Gravitational interaction of the cosmic string with pointlike particles

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- *P.Spirin*, Universe **6**, (2020) no.10, 184
- *P.Spirin*, Universe **7**, (2021) no.7, 206

Content:

- Cosmic string: metric
- Gravitational interaction
- Particle's dynamics
- Cosmic String's excitations
- Transversal string's perturbations
- p- Longitudunal string's perturbations
- 2nd order of PT: radiation
- Restrictions

Metric (cylindric coords):

$$ds^2 = dt^2 - dx^2 - d\varrho^2 - \beta^2\varrho^2 d\varphi^2\,, \qquad 0 < \beta < 1$$

Geometry:

$$R = 2(1-\beta)\delta_+(\varrho)/\varrho, \qquad \delta\varphi = 2\pi(1-\beta)$$

Phase transition energy scale:

$$\eta^2 = \frac{1-\beta^2}{8\pi G}$$

For $\eta = \eta_{\mathrm{GUT}} \sim 10^{16}$ GeV one has

$$1-\beta \sim 10^{-5} \qquad d \sim 10^{-29}\mathrm{cm}$$

Complement:

$$\beta' \equiv 1-\beta = \frac{\delta\varphi}{2\pi} \qquad \beta' = 4G\mu$$

Conformally Euclidean ($x = \mathrm{const}$) coords: $\varrho \to r$

$$\beta\varrho = R_0 (r/R_0)^\beta \,,$$

Metric (conformal coords):

$$ds^2 = dt^2 - dx^2 - \mathrm{e}^{-2(1-\beta)\ln(r/R_0)}(dy^2 + dz^2) \,,$$

where $r^2 = y^2 + z^2$.

Gravitational interaction

Action:

$$S = -\frac{\mu}{2}\int X^\mu_a X^\nu_b g_{\mu\nu}\gamma^{ab}\sqrt{\tilde\gamma}\,d^2\sigma - \frac{1}{\varkappa^2}\int R\sqrt{|g|}\,d^4x - \\ -\frac{1}{2}\int\left(e\,g_{\mu\nu}\dot Z^\mu \dot Z^\nu + \frac{m^2}{e}\right)ds$$

Einstein equations:

$$R^{\mu\nu} - \frac{R}{2}g^{\mu\nu} = \frac{1}{2}\varkappa^2\left[T^{\mu\nu} + \bar T^{\mu\nu}(+T^{\mu\nu}_{\rm sc})\right]$$

Sources

$$T^{\mu\nu} = \mu\int X^\mu_a X^\nu_b \gamma^{ab}\,\delta^4(x - X(\sigma))\sqrt{\tilde\gamma}\,d^2\sigma\,,$$

$$\bar T^{\mu\nu} = e\int \frac{\dot Z^\mu \dot Z^\nu \delta^4(x - Z(s))}{(g_{\lambda\rho}\dot Z^\lambda \dot Z^\rho)^{1/2}\sqrt{|g|}}\,ds$$

Zeroth order: particle

$$^{0}Z^{\mu}(s) = u^{\mu}s + b^{\mu}, \quad u^{\mu} = \gamma(1,0,0,v), \quad b^{\mu} = (0,0,b,0)$$

String:

$$^{0}X^{\mu} = \delta^{\mu}_{a}\sigma^{a} \qquad ^{0}\gamma_{ab} = \eta_{ab}$$

$$^{0}T^{\mu\nu} = \mu\,\delta^{\mu}_{a}\delta^{\nu}_{b}\eta^{ab}\delta(y)\,\delta(z) = \mu\,\delta(y)\,\delta(z)\,\mathrm{diag}(1,-1,0,0)$$

