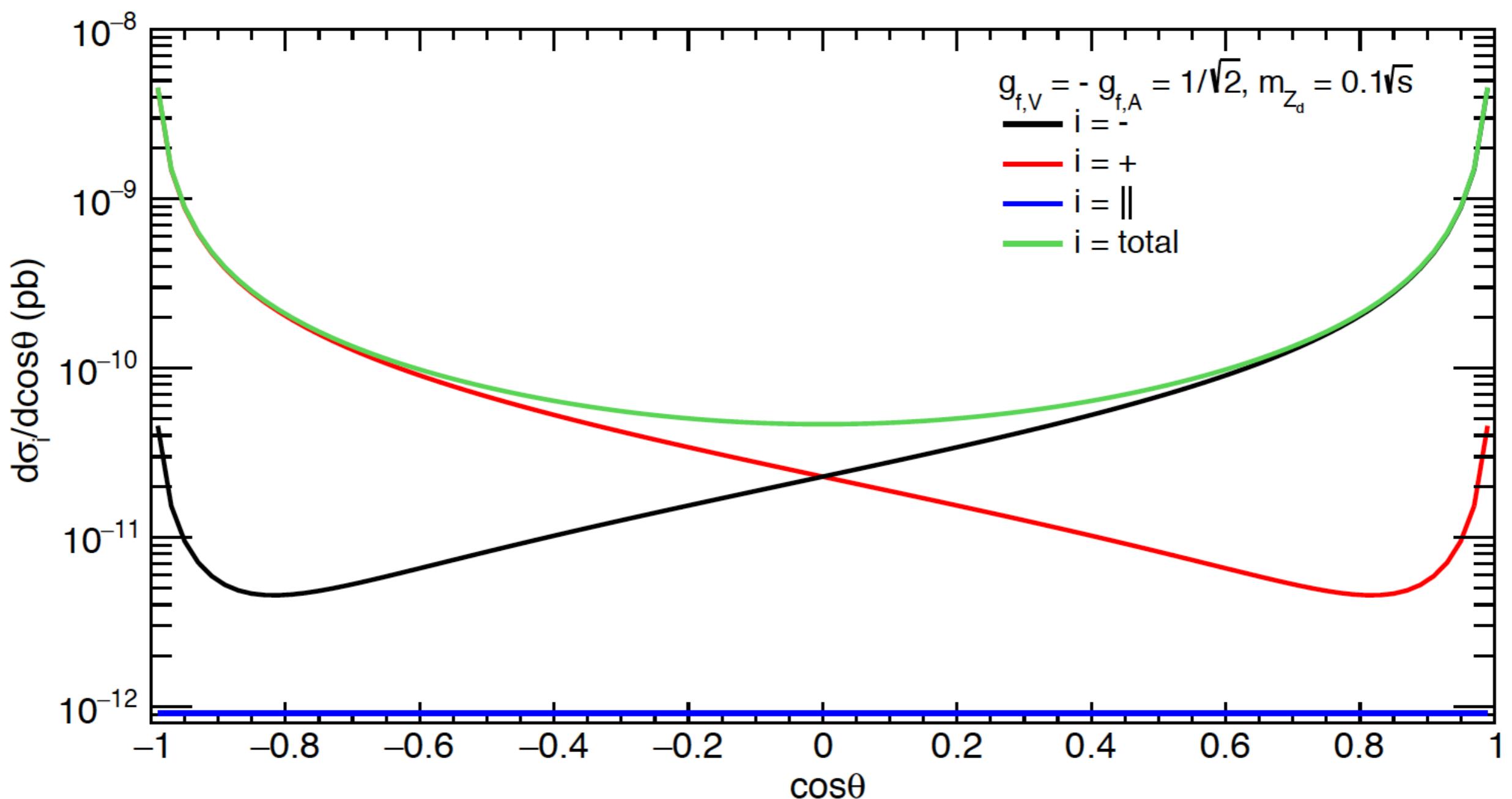
$$\frac{d\sigma_i}{d \cos \theta} = \frac{1}{32\pi s} \left(1 - \frac{m_{Z_d}^2}{s}\right) |\mathcal{M}_i|^2$$

Differential cross section for longitudinal state  
is clearly suppressed by  $m_{Z_d}^2/s$

