

Measuring the muon magnetic anomaly a_{μ} with the Muon g-2 experiment at Fermilab

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Marco Incagli – INFN Pisa

What is *"g-2"*?



$$\vec{\mu}_p = -g_p \frac{e}{2m_p} \vec{S}$$

$$a_p = \frac{g_p - 2}{2}$$

- *g_P*: proportionality constant between spin and magnetic moment for particle P
- *a_P* : magnetic anomaly
- *a*_{*P*} = 0 at tree level (*purely Dirac particle*)
- Using modern language, the term (g-2)/2 reflects the magnitude of the Feynmann diagrams beyond leading order





Standard Model determination of a_{μ}





	092.0 ± 0.00
HLbL Glasgow	10.5 ± 2.6
EW	15.4 ± 0.1
Total SM Davier17	11659181.7 ± 4.2

Theory Initiative White Paper (arXiv 2006:08443) $a_{\mu} = (116\ 591\ 810\ \pm\ 43) \times 10^{-11} \rightarrow 370 \text{ ppb}$



A rich history of g-2 Theory and Experiment



History of muon anomaly measurements and predictions

Tension between theory and experiment



The Fundamental Experimental Principle



• Difference between spin precession and cyclotron revolution for a muon (charged particle with spin) in a magnetic field*:

$$\omega_a = \omega_s - \omega_c = g \frac{e}{2m} B - \frac{e}{m} B = \frac{g-2}{2} \frac{e}{m} B = a_\mu \frac{e}{m} B$$

*s and p are assumed to be in a plane perpendicular to B

- simple classical calculation
- the relativistic approach provides the same result





How do we measure the spin direction?



• Use V-A structure of weak decays to build a polarized beam...



... and to measure the muon polarization looking for energetic positrons





Measuring the spin precession



• The number of observed positrons above a threshold energy oscillates with the $\omega_a/2\pi$ frequency due to spin precession



- exponential decay modulated by spin precession
- note that the x-axis "wraps up" every 100 μ sec for a total of ~700 μ s \rightarrow ~10 muon lifetimes



Extracting a_{μ} (simplified)

 $\omega_a = a_{\mu} (e/m) B \rightarrow a_{\mu} = \omega_a / B (m/e)$ by expressing B in terms of the (shielded) proton precession frequency: $(B = \hbar \omega'_p / 2\mu'_p):$



 $\widetilde{\omega}'_p$ = (shielded) Proton angular velocity weighted for the muon distribution



Additional corrections



• A beam of muons

- 1. requires focusing elements \rightarrow *electrostatic quadrupoles*
- 2. undergoes betatron oscillations around the ideal trajectory
- Additional terms have to be considered in the muon precession formula:

$$\vec{\omega}_{a} = -\frac{e}{mc} \left[a_{\mu} \vec{B} - \left(a_{\mu} - \frac{1}{\gamma^{2} - 1} \right) \vec{\beta} \times \vec{E} - a_{\mu} \left(\frac{\gamma}{\gamma + 1} \right) \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} \right]$$
Term due to ElectroStatic
Quadrupoles (ESQ)
Term due to beam vertical
oscillations: *pitch correction*
Becomes ~0 at magic $\gamma \sim 29.3$
or $p \sim 3.1$ GeV/c



Additional corrections



• The ratio R'_{μ} requires additional corrections related to beam dynamics and to magnetic transient fields:



• $f_{clock} = blinding frequency$; described later in the talk



Four articles on PRx and on arXiv for details



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The key ingredients



• Neglecting the corrections, the three *key ingredients* to measure the muon magnetic anomaly are:





Measuring ω_a : 5 parameters fit function



• Fit with simple positron oscillation:

$$N_e(t) = N_0 \exp\left(-t/\tau_{\mu}\right) \left[1 + A\cos(\omega_a t + \varphi)\right]$$

• This simple fit is clearly not sufficient and well defined resonances are observed in the residuals



The complete 22 parameters fit function



 ω_y, ω_{VW} vertical oscillations $\omega_{CBO}, \omega_{2CBO}$ radial oscillations

 $N_0 e^{-\frac{t}{\gamma r}} \left(1 + \mathbf{A} \cdot A_{BO}(t) \cos(\omega_a t + \phi \cdot \phi_{BO}(t))\right) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$

