To change the flavour of a lepton
EFT for LFV: what to learn from the data?

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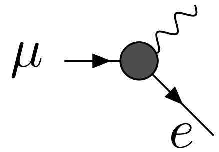
1. LFV is New Physics that $exists$:)
2. just that we don't see it yet: bounds
3. $\mu \leftrightarrow e$ — few, sensitive searches
   - need to account for SM loops
4. $\tau \leftrightarrow \ell$s — more processes, weaker constraints
   - better for disinguishing models?
5. (what can $\mu \leftrightarrow e$ tell us about $\tau \leftrightarrow \ell$?)
   - $[\mu \to \tau] \times [\tau \to e] = [\mu \to e] \Rightarrow$?
6. summary

Lepton Flavour Change is New Physics that *exists*



LFV $\equiv$ contact interaction where charged leptons change flavour.

Lepton Flavour Change is New Physics that *occurs*:

- lepton flavours conserved in SM-without-m_ν
- with $\{m_\nu, U\}$, no conserved lepton flavours
- $\approx$ lower bound on LFV rates:

  if m_ν Dirac, from Yukawa interactions:

$$\mathcal{A}_{LFV} \propto \frac{m_\nu^2}{m_W^2} \quad (\mathrm{GIM-suppressed}) \qquad \Rightarrow BR \lesssim 10^{-48}$$

$\Rightarrow$ if see LFV, lepton flavour sector different from quarks!

Two assumptions in this talk:

$\star$ suppose m_ν NOT Dirac

$\star$ New LFV particles are heavy

$\Rightarrow$ this is a talk about EFT for LFV

Things not in this talk

1. no Lepton Universality Violation: its present in the SM

2. no meson decays: talk of D Guadagnoli

   (exceptional anticipated sensitivities to quark flavour diag. of $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\mu A \to eA$)

   (B,D decays give few restrictive constraints on LFV)

3. no models...

4. no NSI, for lack of time

But not see LFV :(some processes and bounds

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	$10^{-(16\to?)}$ (Mu2e,COMET)
		$10^{-(18\to?)}$ (PRISM/PRIME/ENIGMA)
$\mu \to e\gamma\gamma$	$< 7.2 \times 10^{-11}$	
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$(Belle-II, LHCb?)
$\tau \to \ell\{\pi, \rho, \phi, K, ...\}$	$\lesssim$ few $\times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to ...$		
$h \to \tau^\pm\ell^\mp$	$< 4.7, 2.5 \times 10^{-3}$	
$h \to \mu^\pm e^\mp$	$< 6.1 \times 10^{-5}$	
$Z \to e^\pm\mu^\mp$	$< 7.5 \times 10^{-7}$	

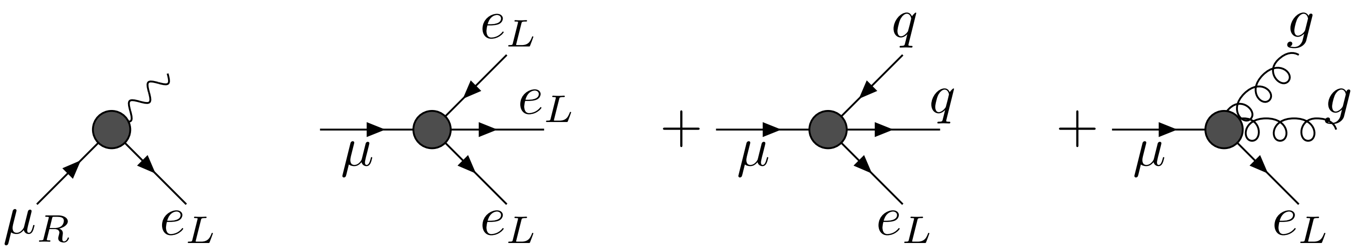
$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

BR $\equiv$ Branching Ratio: (rate for process)/(total decay rate)

As a theorist, to parametrise all those LFV processes...

... describe as contact interactions among 3 or 4 SM particles, should respect gauge symmetries present (QED*QCD at low E, SM above m_W),

Kuno-Okada for $\mu \to e$



$+$ h.c.

building operators

eg: 1008.4884,2005.00059,...