# Polarized differential cross sections-numerical results

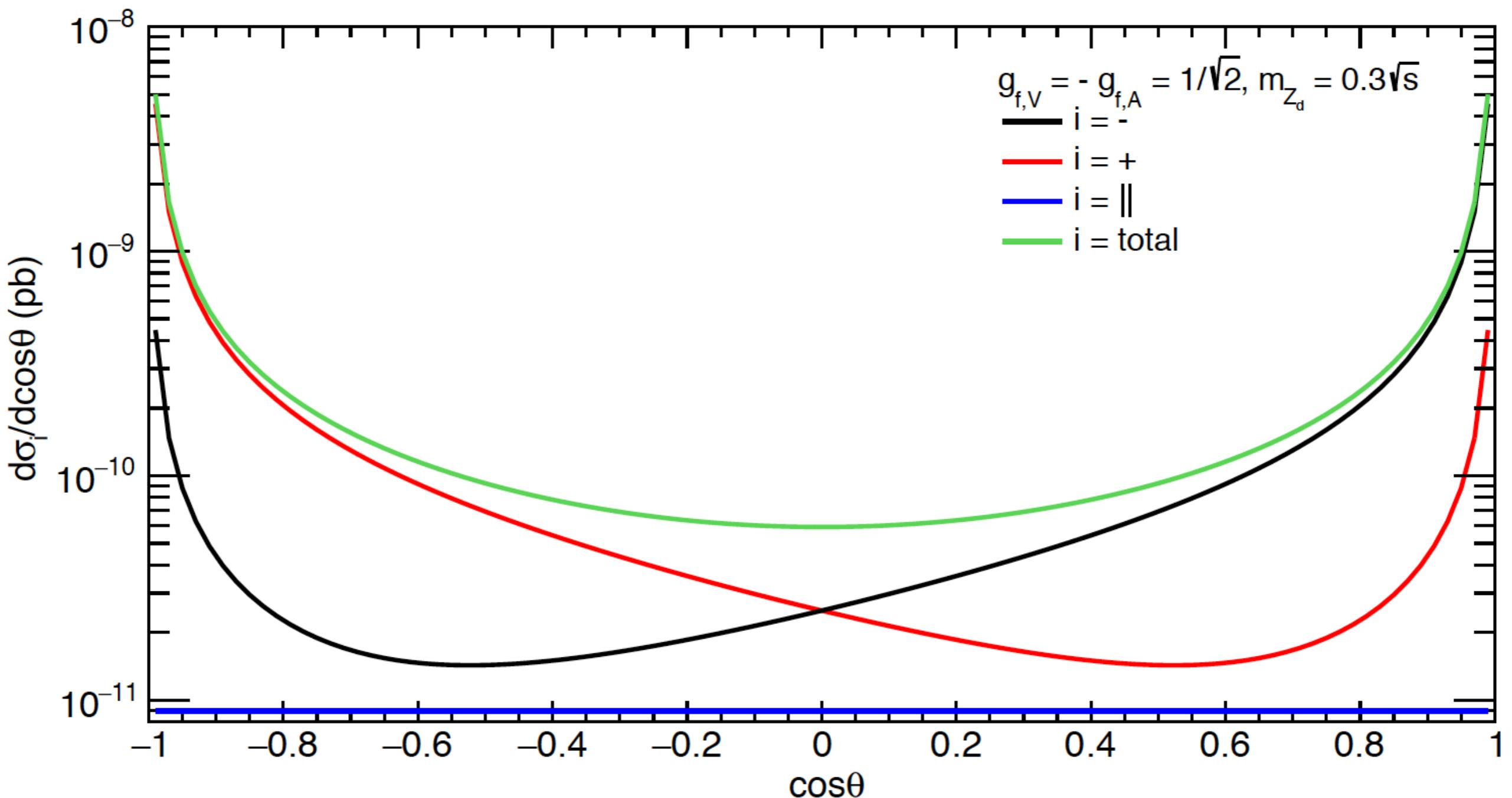
$\varepsilon = 7 \times 10^{-4}$ ;  $\sqrt{s} = 10.58$  GeV CM frame

**V-A coupling**

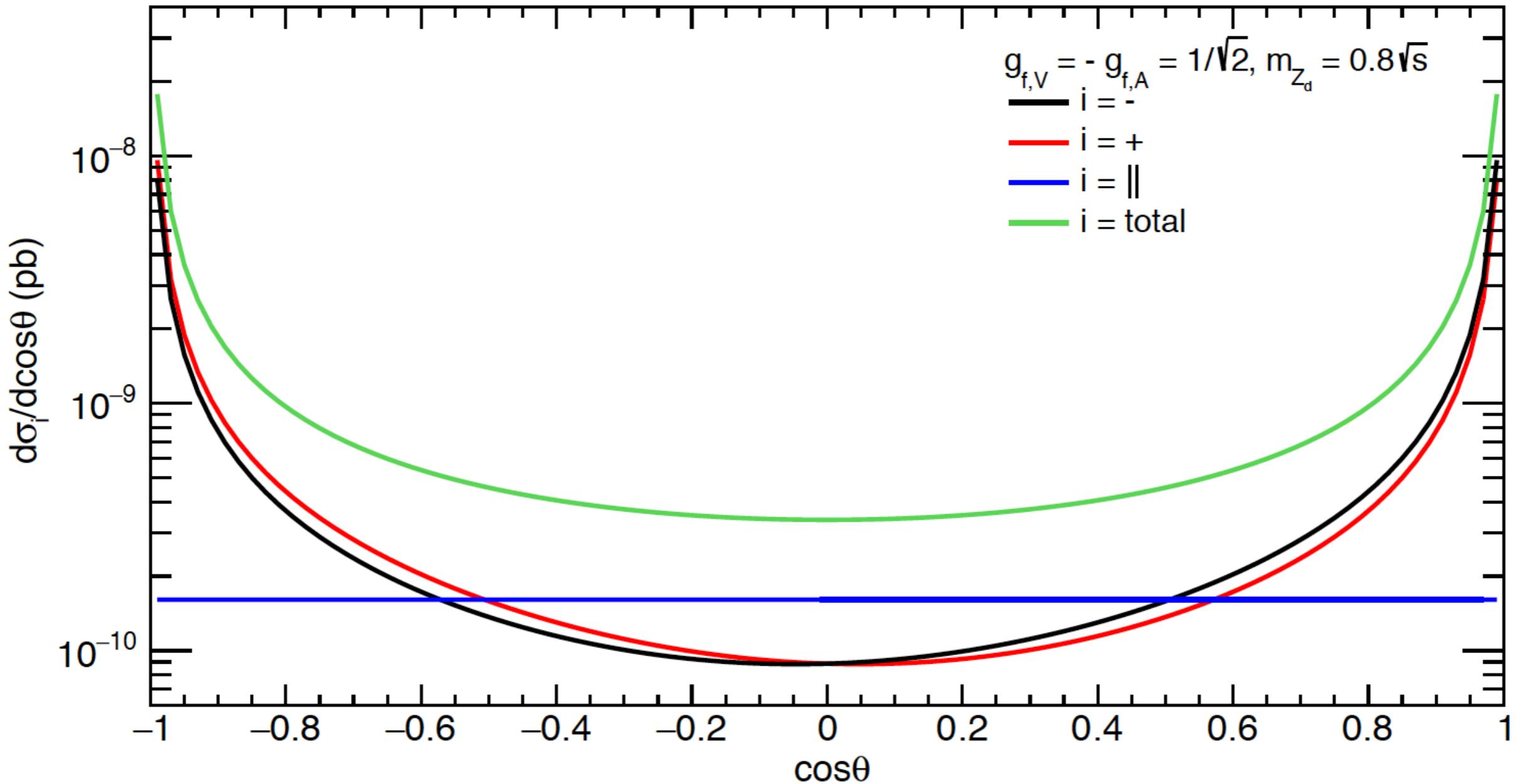


# Polarized differential cross sections-numerical results

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# Polarized differential cross sections-numerical results