First order: linearized fields

$$\bar{h}_{\mu\nu}(x) = -\frac{\varkappa m}{4\pi}\left(u_{\mu}u_{\nu} - \frac{1}{2}\eta_{\mu\nu}\right)\left[\gamma^{2}(z-vt)^{2} + x^{2} + y^{2}\right]^{-1/2}$$

$$h_{\mu\nu}(q) = -\frac{(2\pi)^{2}\varkappa\mu\delta^{2}(q^{0,1})}{\delta_{\alpha\beta}q^{\alpha}q^{\beta}}\Sigma_{\mu\nu} \qquad h_{\mu\nu}(x) = \frac{\varkappa\mu}{2\pi}\Sigma_{\mu\nu}\ln\frac{r}{R_{0}},$$

where $\Sigma_{\mu\nu} \equiv \mathrm{diag}\,(0,0,1,1)$

Particle's Dynamics

Integration of EoM with IC ${}^{1}\dot{Z}^{\mu}(s=0)=0$

$$ {}^{1}\dot{Z}^{0} = {}^{1}\dot{Z}^{x} = 0 \qquad {}^{1}\dot{Z}^{y} = -4G\mu\gamma v \,\mathrm{arctg}\frac{\gamma v s}{b} $$

$$ {}^{1}\dot{Z}^{z} = 2G\mu\gamma v \ln\frac{b^2+\gamma^2 v^2 s^2}{b^2} $$

Identification of R_0:

$$ R_0 = b $$

Scattering angle

$$ \alpha_{\mathrm{sc}} \simeq 4\pi G\mu = \pi\beta' $$

Trajectory

$$ y(z) = b - \beta'\left[z\,\mathrm{arctg}\frac{z}{b} - \frac{b}{2}\ln\frac{b^2+z^2}{b^2}\right] $$

String's EoM

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right) {}^1X^\mu = \varkappa \Pi^{\mu\nu} \delta_a^\lambda \delta_b^\rho \eta^{ab} \left(\frac{1}{2}\,\bar{h}_{\lambda\rho,\nu} - \bar{h}_{\nu\lambda,\rho}\right)_{y=z=0}$$

Gauge ${}^1X^a = 0$ $(a = t, x)$: only the transverse deflections survive!

$$\Box_2 \Phi^\alpha(t,x) = j^\alpha(t,x)$$

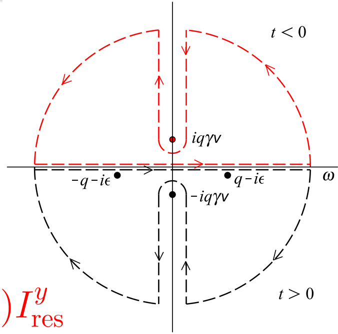
Formal retarded solution $(\alpha = y, z)$

$$\Phi^\alpha(t,x) = -\frac{1}{(2\pi)^2} \int \frac{\mathrm{e}^{-iqx} j^\alpha(\omega, q)}{\omega^2 - q^2 + 2i\epsilon\omega}\, d\omega\, dq$$

Sources

$$j^\alpha(\omega, q) \sim \exp\left(-b\sqrt{q^2 + (\omega/\gamma v)^2}\right)$$

Formal retarded solution



Total solutions:

$$\Phi^{z,y} = \Phi^{z,y}_{\rm cut} + \Phi^{z,y}_{\rm res}$$

Partial solutions:

$$\Phi^z_{\rm cut} = -\Lambda \,{\rm sgn}\, t \, I^z_{\rm cut} \qquad \Phi^y_{\rm cut} = \Lambda v I^y_{\rm cut}$$

$$\Phi^z_{\rm res} = 2\Lambda\theta(t) I^z_{\rm res} \qquad \Phi^y_{\rm res} = 2\Lambda v\theta(t) I^y_{\rm res}$$

Amplitudes:

$$I^{z,y}_{\rm cut} \equiv \frac{\sqrt{2/\pi}}{b^{\frac{D-5}{2}}} \int\limits_0^\infty dq \int\limits_{q\gamma v}^\infty du \, \frac{u e^{-u|t|} \cos qx}{u^2+q^2} \, \frac{J_{\frac{D-5[3]}{2}}\left(b\sqrt{(u/\gamma v)^2 - q^2}\right)}{(\gamma^{-2}v^{-2}u^2 - q^2)^{-\frac{D-5[3]}{4}}}$$

$$I^{z,y}_{\rm res} \equiv \frac{\sqrt{2/\pi}}{(bv)^{\frac{D-5}{2}}} \int\limits_0^\infty dq \begin{matrix} \cos qt \\ \sin qt \end{matrix} \; \cos qx \, q^{\frac{D-5}{2}} K_{\frac{D-5[3]}{2}}(bq/v)$$

Transverse amplitudes:

$$I^y_{\rm res} = \frac{1}{2}\left[\mathrm{arctg}\,\frac{t^2 - x^2 - b^2/v^2}{2bt/v} + \frac{\pi}{2}\right], \qquad (t > 0)$$

$$I^y_{\rm cut} = \frac{1}{2}\left[\mathrm{arctg}\,\frac{v^2t^2 - v^2x^2 - (2v^2-1)\,b^2}{2bv\sqrt{v^2t^2 + (b^2+x^2)/\gamma^2}} - \mathrm{arctg}\,\frac{t^2 - x^2 - b^2/v^2}{2b|t|/v}\right]$$

Both z-amplitudes diverge logarithmically!