 $A_{\rm BO}(t) = 1 + A_A \cos(\omega_{\rm CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{\rm CBO}}}$ $\phi_{\rm BO}(t) = 1 + A_{\phi} \cos(\omega_{\rm CBO}(t) + \phi_{\phi}) e^{-\frac{t}{\tau_{\rm CBO}}}$ $N_{\rm CBO}(t) = 1 + A_{\rm CBO}\cos(\omega_{\rm CBO}(t) + \phi_{\rm CBO})e^{-\frac{t}{\tau_{\rm CBO}}}$ $N_{2CBO}(t) = 1 + A_{2CBO}\cos(2\omega_{CBO}(t) + \phi_{2CBO})e^{-\frac{1}{2\tau_{CBO}}}$ $N_{\rm VW}(t) = 1 + A_{\rm VW} \cos(\omega_{\rm VW}(t)t + \phi_{\rm VW})e^{-\frac{t}{\tau_{\rm VW}}}$ $N_{y}(t) = 1 + A_{y}\cos(\omega_{y}(t)t + \phi_{y})e^{-\frac{t}{\tau_{y}}}$ $J(t) = 1 - k_{LM} \int_{t_0}^{t} \Lambda(t) dt$ Lost muons (μ hitting collimators) Red = free parameters Blue= fixed parameters $\omega_{\rm CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$ $\omega_{y}(t) = F\omega_{\rm CBO(t)}\sqrt{2\omega_{c}/F\omega_{\rm CBO}(t)} - 1$ $\omega_{\rm VW}(t) = \omega_c - 2\omega_u(t)$



Final fit to get ω_a





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Magnetic field \vec{B} determination





- A *Trolley* (Cylinder with 17 NMR probes) runs inside the ring every 2-3 days to map the field experienced by muons
- A set of 378 fixed probes, located in 72 azimuthal positions, continuosly measures the field
- Absolute probes for calibration tested at Argonne (ANL) magnet



$\omega'_p \to \widetilde{\omega}'_p$: muon distribution inside the Ring

- Two tracker stations, made of straw tube modules, placed at φ~180° and φ~270°, are used to trace back positrons and get the muon distribution
- Use Beam Dynamics models to extrapolate the distribution all around the ring
- Systematic uncertainties mostly due to Beam Dynamics models used for extrapolation and to tracker alignment







Final uncertainties from Run 1, ~8×10⁹ positrons

Quantity	Correction Terms	Uncertainty
	(ppb)	(ppb)
ω_a^m (statistical)		434
ω_a^m (systematic)	-	56
$\overline{C_e}$	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}}\langle \omega_p(x,y,\phi) \times M(x,y,\phi) \rangle$	-	56
B_k	-27	37
B_q	-17	92
$\mu_p'(34.7^\circ)/\mu_e$:	10
m_{μ}/m_e	_	22
$g_e/2$	—	0
Total systematic	() 	157
Total fundamental factors	-	25
Totals	544	462

462 ppb overall error 434 ppb statistical 157 ppb systematic 25 ppb external inputs Results for Run 1 are vastly dominated by statistical error At 157 ppb systematic error

- Nearly half of BNL
- Not quite to 100 ppb goal



Final uncertainties from Run 1



- Correction larger than total error
- Dominated by Electric Field correction *C*_e
- Related to non-centered radial distribution due to low kicker field (sparks!)



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- Two largest systematics:
 - phase acceptance
 - quadrupole field



ElectroStatic Quadrupoles (ESQ) transient field Ba

Relative Field [ppb]

- Mechanical vibration in the plates due to the ESQ charge/discharge which generates magnetic perturbations
- Special NMR probes measure the perturbation B_a at several positions
- Uncertainty dominated by limited lacksquarenumber of measurements in Run-1
- Reduced in Run-2 by more ۲ measurements

 $B_q \sim 20 \ ppb$, $\delta_{B_q} \sim 90 \ ppb$







The blinding



$$R'_{\mu} = \underbrace{f_{clock}}_{\widetilde{\omega}'_{p}} \cdot \frac{\omega_{a}}{\widetilde{\omega}'_{p}} \cdot \frac{1 + C_{e} + C_{p} + C_{ml} + C_{pa}}{1 + B_{k} + B_{q}}$$

- Clock frequency *f_{clock}* uncalibrated by Joe Lykken and Greg Bock (FNAL Directorate) Feb 22 2018
 - stop in each week to check clock and sealing
- Secret envelopes kept until physics analysis complete and ready to be revealed Feb 25 2021













On February 25 2021 the Collaboration met for the unblinding:

- 1) The *box* (envelope) was opened
- 2) The number was plugged into two independent programs
- 3) And the result was....

-2 binding number Secret offset



a_{μ} : Unblinding and result





A new prediction based on lattice



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- a recent (published April 7!) result has reduced the uncertainty of the lattice calculation from 2-3% to 0.8%!
- breakthrough which requires further scrutiny
- uncertainty similar to the standard approach based on e⁺e⁻
 →hadrons data via dispersion relation (0.6%)



The Theory Initiative



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PHYSICS REPORTS

 ~130 physicists collaborated for 3 years [June 2017 – June 2020] in seven workshops to produce a reference number for am to be used by FNAL g-2 experiment as a benchmark

Muon g-2 Theory Initiative defines SM benchmark value that our collaboration will use for comparison. We don't "pick and choose" other individual results.