Longitudinal polarization is now equally important  
Helicity +1 and -1 states getting closer to each other

# $Z_d$ decay distributions and the parity violation parameter $\rho \equiv 4g_{l,V}g_{l,A}$

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## Angular distributions of $Z_d$ decays

### Helicity +1 state

$$\frac{d\Gamma_{l^+l^-}^+}{d \cos \theta_d} = \frac{\alpha \varepsilon^2 y}{2m_{Z_d}} [2g_{l,V}^2 m_l^2 + (1 + \cos^2 \theta_d)p_l^2 + \rho \cos \theta_d E_l p_l]$$

### Helicity -1 state

$$\frac{d\Gamma_{l^+l^-}^-}{d \cos \theta_d} = \frac{\alpha \varepsilon^2 y}{2m_{Z_d}} [2g_{l,V}^2 m_l^2 + (1 + \cos^2 \theta_d)p_l^2 - \rho \cos \theta_d E_l p_l]$$

### Longitudinal state

$$\frac{d\Gamma_{l^+l^-}^{\parallel}}{d \cos \theta_d} = \frac{\alpha \varepsilon^2 y}{m_{Z_d}} [g_{l,V}^2 m_l^2 + \sin^2 \theta_d p_l^2]$$

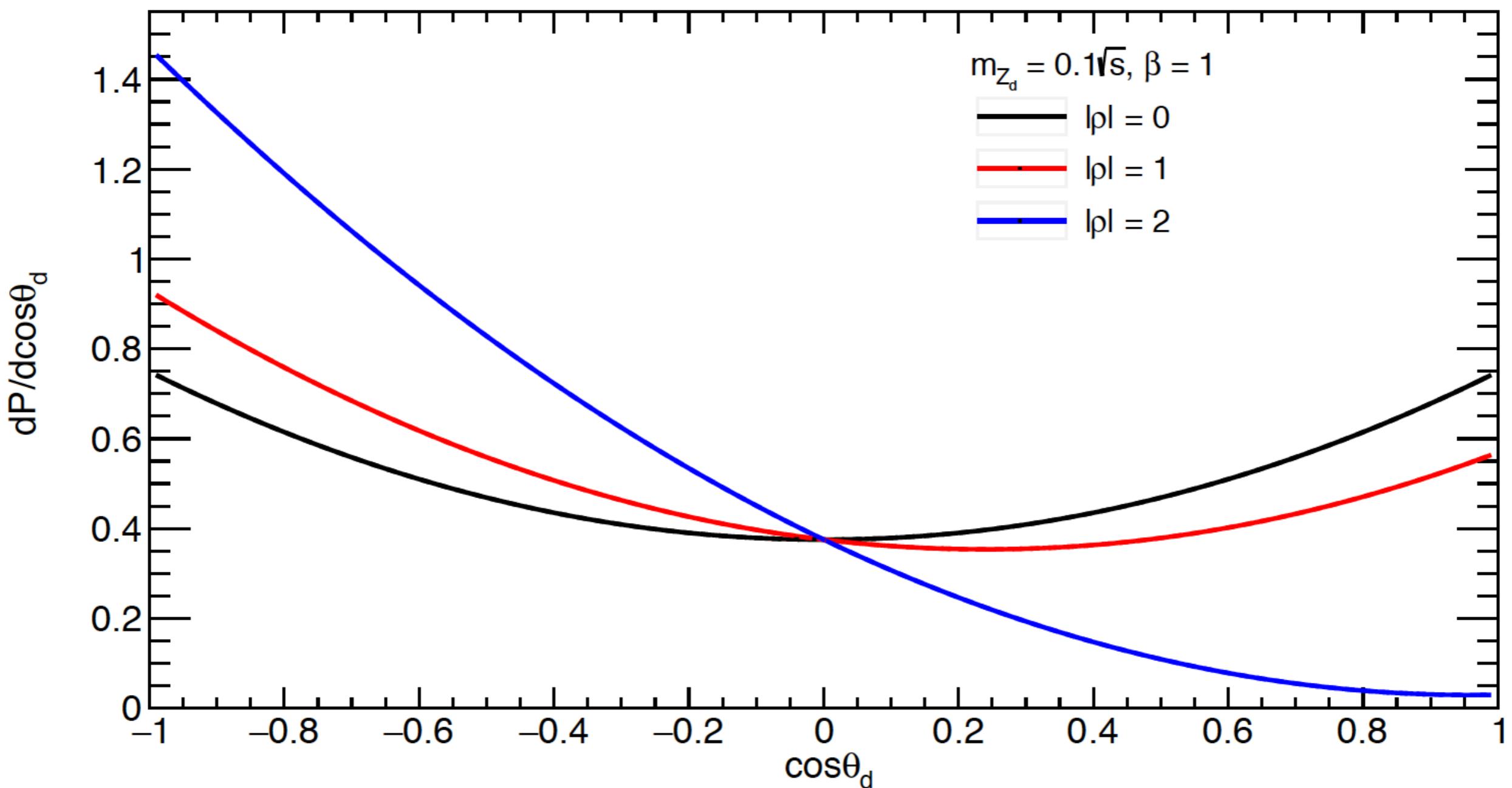
$\theta_d$  the angle between  $l$ -direction in the  $Z_d$  rest frame and the  $Z_d$  boost direction

$$y = \sqrt{1 - 4m_l^2/m_{Z_d}^2}$$

Forward-backward asymmetry of leptons from  $Z_d$  decays  
 $Z_d$  produced in the backward direction  $-1 \leq \cos \theta \leq 0$

