Polarization effects in the search for dark vector boson in $e^+e^-$ colliders

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Phys.Rev.D 103 (2021) 1, 015016
Introduction and motivations

- Growing interests in searching for DM related phenomenon with high statistics and high precision measurements.

- Such phenomenon has to do hidden sector*, assumed to interact with the visible sector through a messenger particle.

- A popular proposal for such a messenger is the so-called dark photon**, which mixes with $U(1)_Y$ in SM.

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Introduction and motivations

- Such a mixing induces EM couplings between dark photon and SM fermions, which generate rich phenomenology.

- The search for light boson with the reaction $e^+e^- \rightarrow A' + \gamma$ has been proposed*.

- Many new proposals to search for dark photons with the above process—see the list next page

- These proposals are based upon either fixed target or electron-positron collider

Introduction and motivations

- B. Wojtsekhowski et al., JINST 13, no. 02, P02021 (2018)
Introduction and motivations

• The dark photon interaction with EM current is given by
  \[ \mathcal{L}_{\text{int}} = \varepsilon_\gamma e J_{\text{em}}^{\mu} A'_\mu \quad \varepsilon_\gamma \equiv \varepsilon \text{ in} \]
  \[ \mathcal{L}_{\text{gauge}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} \frac{\varepsilon}{\cos \theta_W} B^{\mu\nu} A'_\mu A'_\nu - \frac{1}{4} A'_{\mu\nu} A'^{\mu\nu} \]

• The neutral current interaction is suppressed in the limit \( M_{A'} \ll M_Z \)

• The detection of \( A' \) determines the mixing parameter and the mass of the dark photon.

• On the other hand, there could be other mixing between dark boson and SM gauge bosons, such as:

  \[ \mathcal{L}_{\text{mass}} = \frac{1}{2} M_Z^2 Z_\mu^0 Z^{0\mu} - \delta m^2 Z_\mu^0 A'^{\mu} + \frac{1}{2} M_{A'}^2 A'_\mu A'^{\mu} \]

Introduction and motivations

• With the above mass mixing, an independent neutral current coupling between dark boson and SM fermions is induced:

$$\mathcal{L}_{\text{int}} = \varepsilon_Z \frac{g}{\cos \theta_W} J_{\text{NC}}^\mu A'_\mu \text{ with } \varepsilon_Z \equiv \delta m^2 / M_Z^2$$

• Considering both mixings, the interaction between dark boson (renamed as $Z_d$ from now on) and SM fermions becomes

$$e\varepsilon \bar{f}(g_f, V \gamma_\mu + g_f, A \gamma_\mu \gamma_5) f Z_d^\mu$$

• In the search for $Z_d$ with $e^+e^- \rightarrow Z_d +\gamma$, can one determine the relative strength of vector and axial-vector couplings?

• The key is on the polarization of $Z_d$
Outline

• Heuristic derivation of $Z_d$-fermion interactions

• Ward-Takahashi identity and the polarization of $Z_d$ in $e^+e^- \rightarrow Z_d + \gamma$

• Differential cross section of $e^+e^- \rightarrow Z_d + \gamma$ for each polarization of $Z_d$ and the decay distribution of $Z_d \rightarrow l^+l^-$

• Searching for $Z_d$ by $e^+e^- \rightarrow Z_d + \gamma$ and $Z_d$ decaying to muon pairs in BaBar and Belle II

• Summary
Heuristic derivation of $Z_d$-fermion interactions

The mixing terms give two point functions

\[ i \Pi^{\mu\nu}_{A_{Z_d}} = i \varepsilon k^2 g^{\mu\nu}, \]
\[ i \Pi^{\mu\nu}_{Z_{Z_d}} = -i (\varepsilon \tan \theta_W k^2 + \delta m^2) g^{\mu\nu}, \]

The $EM$ interactions of dark boson

\[ i e J^\alpha_{em} \frac{-ig_{\alpha\mu}}{k^2} i \varepsilon k^2 g^{\mu\nu} Z_{d\nu} = i e \varepsilon J^\nu_{em} Z_{d\nu}. \]

The $Neutral-Current$ interactions of dark boson

\[ \frac{ig}{\cos \theta_W} J^\alpha_{NC} \frac{-i}{k^2 - M_Z^2} (g_{\alpha\mu} - \frac{k_{\alpha} k_{\mu}}{M_Z^2}) \cdot (-i) (\varepsilon \tan \theta_W k^2 + \delta m^2) g^{\mu\nu} Z_{d\nu} \]
\[ = \frac{-ig}{\cos \theta_W} J^\nu_{NC} Z_{d\nu} \frac{(\varepsilon \tan \theta_W M_{Z_d}^2 + \delta m^2)}{(M_{Z_d}^2 - M_Z^2)}. \]
Heuristic derivation of $Z_d$-fermion interactions