Dimensional regularization: $D = 4 + 2\epsilon$ $(\epsilon \geqslant 0)$

$$\mathrm{ren}\; I^z_{\rm res} = -\frac{v}{4}\ln\frac{[v^2(t+x)^2 + b^2][v^2(t-x)^2 + b^2]}{4r^4_{\rm res}}$$

Cut-amplitude after the q-integration: $(u = q\gamma v\sqrt{\tilde{u}^2 + 1})$

$$\mathrm{reg}\; I^z_{\rm cut} = \frac{\Gamma(\epsilon)}{2\pi}\sum_{\pm}\int\limits_0^\infty d\tilde{u}\,\frac{\mu_{\rm cut}^{-2\epsilon}\tilde{u}^{2\epsilon}}{\tilde{u}^2 + v^{-2}}\left[\left(\gamma v|t|\sqrt{\tilde{u}^2+1} \pm ix\right)^2 - b^2\tilde{u}^2\right]^{-\epsilon}$$

Finally:

$$\text{ren}\ I^z_{\text{cut}} = -\frac{v}{2}\ln\frac{v^2b^2x^2 + \left(\gamma^2v^2t^2 + b^2 + \gamma v^2|t|\sqrt{\gamma^2v^2t^2 + b^2 + x^2}\right)^2}{r^2_{\text{cut}}(\gamma^2v^2t^2 + b^2)}$$

Properties of dalembertians:

$$\Box_2 I^y_{\text{res}} = \Box_2 I^z_{\text{res}} = 0$$

Continuity requirements:

$$\dot{I}^y_{\text{cut}}(0^+, x) = -\dot{I}^y_{\text{cut}}(0^-, x) = -\frac{bv}{v^2x^2 + b^2}$$

$$I^z_{\text{res}}(0, x) = I^z_{\text{cut}}(0, x)$$

Length-scale factors relation:

$$r_{\text{cut}} = \sqrt{2}r_{\text{res}} \equiv r_0$$

Summarizing the renormalized solutions

The constructed solutions Φ^y and Φ^z satisfy:

- continuity with respect to t and x;
- satisfy the initial EoM;
- reflect the causal structure;
- vanish in the limit $v \to 0$;
- their value has characteristic length parameter $2Gm\gamma = r_\mathcal{E}$;
- Φ^z depends logarithmically upon the single scale factor r_0.

Totally transverse string's perurbation

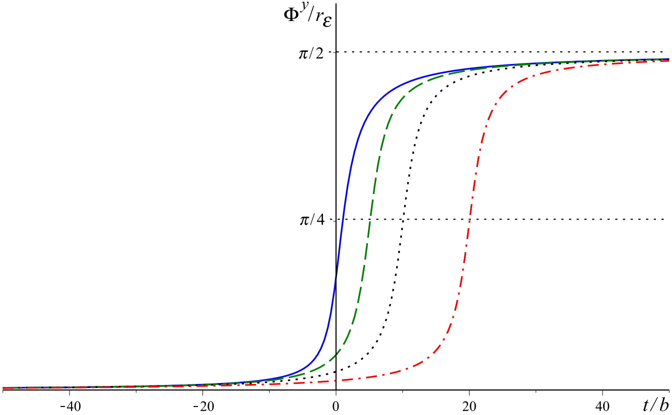


Figure: Transverse string's perurbation Φ^y (in units $r_\mathcal{E} = 1$) as a function of time (normalized by b) at $x/b = 0$ (blue solid), $x/b = 5$ (green dashed), $x/b = 10$ **(black dotted)**, $x/b = 20$ (red dashdotted) for $v = 0.5$