Represent contact interactions in $\mathcal{L}$ as operators $\mathcal{O}_X^\zeta$, classified by dimension
coupling constant $\Leftrightarrow$ (Wilson) coefficient C_X^ζ (dimless, predicted by models, constrained by data)

$$\delta\mathcal{L} \;=\; \sum_{n=1}^{3} \frac{1}{\Lambda^n} \sum_{X,\zeta} C_X^\zeta \mathcal{O}_X^\zeta + h.c.$$

X = Lorentz structure, ζ = flavour labels

Λ = mass scale $\left\{ \begin{array}{ll} \Lambda = v \approx m_t & G_F = 1/v^2 \;\; \text{convenient} \\ \Lambda = \Lambda_{\rm LFV} & C = 1 \; \text{explore reach} \end{array} \right\}$ options in this talk

$BR(\mu \to e...) < 10^{-12}$ **is restrictive**

LFV Branching Ratios normalised to *weak* muon decay

$$BR(\mu \to e\bar{e}e) \equiv \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \quad , \quad \Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{1536\pi^3 v^4}$$

$v = 174$ GeV

$$\text{...so if } \Gamma(\mu \to e\bar{e}e) \simeq \frac{m_\mu^5}{1536\pi^3 \Lambda_{LFV}^4} \quad \text{then } BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{cases}$$

Compare: $\boldsymbol{BR(\tau \to 3\ell) \lesssim 2 \times 10^{-8} \Rightarrow \Lambda_{\rm LFV} \sim 50v \simeq 10}$ **TeV**

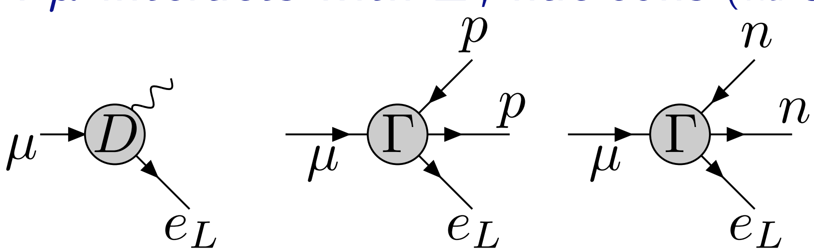
For ex: $\mu A \to eA$ — best future exptal sensitivity

- μ^- captured by Al nucleus, tumbles down to $1s$. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit

For ex: $\mu A \to eA$ — best future exptal sensitivity

- μ^- captured by Al nucleus, tumbles down to $1s$. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV: μ interacts with $\vec{E}$, nucleons (via $\tilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

($E_e \approx m_\mu$ so e_L/e_R)



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ , A\ , T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

KitanoKoikeOkada

$$BR_{SI} \sim \quad Z^2|\sum ...\tilde{C}_{SI}|^2 \quad , \quad \tilde{C}_{SI} \in \{\tilde{C}^p_V, \tilde{C}^p_S, \tilde{C}^n_V, \tilde{C}^n_S, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over nucleons $\propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim ...|\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

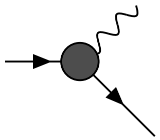
CiriglianoDavidsonKuno

Interesting : most sensitive probe, due to A^2 enhancement + future exptal reach($BR \to 10^{-18+}$)

many steps of theory caln could be improved

$\mu \leftrightarrow e$ data sensitive to multiloop effects from scales beyond LHC

Two dipole operators contribute to $\mu \to e\gamma$:



$$\begin{aligned}
\delta\mathcal{L}_{meg} &= -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,L}\overline{\mu_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,R}\overline{\mu_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta}\right) \\
BR(\mu \to e\gamma) &= 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.2 \times 10^{-13} \\
&\Rightarrow |C_{D,X}| \lesssim 10^{-8}
\end{aligned}$$

MEG expt, PSI

How big does one expect $C_{D,X}$ to be? Suppose operator coefficient

			$n=1$	$n=2$
$C_{D,X}\frac{m_\mu}{v^2} \sim \frac{e y_\mu v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 100$ TeV	10 TeV
$C_{D,X}\frac{m_\mu}{v^2} \sim \frac{e v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3000$ TeV	300 TeV