Group photo from the Seattle workshop in September 2019, https://indico.fnal.gov/event/21626/



WP20

Organizers: Aida El-<u>Khadra</u> Martin <u>Hoferichter</u> DWH

Contribution	Section	Equation	Value ×10 ¹¹	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2-7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, udsc)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9-17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18-30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18-30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP $(e^+e^-, LO + NLO + NNLO)$	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2-8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18-32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2-8, 18-24, 31-36]
Difference: $\Delta a_{\mu} := a_{\mu}^{\exp} - a_{\mu}^{SM}$	Sec. 8	Eq. (8.14)	279(76)	

The future of E989 Muon g-2 (Fnal)

- RUN-1 is only 6% of the final dataset
- Analysis of RUN-2/3 in progress (factor ~2 in precision)
- RUN-4 (November 2020-July 2021), which was recently completed, and Run5 (2021-2022) will allow to reduce by a factor ~4 current error







Conclusions: experiment



- 1. FNAL Muon g-2 :
 - a_{μ} measured at 0.46 ppm \rightarrow 4.2 σ discrepancy!
 - data already available to reduce error to ~0.25 ppm
 - Run4 (just finished) and Run5, starting later this year 2021, will allow to reach ~0.15 ppm
- 2. A new type of experiment projected at J-Parc using low energy muons (p~300 MeV/c)
 - new technique
 - under construction
 - final goal ~0.4 ppm





Conclusions: theory



- 1. close scrutiny of lattice calculations to establish its solidity
 - how to reconcile it with dispersion approach? and with Standard Model global EW fit? \rightarrow next talks
- 2. Use the dispersive approach with t-channel data (*muon-electron* scattering), instead of the standard s-channel
 - Letter Of Intents submitted at CERN: Muone (mu-on-e scattering)







BACKUP SLIDES



a_{μ}^{HLO} : dispersion integral

$$a_{\mu}^{HLO} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} \sigma_{e^+e^- \to hadr}(s) K(s) ds$$

- Kernel function: $K(s) \propto 1/s$
- Due to the 1/s term, the low energies most important





a_{μ}^{HAD} from $\mu + e \rightarrow \mu + e$ scattering

- NEW IDEA:
 - measure $\mathbf{a}_{\mu}^{\text{HAD}}$ using $\mu e \rightarrow \mu e$ (t-channel) instead of $ee \rightarrow \pi \pi$!



ω_a principle of measurement



 $\mu^{+} \text{ Center of Mass}$ $V \xrightarrow{\mu^{+}} \longrightarrow e^{+} \text{ Max E: } e^{+} \text{ parallel to } \mu \text{ spin}$ $V \xrightarrow{\bullet} \longrightarrow \overline{V} \text{ Min E: } e^{+} \text{ antiparallel to } \mu \text{ spin}$

$$N(t) = N_0 e^{-t/\tau} \left(1 + A \cos(\omega_a t + \varphi) \right)$$

- positron emission correlated with muon spin direction
- correlation depends on *positron energy* : <u>the Asymmetry A(E)</u> <u>can be positive, null or negative</u>



Optimizing the statistical sensitivity



- T-method = counting positrons above E_{thr} vs time
 - by decreasing the threshold the asymmetry decreases but the number of events increases \rightarrow max sensitivity for $E_{thr} \sim 1.7 \, GeV$
- A-method = each positron is weighted by the value of its asymmetry A(E)
 - optimize statistical sensitivity
- In theory the A-method can use *all decay positrons*, in practice, due to calorimeter acceptance and to low A(E) value at low energies, only positrons with $E_{thr} > 1.1 \text{ GeV}$ (y>0.3) are used





C_{PA} – Phase acceptance error

 $f(t) \simeq N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)]$

- But if the phase of the muon population changes in time : $\cos(\omega_a t + \phi(t)) \approx \cos(\omega_a t + \phi' t + \phi_0) = \cos((\omega_a + \phi')t + \phi_0)$
- The extracted ω_{a} is shifted by ϕ' !



- The phase depends on the muon decay position (x,y)
- Not a big issue if the muon distribution remains stable with time



C_{PA} – Phase acceptance error av [kv]

- 2 quadrupoles HV resistors failed \rightarrow changing E-field \rightarrow beam vertical mean and width changed
- $C_{PA} = -158 \, ppb, \, \delta_{PA} = 75 \, ppb$



GOOD

BAD

Nominal 1-Step

Nominal 2-Step **Beam Injection**

Fit Start Time Damaged 1-Step Damaged 2-Step

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20

18

16

14

12

10