$$\sqrt{x^2 + b^2} \equiv t_x$$

Relaxation: $t = 2t_x$

Totally transverse string's perurbation

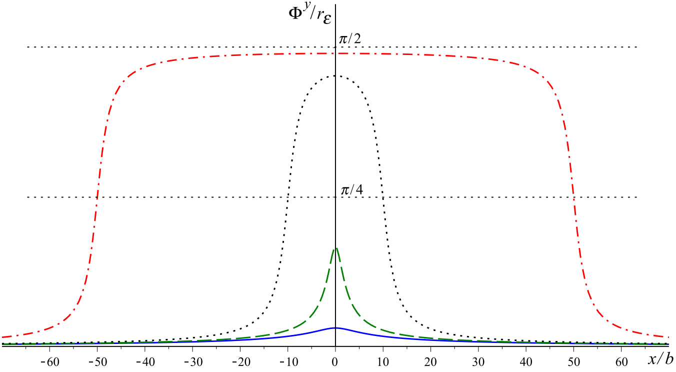


Figure: The profile of the transverse string's perurbation Φ^y (in units $r_\mathcal{E} = 1$) at $t/b = -5$ (blue solid), $t/b = 0$ (green dashed), $x/b = 10$ **(black dotted)**, $x/b = 50$ (red dashdotted) for $v = 0.5$

$$x_{1/2}(t) \equiv \sqrt{t^2 - b^2} \qquad V = \frac{\partial x_{1/2}}{\partial t} = \frac{t}{\sqrt{t^2 - b^2}}$$

From $|\ln(r/b)| \ll 1/\beta'$ there exists $r_{\max}$ (or $t_{\max}$)

$$t_{\max} \sim \beta'^{-2}$$

Other restrictions:

$$b \gg r_g = 2Gm\,, \qquad\qquad b \gg d = \eta^{-1}$$

Hypothesis of the absence of interaction beyond $t_{\max}$

$$r_0 = 2\gamma v t_{\max}$$

Before the collision $t < 0$:

$$\Phi^z(t,x) < \Phi^z(0,x) = r_{\mathcal{E}} v \left(\overbrace{2\ln\frac{1}{\beta'} + \ln 2\gamma}^{\Phi^z_{\rm bg}} - \frac{1}{2}\ln\frac{v^2x^2+b^2}{b^2} \right)$$

Eventual value

$$\Phi^z(t_{\max},x) \simeq r_{\mathcal{E}} v \left[2\ln\gamma + \ln(1+v) + \ln 2\right] \equiv \Phi^z_\infty$$

p-Longitudinal string's perurbation

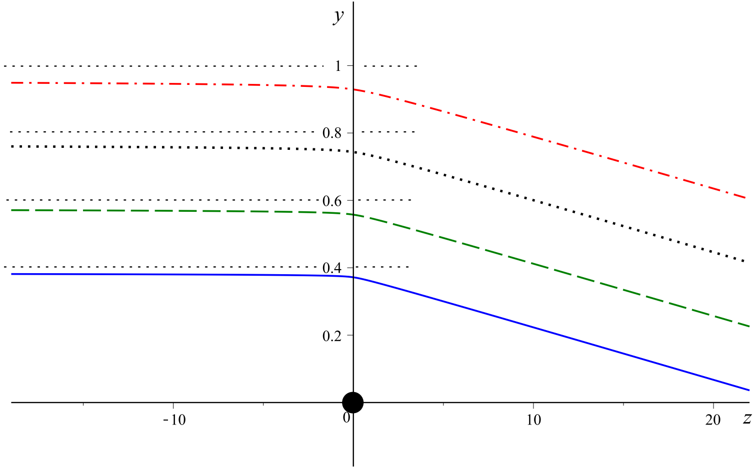


Figure: Gravitational lensing by a string with angular deficit $\beta' = 0.005$ for $b = 0.4$ (blue solid), $b = 0.6$ (green dashed), $b = 0.8$ (black dotted), $b = 1$ (red dashdotted)