$\Rightarrow \mu \to e$ expts probe multi-loop effects in NP theories with $\Lambda_{NP} \gg$ reach of LHC

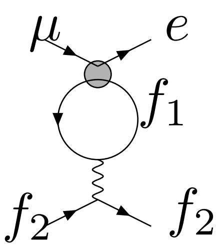
($\tau \to \ell\gamma$ only sensitive to $\Lambda_{\rm LFV} >$3 TeV if @1 loop, *without* Yukawa suppression)

Summary so far

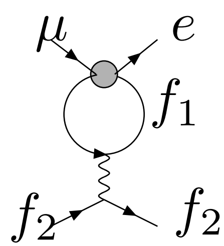
current $\left\{ \begin{array}{c} \mu \to e\gamma \\ \mu A \to eA \\ \mu \to e\bar{e}e \end{array} \right\}$ bounds restrictive: probe $\left\{ \begin{array}{l} \text{up to } \Lambda_{LFV} \lesssim 10^2 \to 10^3 \text{ TeV} \\ \text{2loop effects from beyond the LHC} \end{array} \right.$

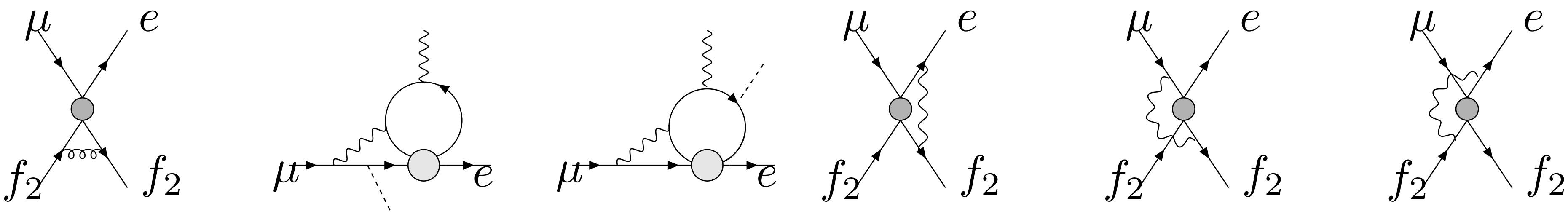
$\tau \to \ell$ searches can probe $\left\{ \begin{array}{l} \text{up to } 10 \to 30 \text{ TeV at tree} \\ \text{rarely sensitive to} \Lambda_{LFV} > 4 \text{ TeV at loop} \end{array} \right.$

$\Rightarrow$**must account for loop corrections in $\mu \to e$**



Including SM loop corrections to operators
eg below m_W: 1-loop QED + QCD (+2-loop QED V→D)

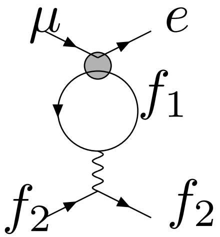




$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\boldsymbol{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\boldsymbol{\Gamma}$$

solve (analytically/numerically):

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV})\boldsymbol{G} \quad , \quad \boldsymbol{G} = \text{fn of SM parameters}, \log(\Lambda_{\rm LFV}/\Lambda_{exp})$$



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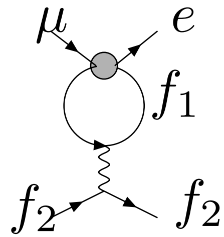
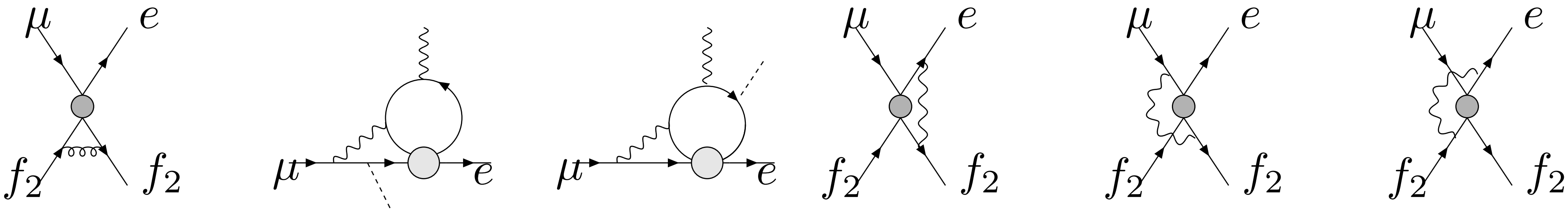