In the limit $M_{Z_d} \ll M_Z$

$$\mathcal{L}_{\text{int}} = \left( \varepsilon_\gamma e J^\mu_{\text{em}} + \varepsilon_Z \frac{g}{\cos \theta_W} J^\mu_{\text{NC}} \right) Z_{d\mu},$$

$$\varepsilon_Z \equiv \delta m^2 / m^2_Z.$$
Ward-Takahashi identity and $Z_d$ polarization

\[ \epsilon^\mu(k_1) = (|\vec{k}_1|, E_{Z_d} \hat{k}_1)/m_{Z_d} \]

\[ = k_1^\mu/m_{Z_d} + \mathcal{O}(m_{Z_d}/E_{Z_d}) \]

$Z_d$ is expected to be transversely polarized for dark boson mass much less than CM energy

$= 0$ for $m_e \rightarrow 0$
BaBar search result and Belle II sensitivity to

\[ A' \rightarrow e^+ e^-, \mu^+ \mu^-, hh \]


Take \( \varepsilon = 7 \times 10^{-4}; m_{Z_d}/\sqrt{s} = 0.1, 0.3, \) and 0.8.

Polarized amplitudes

\[ \theta \] the direction of \( Z_d \) with respect to \( e^- \) direction in CM frame

\[ |\vec{M}|_+^2 = \frac{8\pi^2 \alpha^2 \varepsilon^2}{(t - m_e^2)(u - m_e^2)} \left[(1 + \cos^2 \theta)(s^2 + m_{Z_d}^4) + \rho \cos \theta (s - m_{Z_d}^2)^2\right], \]

\[ |\vec{M}|_-^2 = \frac{8\pi^2 \alpha^2 \varepsilon^2}{(t - m_e^2)(u - m_e^2)} \left[(1 + \cos^2 \theta)(s^2 + m_{Z_d}^4) - \rho \cos \theta (s - m_{Z_d}^2)^2\right], \]

\[ |\vec{M}|_{\parallel}^2 = \frac{8\pi^2 \alpha^2 \varepsilon^2}{(t - m_e^2)(u - m_e^2)} (4m_{Z_d}^2 s \sin^2 \theta), \]

where \( \rho = 4g_{f,V}g_{f,A} \).

\[ g_{f,V}^2 + g_{f,A}^2 = 1 \]

\( m_e \) is neglected except in the denominator.
Polarized differential cross sections

\[
\frac{d\sigma_i}{d\cos \theta} = \frac{1}{32\pi s} \left(1 - \frac{m_{Z_d}^2}{s}\right) |\mathcal{M}|_i^2
\]

Differential cross section for longitudinal state is clearly suppressed by \(m_{Z_d}^2/s\)
Polarized differential cross sections—numerical results

\[ \varepsilon = 7 \times 10^{-4}; \sqrt{s} = 10.58 \text{ GeV CM frame} \]

V-A coupling
Polarized differential cross sections-numerical results

$g_{f,V} = -g_{f,A} = 1/\sqrt{2}$, $m_{Z_t} = 0.3\sqrt{s}$
Longitudinal polarization is now equally important. Helicity +1 and -1 states getting closer to each other.
Z\textsubscript{d} decay distributions and the parity violation parameter \( \rho \equiv 4g_{l,V}g_{l,A} \)

**Angular distributions of Z\textsubscript{d} decays**

**Helicity +1 state**
\[
\frac{d\Gamma_{l+l-}^{+}}{d\cos \theta_{d}} = \frac{\alpha \varepsilon^2 y}{2m_{Z_d}} \left[ 2g_{l,V}m_{l}^2 + (1 + \cos^2 \theta_{d})p_{l}^2 + \rho \cos \theta_{d}E_{l}p_{l} \right]
\]

**Helicity -1 state**
\[
\frac{d\Gamma_{l+l-}^{-}}{d\cos \theta_{d}} = \frac{\alpha \varepsilon^2 y}{2m_{Z_d}} \left[ 2g_{l,V}m_{l}^2 + (1 + \cos^2 \theta_{d})p_{l}^2 - \rho \cos \theta_{d}E_{l}p_{l} \right]
\]

**Longitudinal state**
\[
\frac{d\Gamma_{l+l-}^{\parallel}}{d\cos \theta_{d}} = \frac{\alpha \varepsilon^2 y}{m_{Z_d}} \left[ g_{l,V}m_{l}^2 + \sin^2 \theta_{d}p_{l}^2 \right]
\]

\[ y = \sqrt{1 - \frac{4m_{l}^2}{m_{Z_d}^2}} \]
Forward-backward asymmetry of leptons from $Z_d$ decays
$Z_d$ produced in the backward direction $-1 \leq \cos \theta \leq 0$

\[ m_{Z_d} = 0.1 \sqrt{s}, \beta \equiv p_l/E_l = 1 \]