p-Longitudinal string's perurbation

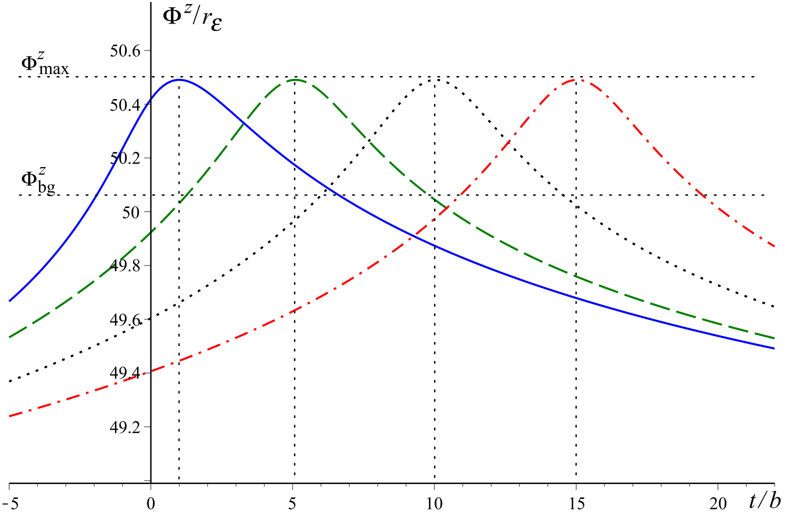


Figure: String's z-deflection (in units $r_\mathcal{E} = 1$) as a function of time (in units $b = 1$) at $x/b = 0$ (blue solid), $x/b = 5$ (green dashed), $x/b = 10$ (**black dotted**), $x/b = 15$ (red dashdotted) for $v = 0.5$ and $\beta' = 10^{-4}$

String's profile in z-direction

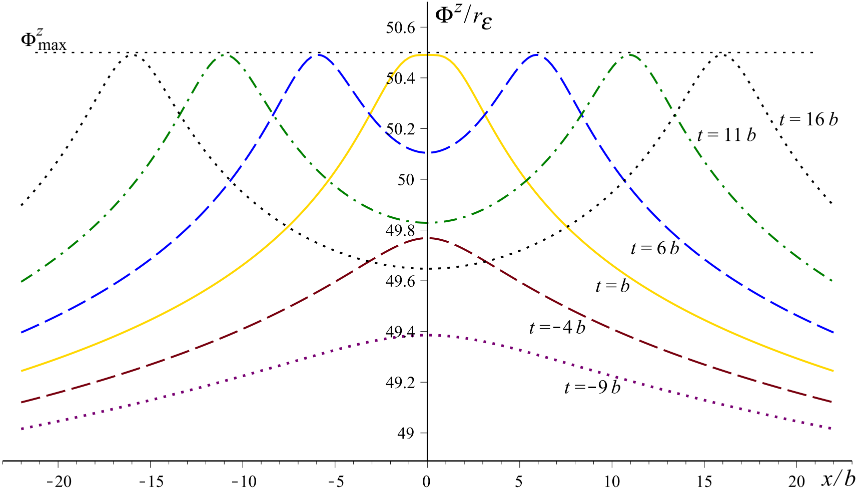


Figure: String's z-profile on a length scale or order of b for $v = 0.5$

Maxima: $x = \pm\sqrt{t^2 - b^2}$

Evolution of string's apex

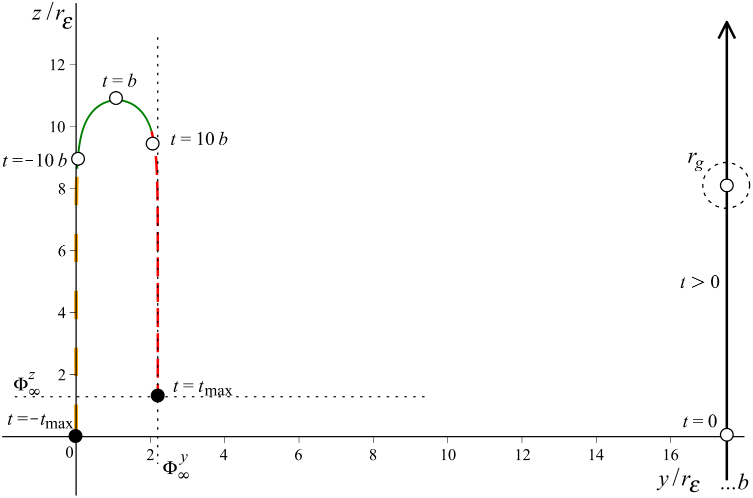


Figure: Trajectory of a string's apex in the particle's scattering plane for $v = 0.7$ and $\beta' = 0.005$