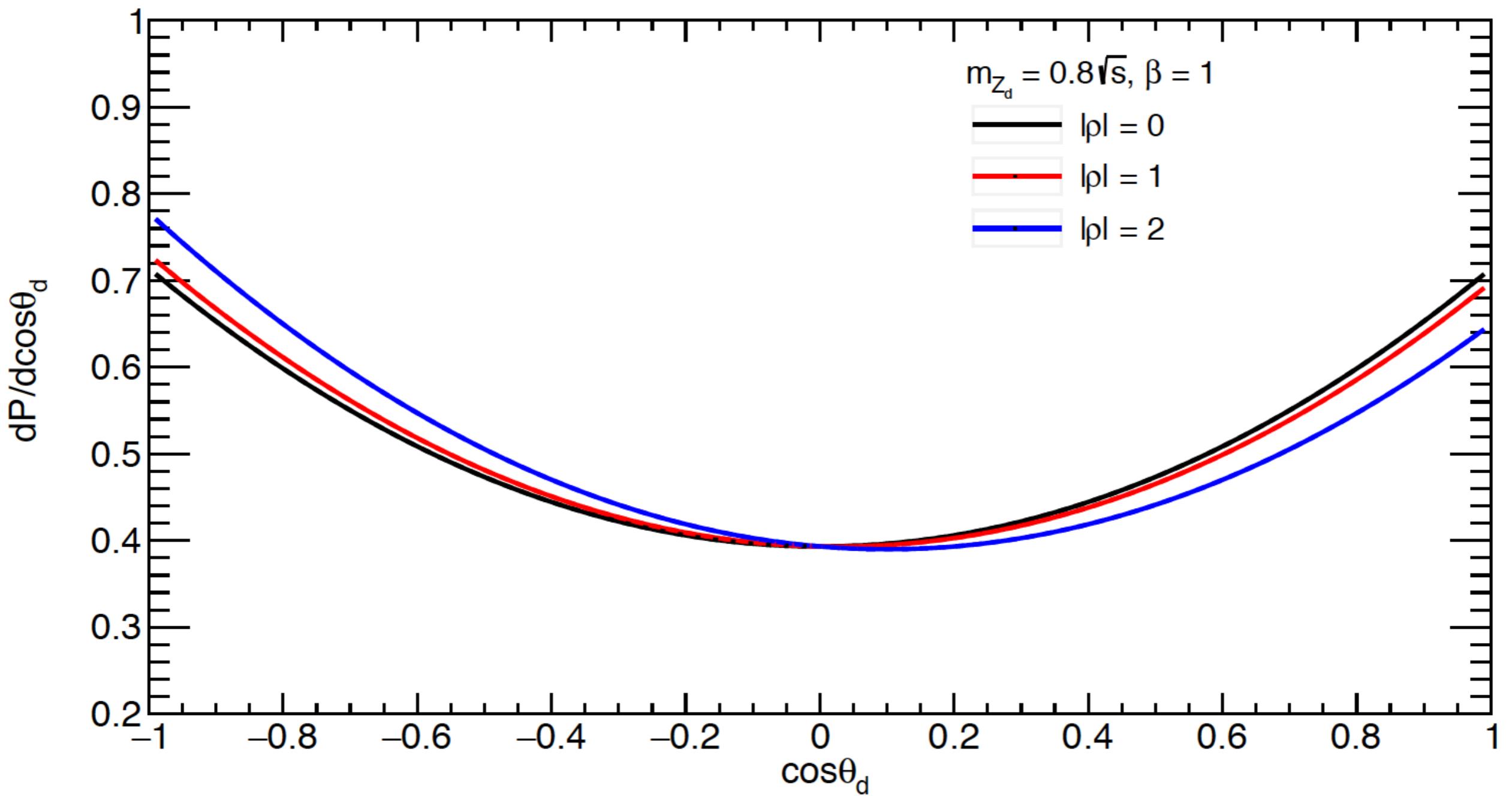
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$$m_{Z_d} = 0.1\sqrt{s}, \beta \equiv p_l/E_l = 1$$



# Forward-backward asymmetry of leptons from $Z_d$ decays

$$m_{Z_d} = 0.8\sqrt{s}, \beta \equiv p_l/E_l = 1$$



# Double angular distributions; correlation between $Z_d$ and lepton directions

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$$\frac{d^2 P}{d\kappa d\xi} = \frac{1}{\sigma_T \cdot \Gamma_{l^+l^-}} \sum_i \left( \frac{d\sigma^i}{d \cos \theta} \right) \cdot \left( \frac{d\Gamma_{l^+l^-}^i}{d \cos \theta_d} \right) \quad i: \text{polarization index}$$
$$= Q_0(\kappa, \xi) + Q_2(\kappa, \xi)\rho^2 \qquad \qquad \kappa = \cos \theta, \quad \xi = \cos \theta_d$$

$Q_0$ : even in both  $\kappa$  and  $\xi$

$Q_2$ : odd in both  $\kappa$  and  $\xi$

$$Q_2 \rho^2 \sim (p_l/E_l) \rho^2 (1 - m_{Z_d}^2/s)^2 \kappa \xi / (1 - \kappa^2)$$

Changes sign when  $\kappa \cdot \xi$  changes sign;  
Reaching to maximum for ultra-relativistic  
lepton and the limit  $s \gg m_{Z_d}^2$

# Signal event asymmetry

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$$\kappa = \cos \theta, \xi = \cos \theta_d$$

$$\mathcal{A}_{\text{PN}} \equiv \frac{S(\kappa \cdot \xi > 0) - S(\kappa \cdot \xi < 0)}{S(\kappa \cdot \xi < 0) + S(\kappa \cdot \xi > 0)} = \frac{3}{4} \left( \frac{\rho^2}{4} \right) \frac{-\ln(1 - \kappa_m^2)}{\ln\left(\frac{1+\kappa_m}{1-\kappa_m}\right) - \kappa_m}$$