![Graph showing the distribution of $dP/d\cos\theta_d$ vs $\cos\theta_d$ for $m_{Z_d} = 0.1 \sqrt{s}, \beta = 1$ with different $|p_l|$ values: $|p_l| = 0$, $|p_l| = 1$, $|p_l| = 2$.](image-url)
Forward-backward asymmetry of leptons from \( Z_d \) decays

\[ m_{Z_d} = 0.8\sqrt{s}, \beta \equiv p_l/E_l = 1 \]
Double angular distributions; correlation between $Z_d$ and lepton directions

\[
\frac{d^2 P}{dkd\xi} = \frac{1}{\sigma_T \cdot \Gamma_{l+l^-}} \sum_i \left( \frac{d\sigma^i}{d\cos\theta} \right) \cdot \left( \frac{d\Gamma^i_{l+l^-}}{d\cos\theta_d} \right)
\]

\[
= Q_0(\kappa, \xi) + Q_2(\kappa, \xi) \rho^2
\]

\[\kappa = \cos\theta, \quad \xi = \cos\theta_d\]

$Q_0$: even in both $\kappa$ and $\xi$

$Q_2$: odd in both $\kappa$ and $\xi$

\[
Q_2 \rho^2 \sim \left( \frac{p_l}{E_l} \right) \rho^2 (1 - m^2_{Z_d}/s)^2 \kappa\xi/(1 - \kappa^2)
\]

Changes sign when $\kappa \cdot \xi$ changes sign;

Reaching to maximum for ultra-relativistic lepton and the limit $s \gg m^2_{Z_d}$
Signal event asymmetry

\[ \kappa = \cos \theta, \quad \xi = \cos \theta_d \]

\[ A_{\text{PN}} \equiv \frac{S(\kappa \cdot \xi > 0) - S(\kappa \cdot \xi < 0)}{S(\kappa \cdot \xi < 0) + S(\kappa \cdot \xi > 0)} = \frac{3}{4} \left( \frac{\rho^2}{4} \right) \frac{-\ln \left( 1 - \kappa_m^2 \right)}{\ln \left( \frac{1+\kappa_m}{1-\kappa_m} \right) - \kappa_m} \]

\( \kappa_m \) : maximum of \( \kappa \) \quad \(-\kappa_m\) : minimum of \( \kappa \)

\( \xi \) : fully integrated

\( \kappa_m = 0.95 \Rightarrow A_{\text{PN}} = 0.64 \times \left( \frac{\rho^2}{4} \right) \quad \varepsilon_\gamma = \varepsilon_Z \quad \rho = 1.74 \)

\( \kappa_m = 0.80 \Rightarrow A_{\text{PN}} = 0.55 \times \left( \frac{\rho^2}{4} \right) \quad \varepsilon_\gamma = \varepsilon_Z \tan \theta_W \quad \rho = -2 \frac{V - A}{V + A} \)

This parameter has to be calculated with actual detector acceptance
Prospect of probing parity violation parameter $\rho$ at Belle II

Belle II calorimeter angular coverage*

Corresponding photon rapidity range

Boost velocity from LAB to CM

$\beta_{\text{CM}} = (E_{e^-} - E_{e^+})/(E_{e^-} + E_{e^+}) = 3/11$

7GeV 4GeV

$\eta_{\gamma}^{\text{CM}} = \eta_{\gamma}^{\text{lab}} + \ln((1 - \beta_{\text{CM}})/(1 + \beta_{\text{CM}}))/2 \Rightarrow -1.79 \leq \eta_{\gamma}^{\text{CM}} \leq 1.94$

$K_\Lambda$-muon detector angular coverage

$25^\circ \leq \theta_{\mu}^{\text{lab}} \leq 150^\circ \Rightarrow -1.60 \leq \eta_{\mu}^{\text{CM}} \leq 1.23$

BaBar search result and Belle II sensitivity to

\[ A' \rightarrow e^+ e^-, \mu^+ \mu^-, hh \]


Belle II sensitivity is comparable to BaBar results for the same integrated luminosity
Calculating $\mathcal{A}_{PN}$ in Belle II

$$\mathcal{A}_{PN} \equiv \frac{S(\kappa \cdot \xi > 0) - S(\kappa \cdot \xi < 0)}{S(\kappa \cdot \xi < 0) + S(\kappa \cdot \xi > 0)}$$

$$\sigma_{\mathcal{A}_{PN}} = \sqrt{1 + \mathcal{A}_{PN}^2(\sqrt{B}/S)}$$

Assume a 5 $\sigma$ detection of dark boson signature at 50 ab$^{-1}$

$$S = 5\sqrt{B}$$

CalcHEP version 3.7.5, A. Pukhov, A. Belyaev, and N. Christensen, 2019
Results
Detection significance and asymmetry parameter

\[ \chi^2 = 2 \left( n \ln\left( \frac{n}{w} \right) + w - n \right) \]