$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}$$

solve (analytically/numerically):

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV})\boldsymbol{G} \quad , \quad \boldsymbol{G} = \text{fn of SM parameters}, \log(\Lambda_{\rm LFV}/\Lambda_{exp})$$

For ex: $BR(\mu\to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.1\times 10^{-13} \Rightarrow C_{D,X} \lesssim 10^{-8}$

$$C_{D,X}(m_\mu) = C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) - \frac{\alpha_e}{4\pi e}\left(C^{\mu\mu}_{S,XX} - 8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C_{2loop}\right)\ln\frac{m_W}{m_\mu}$$
$$+16\frac{\alpha_e^2}{2e(4\pi)^2}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right)\ln^2\frac{m_W}{m_\mu} - 8\lambda^{aT}f_{TD}\frac{\alpha_e}{4\pi e}\left(\frac{2m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_s}{m_\mu}C^{ss}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)\ln\frac{m_W}{2\ \mathrm{Ge}}$$
$$+16\frac{\alpha_e^2}{3e(4\pi)^2}\left(\sum_{u,c}4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)\ln^2\frac{m_W}{2\mathrm{GeV}}$$

$C^{\zeta}_{Lor}(m_W)$ on right. $\lambda = \alpha_s(m_W)/\alpha_s(2\mathrm{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

Can calculate "sensitivities" = one-at-a-time-bounds

2010.00317

coefficient	$\mu \to e\gamma$	$\mu \to e\bar{e}e$	$\mu A \to eA$
$\lvert C_{D,X}\rvert$	1.12×10^{-8}	4.30×10^{-7}	2.35×10^{-7}
$\lvert C^{ee}_{V,XX}\rvert$	1.10×10^{-4}	7.80×10^{-7}	1.86×10^{-5}
$\lvert C^{ee}_{V,XY}\rvert$	2.55×10^{-4}	9.34×10^{-7}	3.77×10^{-5}
$\lvert C^{ee}_{S,XX}\rvert$	1.73×10^{-4}	2.8×10^{-6}	(3.64×10^{-3})
$\lvert C^{\mu\mu}_{V,XX}\rvert$	1.10×10^{-4}	5.60×10^{-5}	1.85×10^{-5}
$\lvert C^{\mu\mu}_{V,XY}\rvert$	2.56×10^{-4}	1.12×10^{-4}	3.77×10^{-5}
$\lvert C^{\mu\mu}_{S,XX}\rvert$	8.24×10^{-7}	(1.58×10^{-5})	(1.73×10^{-5})
$\lvert C^{\tau\tau}_{V,XX}\rvert$	3.80×10^{-4}	1.95×10^{-4}	1.24×10^{-5}
$\lvert C^{\tau\tau}_{V,XY}\rvert$	4.40×10^{-4}	1.91×10^{-4}	1.25×10^{-5}
$\lvert C^{\tau\tau}_{S,XX}\rvert$	5.33×10^{-6}	1.02×10^{-4}	1.12×10^{-4}
$\lvert C^{\tau\tau}_{S,XY}\rvert$	—	—	—
$\lvert C^{\tau\tau}_{T,XX}\rvert$	1.10×10^{-8}	(4.20×10^{-7})	(2.30×10^{-7})

use: if model gives smaller coefficients, it is consistent with data.
If it generates larger coefficients, need to arrange a cancellation...

$\Rightarrow$(almost) every $\mu \to e$ interaction with ≤ 4 legs(below m_W), otherwise flav. diag., contributes at $\gtrsim \mathcal{O}(10^{-3})$ to $\mu \to e\gamma$ and/or $\mu \to e\bar{e}e$ and/or $\mu A \to eA$

(possible exceptions :$\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)

$\Leftrightarrow$ excellent for discovery = modulo cancellations, can find $\mu \leftrightarrow e$

But only ~12(future ~20?) bds on ~90 coefficients ?