$\kappa_m$  : maximum of  $\kappa$        $-\kappa_m$  : minimum of  $\kappa$

$\xi$  : fully integrated

$$\kappa_m = 0.95 \Rightarrow \mathcal{A}_{\text{PN}} = 0.64 \times (\rho^2/4) \quad \varepsilon_\gamma = \varepsilon_Z \quad \rho = 1.74$$

$$\kappa_m = 0.80 \Rightarrow \mathcal{A}_{\text{PN}} = 0.55 \times (\rho^2/4) \quad \varepsilon_\gamma = \varepsilon_Z \tan \theta_W \quad \rho = -2 \\ V - A$$

This parameter has to be calculated with actual detector acceptance

# Prospect of probing parity violation parameter $\rho$ at Belle II

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Belle II calorimeter angular coverage\*  $12.4^\circ \leq \theta_\gamma^{\text{lab}} \leq 155.1^\circ$   
Corresponding photon rapidity range  $-1.51 \leq \eta_\gamma^{\text{lab}} \leq 2.22$

Boost velocity from LAB to CM

$$\beta_{\text{CM}} = (E_{e^-} - E_{e^+})/(E_{e^-} + E_{e^+}) = 3/11$$

7GeV 4GeV

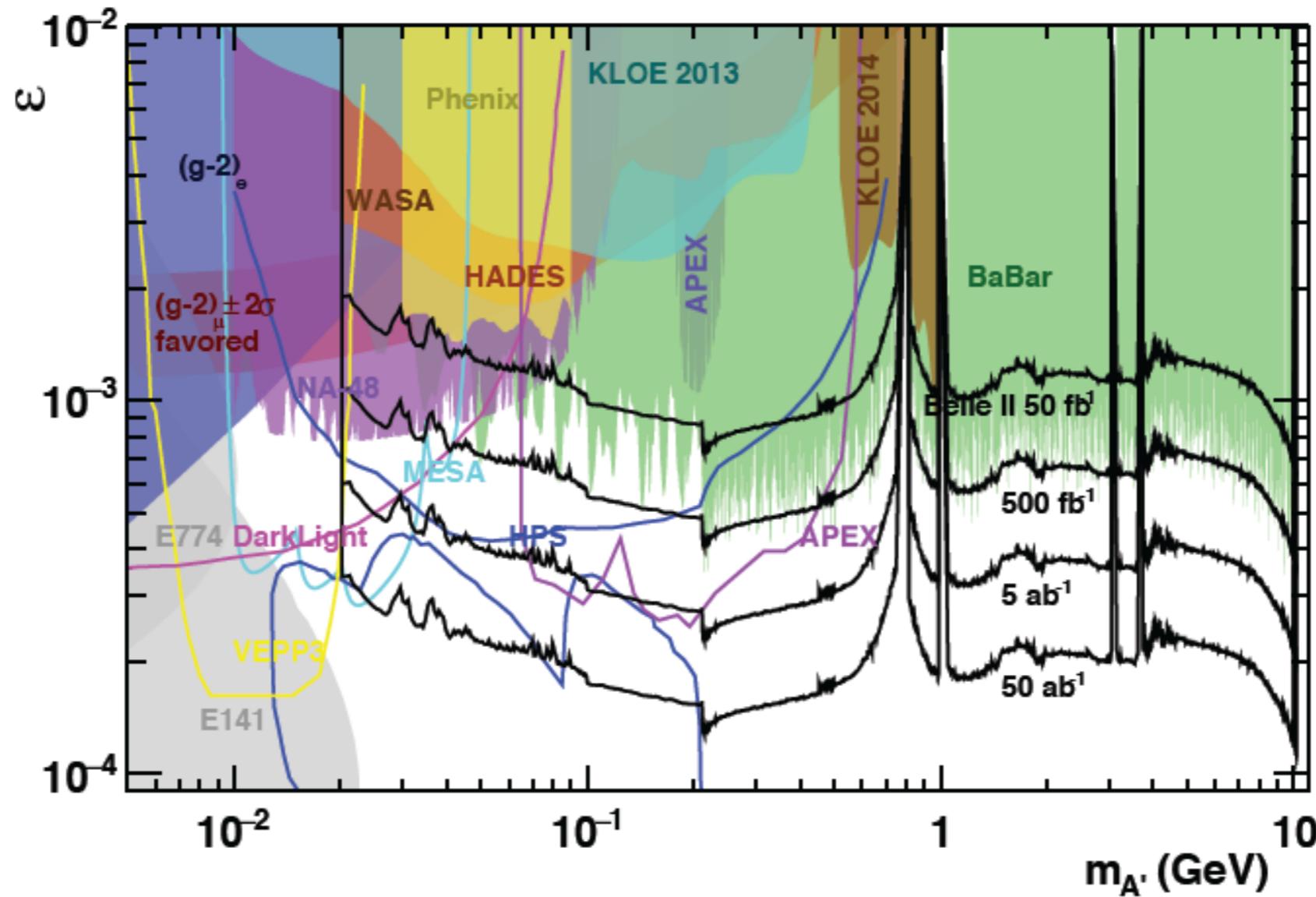
$$\eta_\gamma^{\text{CM}} = \eta_\gamma^{\text{lab}} + \ln((1 - \beta_{\text{CM}})/(1 + \beta_{\text{CM}}))/2 \Rightarrow -1.79 \leq \eta_\gamma^{\text{CM}} \leq 1.94$$

K<sub>L</sub>-muon detector angular coverage  $25^\circ \leq \theta_{\mu^\pm}^{\text{lab}} \leq 150^\circ \Rightarrow -1.60 \leq \eta_{\mu^\pm}^{\text{CM}} \leq 1.23$

\*I. Adachi *et al.* [Belle II], Nucl. Instrum. Meth. A 907, 46-59 (2018)

# BaBar search result and Belle II sensitivity to $A' \rightarrow e^+e^-, \mu^+\mu^-, hh$

The Belle II physics book, arXiv:1808.10567



J. P. Lees et al. [BaBar Collaboration], Phys. Rev. Lett. 113, no. 20, 201801 (2014)

Belle II sensitivity is comparable to BaBar results for the same integrated luminosity