\( n \): observed event number

\( w \): expected event number

\[ n = S + B, \quad w = B \]

Detection significance

Simultaneous fittings to \( \kappa \cdot \xi > 0 \) and \( \kappa \cdot \xi < 0 \) event bins

\[ \chi^2 = 2 \left( n_a \ln\left( \frac{n_a}{w_a} \right) + w_a - n_a \right) + 2 \left( n_b \ln\left( \frac{n_b}{w_b} \right) + w_b - n_b \right) \]

\[ n_{a,b} = S_{a,b} + B_{a,b} \quad (S_a + S_b = S, \quad B_a + B_b = B) \]

\[ \mathcal{A}_{PN} = (S_a - S_b) / (S_a + S_b) \]

Detection significance

\[ \frac{S}{\sqrt{B}} \sqrt{1 + \mathcal{A}_{PN}^2} \cdot \sigma \]
Numerical results with Belle II detector angular coverage

Cross section* for QED background $e^+e^- \rightarrow \gamma \mu^+\mu^-$ with photon and muon rapidity ranges and $\sim$5 MeV energy resolution for the invariant mass $M_{\mu^+\mu^-}$

\begin{align*}
\sim 7.76 \times 10^{-2} \text{ pb} & \quad \text{for } M_{\mu^+\mu^-} \approx 0.5 \text{ GeV} \\
\sim 2.48 \times 10^{-2} \text{ pb} & \quad \text{for } M_{\mu^+\mu^-} \approx 2.0 \text{ GeV}
\end{align*}

Assume a 5$\sigma$ detection of dark boson signature at 50 ab$^{-1}$

\begin{align*}
S &= 9850, \quad B = 3.88 \cdot 10^6 & m_{Z_d} &= 0.5 \text{ GeV} \\
S &= 5700, \quad B = 1.30 \cdot 10^6 & m_{Z_d} &= 2.0 \text{ GeV}
\end{align*}

*CalcHEP version 3.7.5, A. Pukhov, A. Belyaev, and N. Christensen, 2019
Summary on asymmetry parameters, event numbers and detection significance

| \(|\rho|\) | \(m_{Z_d}/\text{GeV}\) | 0.00 | 1.74 | 2.00 |
|---|---|---|---|---|
| \(|\rho|\) | \(m_{Z_d}/\text{GeV}\) | 0.5 | 2.0 | 0.5 | 2.0 | 0.5 | 2.0 |
| \(A_{\text{PN}}\) | \(A_{\text{PN}}\) | 0.0 | 0.0 | 0.43 | 0.44 | 0.58 | 0.60 |
| \(\text{Det. Sig. (Eq. (15))}\) | \(\text{Det. Sig. (Eq. (15))}\) | 5.0\(\sigma\) | 5.0\(\sigma\) | 5.4\(\sigma\) | 5.5\(\sigma\) | 5.8\(\sigma\) | 5.8\(\sigma\) |
| \(S(\kappa \cdot \xi > 0)\) | \(S(\kappa \cdot \xi > 0)\) | 4925 | 2817 | 7040 | 4053 | 7780 | 4507 |
| \(S(\kappa \cdot \xi < 0)\) | \(S(\kappa \cdot \xi < 0)\) | 4925 | 2817 | 2810 | 1581 | 2070 | 1127 |
| \(\text{Br}(Z_d \rightarrow \mu^+\mu^-)\) | \(\text{Br}(Z_d \rightarrow \mu^+\mu^-)\) | 40\% | 24\% | 21\% | 7.5\% | 17\% | 6.7\% |
| \(\varepsilon \cdot 10^4\) | \(\varepsilon \cdot 10^4\) | 3.3 | 3.2 | 4.6 | 5.7 | 5.1 | 6.1 |
Conclusions

- We have discussed the search for dark boson with the process $e^+e^- \rightarrow Z_d + \gamma$ in the $e^+e^-$ collider.
- The dark boson is shown to be transversely polarized when the dark boson mass is much less than the CM energy.
- We analyze the muon angular distributions from polarized $Z_d$ decays and define the asymmetry parameter $A_{PN}$ which is proportional to the square of parity violation parameter $\rho \equiv 4g_{l,V}g_{l,A}$.
- We calculate the asymmetry parameter with Belle II detector angular coverage and discuss its consequences on the dark boson search.