DKunoYamanaka

$\mu \to e\gamma$: $BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ $\Rightarrow$ **2 constraints**

$\mu \to e\bar{e}e$: (e relativistic $\approx$ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2$$
$$+ |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

$\Rightarrow$ **6 more constraints**

$\mu A \to eA$: (S_A^N, V_A^N = integral over nucleus A of N distribution$\times$lepton wavefns, **different** for diff. nuclei)

$$BR_{SI} \sim Z^2|V_A^p\tilde{C}_{V,L}^p + S_A^p\tilde{C}_{S,R}^p + V_A^n\tilde{C}_{V,L}^n + S_A^b\tilde{C}_{S,R}^n + D_AC_{D,R}|^2 + |L \leftrightarrow R|^2$$
$$BR_{SD} \sim |\tilde{C}_A^N + 2\tilde{C}_T^N|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) $\Rightarrow$ **4 + 2 more constraints**

future: improved theory, 3SI+2SD targets $\Rightarrow$ 6 + 4 constraints

is that enough? Can one distinguish models with ~10% of dim6 coeffs?

?models have more than 10 parameters?

...can some models sit anywhere in the 12-d subspace?

$\tau \leftrightarrow \ell$ for distinguishing models?

(less sensitive than $\mu \leftrightarrow e$: $\frac{\Gamma(\tau\to\ell+...)}{\Gamma(\tau\to\ell\nu\bar{\nu})} \sim 10^{-7} \approx \sqrt{\frac{\Gamma(\mu\to e+...)}{\Gamma(\mu\to e\nu\bar{\nu})}} \to 10^{-8}$!
because can make muon beams: PSI plans $\sim 10^{10}\mu$/sec
But $\tau_\tau \sim 10^{-7}\tau_\mu$, so Belle-II aims to pair-produce $\sim 100\tau$/sec)

but, **?** can one search for $\ell \to \tau$ with intense ℓ beams on target? Cirigliano etal: 2102.06176

more bds in $\tau \leftrightarrow \ell$ sector

$BR \lesssim 10^{-6} \to 10^{-8}$ for $\tau \to \ell + \left\{ \begin{array}{cccccc} \pi & \eta & \rho & \omega & \phi & \dots \\ 1(0^-) & 0(0^-) & 1(1^-) & 0(1^-) & 0(1^-) & \boldsymbol{I(J^P)} \end{array} \right.$ $(\phi = ss)$)

isospin,lorentz of mesons restrict operators that can contribute:

$$\begin{aligned} BR(\tau \to e\pi) &\propto |C_A^{e\tau uu} - C_A^{e\tau dd}|^2 + \#(C_P^{e\tau uu} - C_P^{e\tau dd})|^2 \\ BR(\tau \to e\eta) &\propto |C_A^{e\mu uu} + C_A^{e\mu dd} + \#C_A^{e\mu ss} + \#\text{pseudoscalar}|^2 \\ BR(\tau \to e\rho) &\propto |C_V^{e\mu uu} - C_V^{e\mu dd}|^2 \end{aligned}$$

to compare to $\mu A \to eA$, where everything interferes:

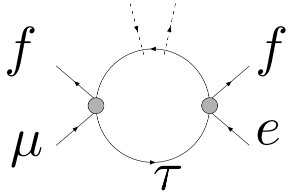
$BR(\mu Ti \to eTi) \propto |C_S^{e\mu uu} + C_S^{e\mu dd} + .2(C_V^{e\mu uu} + C_V^{e\mu dd}) + .05C_S^{e\mu ss} + .01C_S^{e\mu cc} + .002C_S^{e\mu bb}|^2$

$[\mu \to \tau] \times [\tau \to e] = [\mu \to e] \Rightarrow$?

1. if a model contains $(\mu \to \tau)$ and $(\tau \to e)$, then no conserved flavour $\Rightarrow$ "probably" mediates $\mu \to e$. "Expect"
$$\mathcal{A}(\mu \to e) \gtrsim \mathcal{A}(\tau \to \mu) \times \mathcal{A}(\tau \to e)$$

2. can one calculate anything model-independent? In EFT:

$$\Delta C^{e\mu ff} \simeq \frac{1}{16\pi^2} C^{e\tau\cdots} C^{\tau\mu\cdots}$$



3. ...so, eg: if see $\tau \to \mu$, not $\mu \to e$, tells where should not see $\tau \to e$:

$$C^{e\tau\cdots} \lesssim 16\pi^2 \frac{\Delta C^{e\mu ff}}{C^{\tau\mu\cdots}}$$

Summary

LFV must occur, and could give complementary information about the neutrino mass mechanism and New Physics in the lepton sector.