Scalar field addenda

Scalar part of the total action:

$$S_{\rm sc} = \frac{1}{2}\int g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\,\sqrt{|g|}\,d^4x - \phi j$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = -j \qquad j = f\int\sqrt{\dot{Z}^2}\,\frac{\delta^4\left(x - Z(s)\right)}{\sqrt{|g|}}\,ds$$

$$(m + f\phi)\ddot{Z}^\mu = -m\Gamma^\mu_{\nu\lambda}\dot{Z}^\nu\dot{Z}^\lambda + f\,\Pi^{\mu\nu}\phi_{,\nu}$$

Zeroth order: scalar field

$$^0\phi = 0 \qquad ^0j(t,\boldsymbol{x}) = \frac{f}{\gamma}\,\delta(x)\,\delta(y-b)\,\delta(z-vt)$$

First order: field equation $\partial^2\,{}^1\phi = {}^0j$

$$^1\phi(x) = -\frac{f}{4\pi}\left[\gamma^2(z-vt)^2 + x^2 + y^2\right]^{-1/2} \qquad {}^1T^{\mu\nu}_{\rm sc} = 0$$

Scalar correction to the particle's trajectory

$$m\,{}^1\ddot{Z}^\mu_{\rm sc} = f\,{}^0\Pi^{\mu\nu}\,{}^1\phi_{,\nu}\,, \qquad Z^\mu_{\rm sc} = 0\ (\text{self} - \text{action})$$

Radiation formula for the scalar field

$$\frac{dE}{d\omega d\Omega} = \frac{\omega^2}{16\pi^3}\,|j(k)|^2, \qquad \omega = k^0 = |\boldsymbol{k}|$$

Source: $j = {}^0j + {}^1j + \ldots$ (${}^0j$ – uniform motion)

$$\Box^2\phi = -\,{}^1j(x) \qquad {}^1j(x) \equiv \rho(x) + \sigma(x)$$

Partial sources (''local'' and ''non-local''):

$$\rho(x) = -f\int {}^1Z^\mu(s)\,\partial_\mu\delta^4\!\left(x - {}^0Z(s)\right)ds$$

$$\sigma(x) = \varkappa\,\partial_\mu\!\left({}^1h^{\mu\nu}(x)\,\partial_\nu{}^1\phi(x) - \frac{1}{2}\,{}^1h(x)\,\eta^{\mu\nu}\partial_\mu{}^1\phi(x)\right)$$

Radiation amplitudes

Local: $(\psi \equiv 1 - v\cos\vartheta)$

$$^1\rho(k) = -\frac{\varkappa^2 f\mu v}{4\omega\gamma\psi^2}\, e^{i(kb)} e^{-(ku)b/\gamma v}\Big(\cos\vartheta + i\sin\vartheta\sin\varphi\Big)$$

Charact. frequencies/angles: $\omega \sim \gamma^2/b\,, \qquad \vartheta \sim 1/\gamma$

Non-local: $^1\sigma(k) = \ldots e^{i(kb)} e^{-(ku)b/\gamma v} + \ldots e^{-\omega bR}$

$$R \equiv \sqrt{\gamma^{-2}v^{-2} + \sin^2\vartheta\cos^2\varphi}$$

Total source:

$$^1j(k) = -\frac{\varkappa^2 f\mu}{4\gamma\omega R}\,\frac{(1-\sin^2\vartheta\cos^2\varphi)\,\exp(-\omega bR)}{\psi\cos\vartheta - v\sin^2\vartheta\sin^2\varphi - ivR\sin\vartheta\sin\varphi}$$

Charact. frequencies/angles: $\omega bR \lesssim 1$

$$\langle\omega\rangle \sim \frac{\gamma v}{b} \qquad \langle\vartheta\rangle \sim \gamma^{-1} \qquad \langle\varphi\rangle \sim 1$$