## Calculating $\mathcal{A}_{\text{PN}}$ in Belle II

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$$\mathcal{A}_{\text{PN}} \equiv \frac{S(\kappa \cdot \xi > 0) - S(\kappa \cdot \xi < 0)}{S(\kappa \cdot \xi < 0) + S(\kappa \cdot \xi > 0)}$$

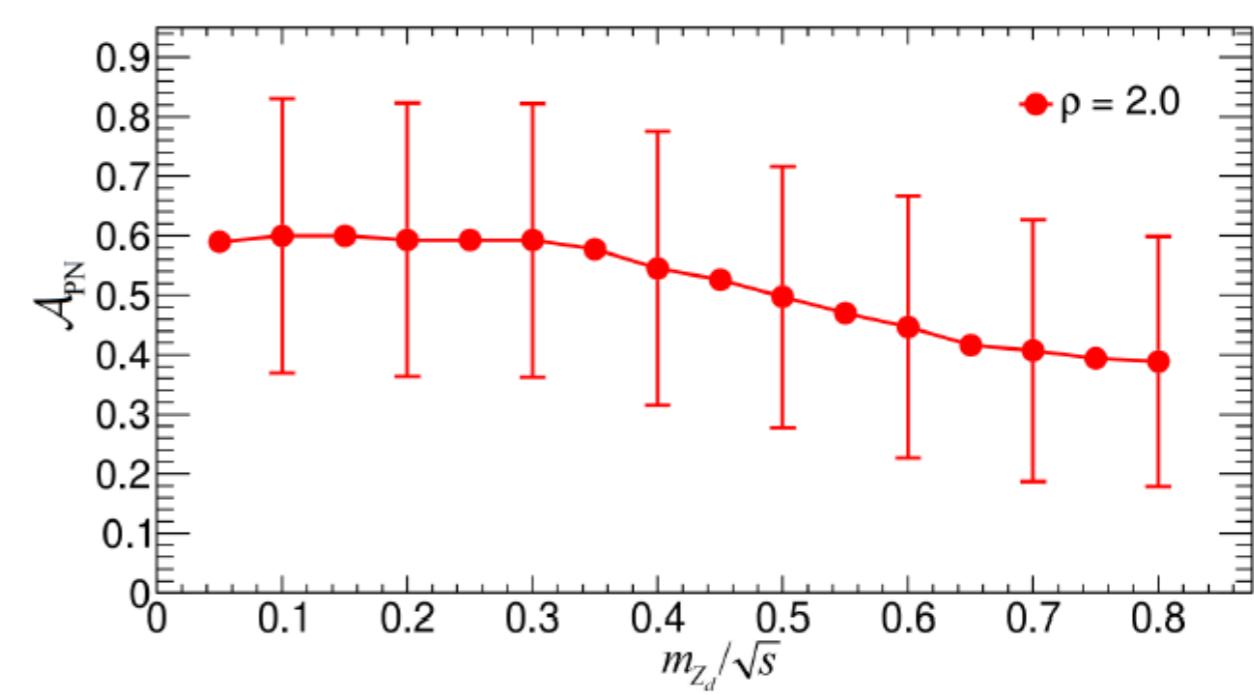
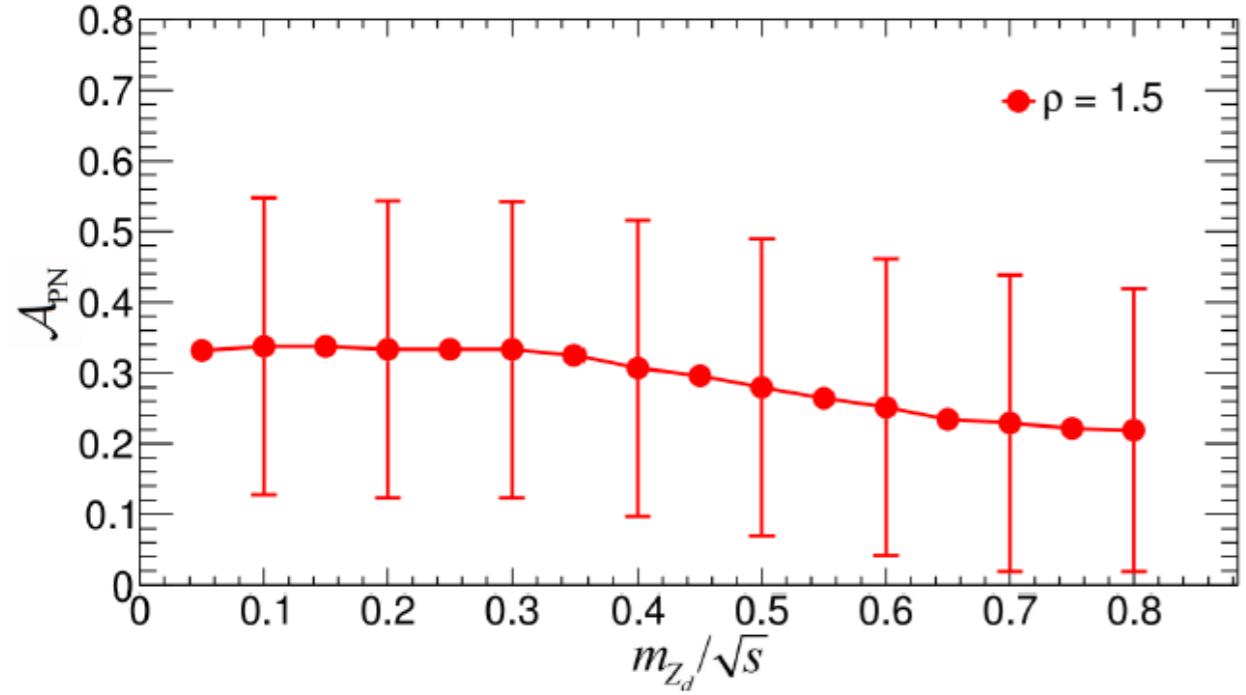
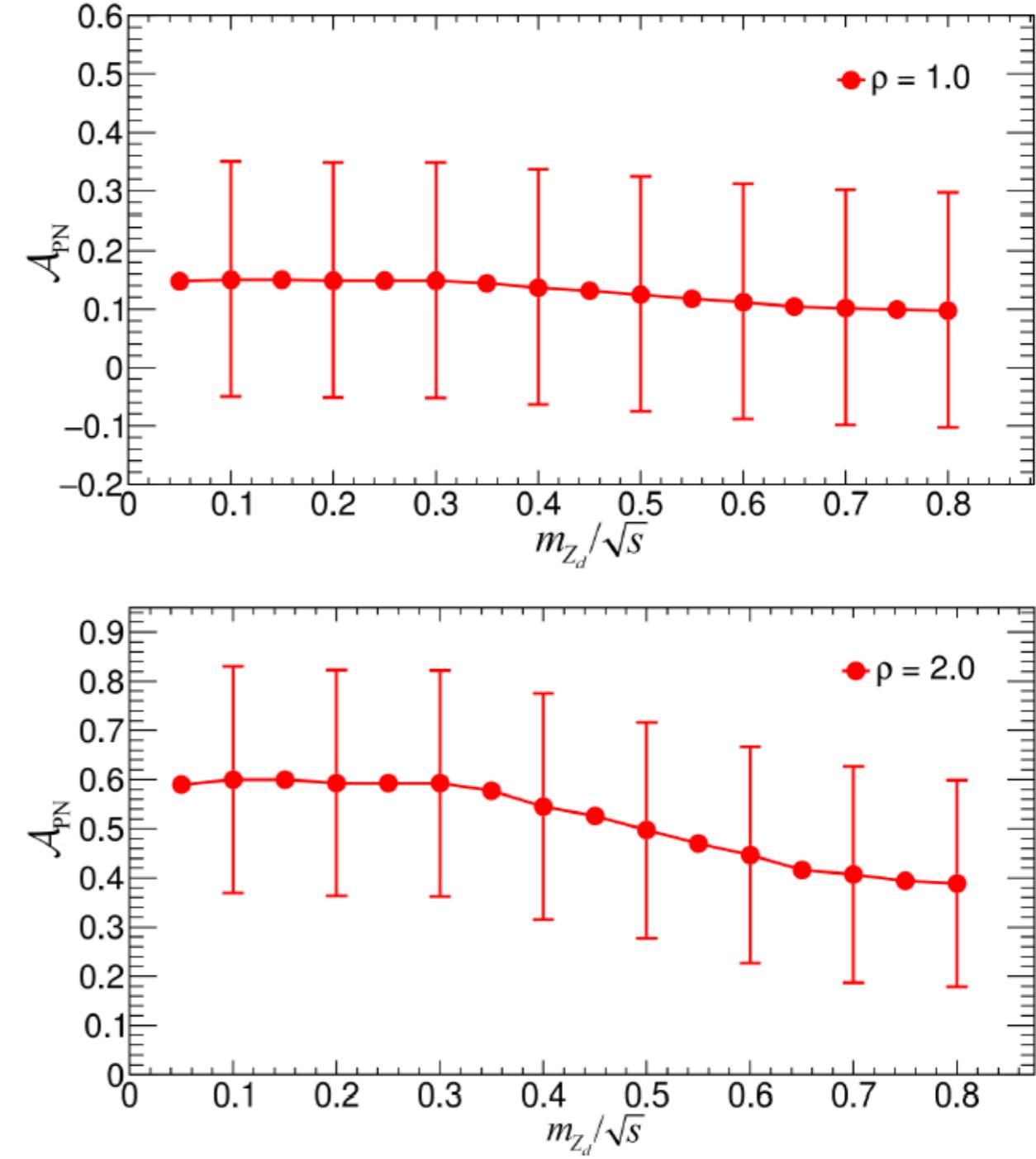
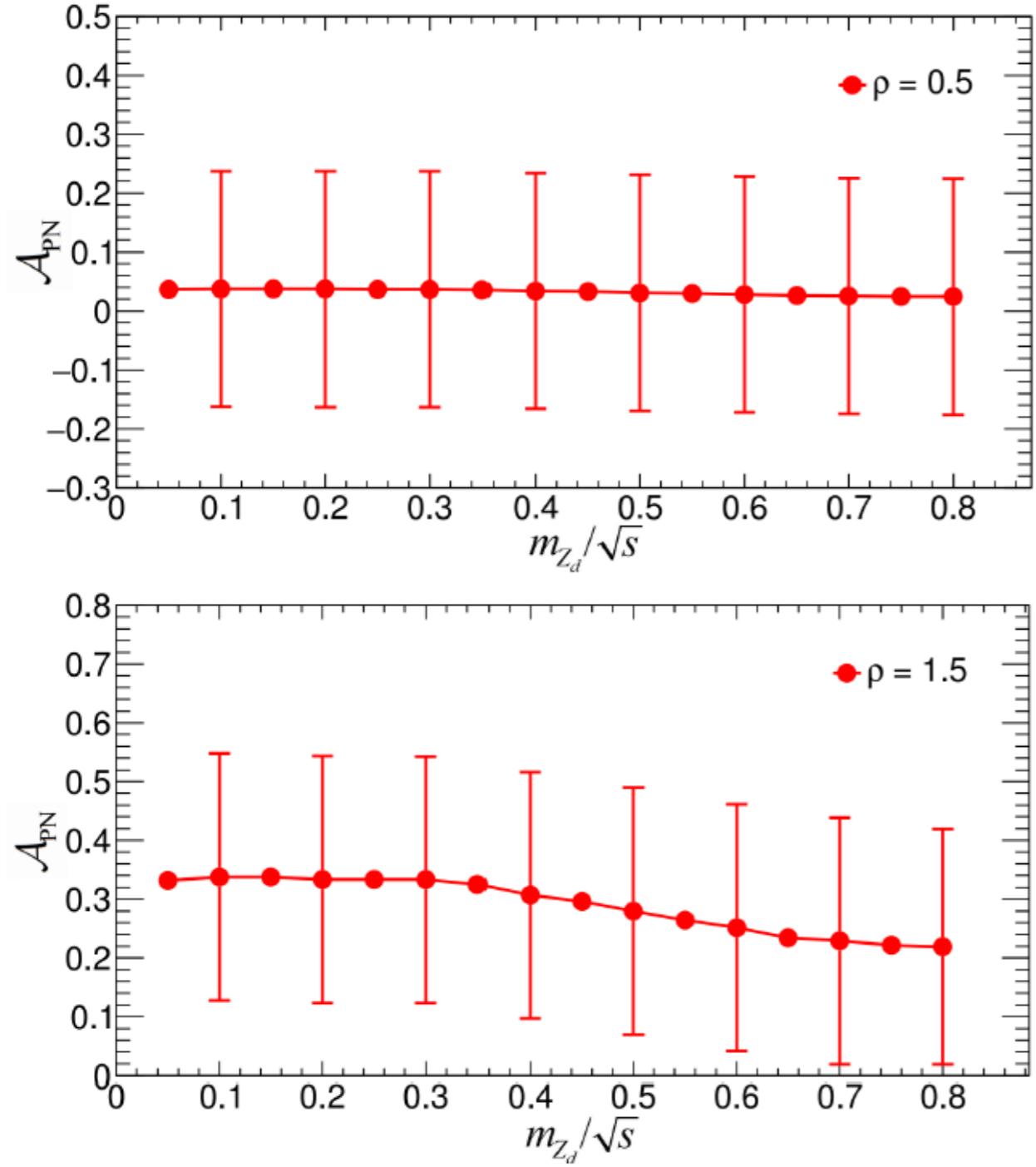
$$\sigma_{\mathcal{A}_{\text{PN}}} = \sqrt{1 + \mathcal{A}_{\text{PN}}^2} (\sqrt{B}/S)$$