Current $\mu \to e$ bounds (BR$\lesssim 10^{-12}$) probe $\Lambda_{\rm LFV} \lesssim 10^2 \to 10^3$ TeV, and upcoming expts aim to improve sensitivity to BR$\sim 10^{16}$.

$\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$ have experimental sensitivity to only $\sim$ a dozen operators at low energy. But including loop corrections via leading order RGEs mixes $\approx$ every operator coefficient (in chiral basis) into the testables ones. So modulo cancellations, these processes should see $\mu \to e$ LFV if its within reach.

Current $\tau \to \ell$ searches probe scales $\sim 10 \to 50$ TeV in many processes, and BelleII should improve the exptal sensitivity by an order of magnitude. The greater number of observables could assist in distinguishing models.

BackUp

Models may *not* generate at tree the operators expts probe...

ex: $\mu A \to eA$ in a model giving tensor $C_T^{uu}(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u)$ at weak scale

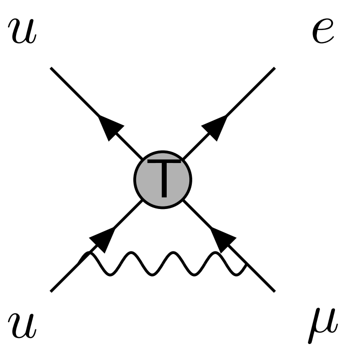
1: forget loops quark tensor matches to nucleon spin $\bar{N}\gamma\gamma_5 N$: $(N \in \{n, p\})$

$$\Rightarrow BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2}|C_T^{uu}|^2$$

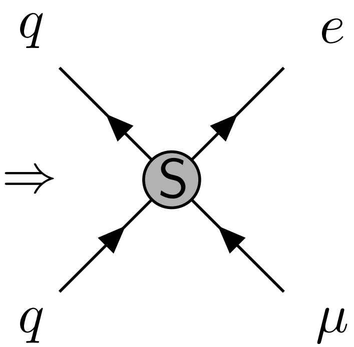
nuclear matrix elements: EngelRTO, KlosMGS

2: include loops

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$



$+ ... \quad \Rightarrow$



$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\; C_T^{uu}\; (\overline{u}u)(\overline{e}P_Y\mu)$$

$$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7}C_T^{uu}(m_W)$$

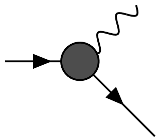
Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}$$

loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained!

loop sensitivity of $\tau \to \ell\gamma$

Two dipole operators contribute to $\tau \to e\gamma$:



$$\delta\mathcal{L}_{teg} = -\frac{4G_F}{\sqrt{2}} m_\tau \left(C_{D,L}\overline{\tau_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,R}\overline{\tau_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta}\right)$$

$$\widetilde{BR}(\tau \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 1.9 \times 10^{-7}$$

$$\Rightarrow \quad |C_{D,X}| \lesssim 7 \times 10^{-6}$$

Babar

How big does one expect $C_{D,X}$ to be? Suppose operator coefficient

			$n=1$	$n=2$
$C_{D,X}\frac{m_\tau}{v^2} \sim \frac{e m_\tau}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3$ TeV	0.2
$C_{D,X}\frac{m_\tau}{v^2} \sim \frac{e v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 30$ TeV	2 TeV

$\Rightarrow$ NP contributing in loops to $\tau \to \ell$ maybe LHC-accessible?

(? can we also forget loops for $\mu \leftrightarrow e$?
... we are in discovery mode; loops are for precision physics, no?)