Emitted energy – angular distribution

$$\frac{dE}{d\vartheta} = \frac{2G^2 f^2 \mu^2 u}{\pi b \psi^2}\left[\boldsymbol{E}(u) - \frac{\boldsymbol{K}(u)}{\gamma^2}\right], \quad u = \frac{\gamma v \sin\vartheta}{\sqrt{1+\gamma^2 v^2 \sin^2\vartheta}}$$

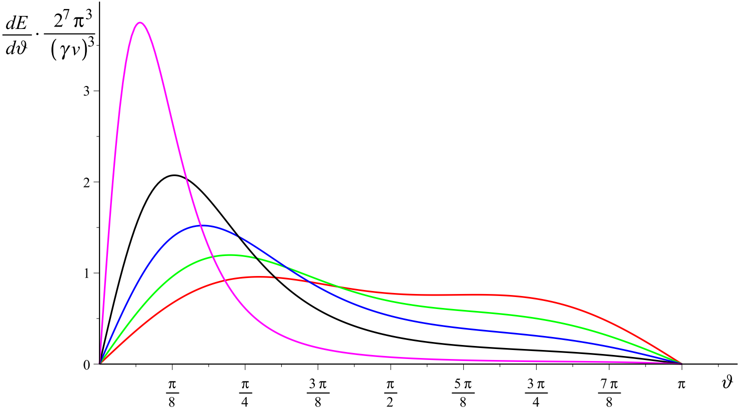


Figure: Plots of ϑ-distribution of the scalar bremsstrahlung, normalized by overall factor $2^7(\pi/\gamma v)^3$, in units $G = \mu = b = f = 1$: $v = 0.1$ (red), $v = 0.3$ (green), $v = 0.5$ (blue), $v = 0.7$ (black), $v = 0.9$ (magenta).

Emitted energy – frequency distribution

$$\frac{dE}{d\omega} \sim \left(\frac{dE}{d\omega}\right)_{\omega=0} \mathrm{e}^{-2b\langle R\rangle\omega},$$

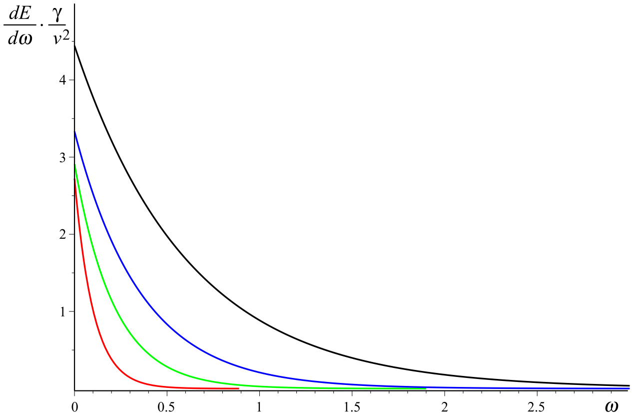


Figure: Plots of the frequency distribution of the scalar bremsstrahlung, normalized by the overall factor γ/v^2, in units $G = \mu = b = f = 1$: $v = 0.2$ (red), $v = 0.4$ (green), $v = 0.6$ (blue), $v = 0.8$ (black).

φ-distribution:

$$\frac{dE}{d\varphi}=\frac{G^2f^2\mu^2\gamma^3}{\pi b\sin^5\varphi}\left[\sin\varphi\left(1+2\cos^2\varphi\right)-\frac{3}{2}\cos^2\varphi\ln\frac{1+\sin\varphi}{1-\sin\varphi}\right]$$

Total emitted energy:

$$E=\frac{3\pi}{8}\frac{G^2f^2\mu^2}{b}\gamma^3$$

Generic case

$$E=\frac{G^2f^2\mu^2}{b}\gamma^3v^3\,\Xi(v^2)$$

where the introduced $\Xi(v^2)$ satisfies

$$1.18\approx\frac{3\pi}{8}\leqslant\Xi(v^2)\leqslant\frac{4}{3}\approx1.33$$

Conclusions - Interesting features

- Dimensional regularization was performed inside the integral directly
- A particle attracts string in the lateral direction (y-axis) and repels it along own motion (z-axis)
- Two induced transverse waves along the string
- (Possible) Identification of the logarithmic lenghth factor with the large-scale parameter involves the conical angular deficit
- The destructive interference (i) is complete, valid in any phase-volume domain; (ii) takes place for any velocity

~~Theorists' lives matter!~~

Thank you!