Assume a  $5\sigma$  detection of dark boson signature at  $50 \text{ ab}^{-1}$

$$S = 5\sqrt{B}$$

# Results

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# Detection significance and asymmetry parameter

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$$\chi^2 = 2 \left( n \ln\left(\frac{n}{w}\right) + w - n \right)$$

$n$ : observed event number  
 $w$ : expected event number

$$n=S+B, w=B \quad \text{Detection significance} \quad \frac{S}{\sqrt{B}} \cdot \sigma$$

Simultaneous fittings to  $\kappa \cdot \xi > 0$  and  $\kappa \cdot \xi < 0$  event bins

$$\chi^2 = 2 \left( n_a \ln\left(\frac{n_a}{w_a}\right) + w_a - n_a \right) + 2 \left( n_b \ln\left(\frac{n_b}{w_b}\right) + w_b - n_b \right)$$

$$n_{a,b} = S_{a,b} + B_{a,b} \quad (S_a + S_b = S, B_a + B_b = B)$$

$$\mathcal{A}_{\text{PN}} = (S_a - S_b)/(S_a + S_b)$$

$$\text{Detection significance} \quad \frac{S}{\sqrt{B}} \sqrt{1 + \mathcal{A}_{\text{PN}}^2} \cdot \sigma$$

# Numerical results with Belle II detector angular coverage

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Cross section\* for QED background  $e^+e^- \rightarrow \gamma\mu^+\mu^-$  with photon and muon rapidity ranges and  $\sim 5$  MeV energy resolution for the invariant mass  $M_{\mu^+\mu^-}$

$\sim 7.76 \times 10^{-2}$ pb	for $M_{\mu^+\mu^-} \simeq 0.5$ GeV	1.5 MeV to 8 MeV energy resolution taken in BaBar analysis
$\sim 2.48 \times 10^{-2}$ pb	for $M_{\mu^+\mu^-} \simeq 2.0$ GeV	

Assume a  $5\sigma$  detection of dark boson signature at  $50 \text{ ab}^{-1}$

$$S = 9850, B = 3.88 \cdot 10^6 \quad m_{Z_d} = 0.5 \text{ GeV}$$

$$S = 5700, B = 1.30 \cdot 10^6 \quad m_{Z_d} = 2.0 \text{ GeV}$$

\*CalcHEP version 3.7.5, A. Pukhov, A. Belyaev, and N. Christensen, 2019

# Summary on asymmetry parameters, event numbers and detection significance

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$ \rho $	0.00	1.74	2.00			
$m_{Z_d}/\text{GeV}$	0.5	2.0	0.5	2.0	0.5	2.0
$\mathcal{A}_{\text{PN}}$	0.0	0.0	0.43	0.44	0.58	0.60
Det. Sig. (Eq. (15))	$5.0\sigma$	$5.0\sigma$	$5.4\sigma$	$5.5\sigma$	$5.8\sigma$	$5.8\sigma$
$S(\kappa \cdot \xi > 0)$	4925	2817	7040	4053	7780	4507
$S(\kappa \cdot \xi < 0)$	4925	2817	2810	1581	2070	1127
$\text{Br}(Z_d \rightarrow \mu^+ \mu^-)$	40%	24%	21%	7.5%	17%	6.7%
$\varepsilon \cdot 10^4$	3.3	3.2	4.6	5.7	5.1	6.1

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# Conclusions

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- We have discussed the search for dark boson with the process  $e^+e^- \rightarrow Z_d + \gamma$  in the  $e^+e^-$  collider
- The dark boson is shown to be transversely polarized when the dark boson mass is much less than the CM energy
- We analyze the muon angular distributions from polarized  $Z_d$  decays and define the asymmetry parameter  $\mathcal{A}_{PN}$  which is proportional to the square of parity violation parameter  $\rho \equiv 4g_{l,V}g_{l,A}$ .
- We calculate the asymmetry parameter with Belle II detector angular coverage and discuss its consequences on the dark boson search.