$\boldsymbol{BR(\tau \to 3\ell) < 2 \times 10^{-8}}$

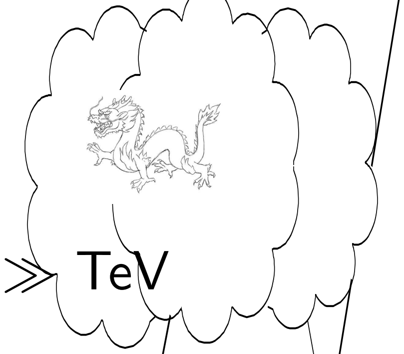
τ has many weak decays : $\to \begin{cases} \nu u \bar{d} \times 3(col.) \\ \nu \mu \bar{\nu} \\ \nu e \bar{\nu} \end{cases} \Rightarrow BR(\tau \to e\bar{\nu}\nu) \simeq 0.175$

to estimate $\Lambda_{\rm LFV}$ for $\Gamma(\tau \to 3\ell)$ define

$$\widetilde{BR}(\tau \to 3\ell) \equiv \frac{\Gamma(\tau \to 3\ell)}{\Gamma(\tau \to e\bar{\nu}\nu)} \sim \frac{v^4}{\Lambda^4_{\rm LFV}} \lesssim 10^{-7} \Rightarrow \Lambda_{LFV} \sim 50v \simeq 10 \text{ TeV}$$

vs $\Lambda_{\rm LFV} \sim 200$ TeV from $BR(\mu \to e\bar{e}e) < 10^{-12}$.

EFT for Heavy LFV Physics...



$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$\mathcal{L}_{SM}$ | $+\mathcal{L}(\text{SM invar. operators, dim6})$

$m_W \sim m_h \sim m_t$

$\{\gamma, g, f\}$

$\mathcal{L}_{QED\times QCD}$ | $+\mathcal{L}(3\to 4 \text{ legged QCDxQED invar. ops})$ $\sim$ 90 of them!

2 GeV $\sim m_c, m_b, m_\tau$

$\mathcal{L}(n, p, \pi, \gamma, e, \mu)$ | $+\mathcal{L}(\text{3or4 legged QED invar. ops})$

data $(\mu\to e\gamma, \mu\to e\bar{e}e, \mu A\to eA)$

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon
+BowmanChengLiMatis

Operator basis $m_\tau \to m_W$: ~ 90 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad \textit{dim } 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad \textit{dim } 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad \textit{dim } 7$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

...zzz...but ~ 90 coeffs!

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

operators at exptal scale

There are dipoles of 2 chiralities

$$D \qquad \overline{e}\sigma^{\alpha\beta}P_L\mu F_{\alpha\beta} \qquad \overline{e}\sigma^{\alpha\beta}P_R\mu F_{\alpha\beta}$$

which also contribute in $\mu \to e\gamma$, $\mu \to e\bar{e}e$.

Six 4-fermions for $\mu \to e\bar{e}e$, $Y, X \in \{L, R\}, Y \neq X$

$$V \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$S \qquad (\overline{e}P_Y\mu)(\overline{e}P_Y e)$$

For $\mu A \to eA$, interactions with nucleons $N \in \{n, p\}$ parametrised by :

$$S,\ V \qquad \overline{e}P_X\mu\overline{N}N \qquad \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N \qquad X \in \{L, R\}$$

$$A,\ T \qquad \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N \qquad \overline{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N$$

$$P,\ Der \qquad \overline{e}P_X\mu\overline{N}\gamma_5 N \qquad \overline{e}\gamma^\alpha P_X\mu(\overline{N}i\overleftrightarrow{\partial}_\alpha\gamma_5 N)$$

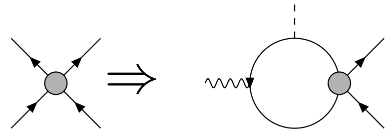
Matching in χPT gives Derivative. But absorb in matching into $G_O^{N,q}$= quark matrix elements in nucleons.

chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

"Accidental cancellations" and "naturalness" in EFT

("accidental" cancellations occur; in 1-loop RGEs give, for coeff.s at $\sim m_W$ of
$\mathcal{O}_{D,L} = m_\mu(\overline{e}\sigma \cdot F P_L \mu)$, $\mathcal{O}^{\tau\tau}_{T,LL} = (\overline{e}\sigma P_L \mu)(\overline{\tau}\sigma P_L \tau)$:

$$BR(\mu \to e\gamma) \approx |0.938 C_{D,L} + 0.981 C^{\tau\tau}_{T,LL} + \ldots|^2$$)



If imagine that NP knows about all the SM parameters, but not about the scale at which you do expts, could argue that RG-stable cancellations in EFT can be "natural".

(caveat: NP does know about all the mass scales in the theory, which often determine the scales in the logs...)

So if resum RGs, cancellations among coeff.s with same anom dim are ok?
If not resum, can allow cancellations among all coeff.s who multiply same log?

Interest of this argument, is that forbidding "unnatural" calcellations transforms a single exptal bound into many bounds...but unnatural cancellations occur, see green parenthese: dipole is tree, tensor is log-enhanced loop.

Quantifying which targets give independent information (on nucleons)

1. neglect Dipole (better sensitivity of $\mu \to e\gamma$ (MEGII) and $\mu \to e\bar{e}e$(Mu3e).
   remain to determine: $\vec{C} \equiv (\widetilde{C}^{pp}_{VR}, \widetilde{C}^{pp}_{SL}, \widetilde{C}^{nn}_{VR}, \widetilde{C}^{nn}_{SL})$

2. recall that
$$BR_{SI}(A\mu \to Ae) \propto \left|\vec{C} \cdot \vec{v}_A\right|^2$$
   where target vector for nucleus A
$$\vec{v}_A \equiv \left(V^{(p)}_A, S^{(p)}_A, V^{(n)}_A, S^{(n)}_A\right)$$

3. So first experimental search (eg on Aluminium) probes projection of $\vec{C}$ of $\vec{v}_{Al}$
   ... next target needs to have component $\perp$ to Aluminium!
   $\Leftrightarrow$ plot misalignment angle θ between target vectors

4. how big does θ need to be?
   overlap integrals have theory uncertainty: $\Delta\theta \left\{ \begin{array}{ll} \text{nuclear} & \sim 5\% (KKO) \\ NLO\ \chi\text{PT} & \sim 10\%(?) \end{array} \right.$
   Both vectors uncertain by $\Delta\theta$; need misaligned by $2\Delta\theta \approx 10 \to 20\%$

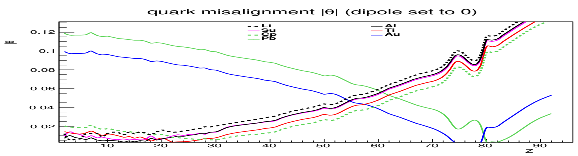
But what happens when match nucleons to quarks?

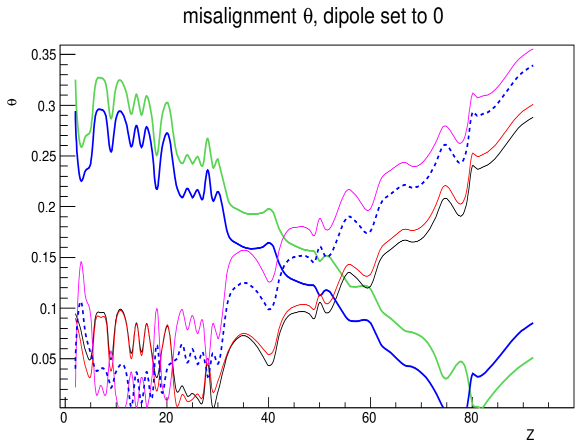
By measuring $\mu A \to eA$ on different targets, could determine coefficients of LFV ops with vector and scalar currents of n or p.

Match to quarks: $(\Gamma_O \in \{I, \gamma_5, \gamma^\alpha, \gamma^\beta\gamma_5, \sigma^{\alpha\beta}\})$

$$\begin{aligned} \langle N(P_f)|\bar{q}(x)\Gamma_O q(x)|N(P_i)\rangle &= G_O^{N,q}\langle N|\bar{N}(x)\Gamma_O N(x)|N\rangle \\ &= G_O^{N,q}\overline{u_N}(P_f)\Gamma_O u_N(P_i)e^{-i(P_f-P_i)x} \end{aligned}$$

But for scalar ops, $G_S^{p,u} = G_S^{n,d} \simeq G_S^{p,d} \simeq G_S^{n,u}$
so need great precision to differentiate LFV ops with scalar currents of u or d :(

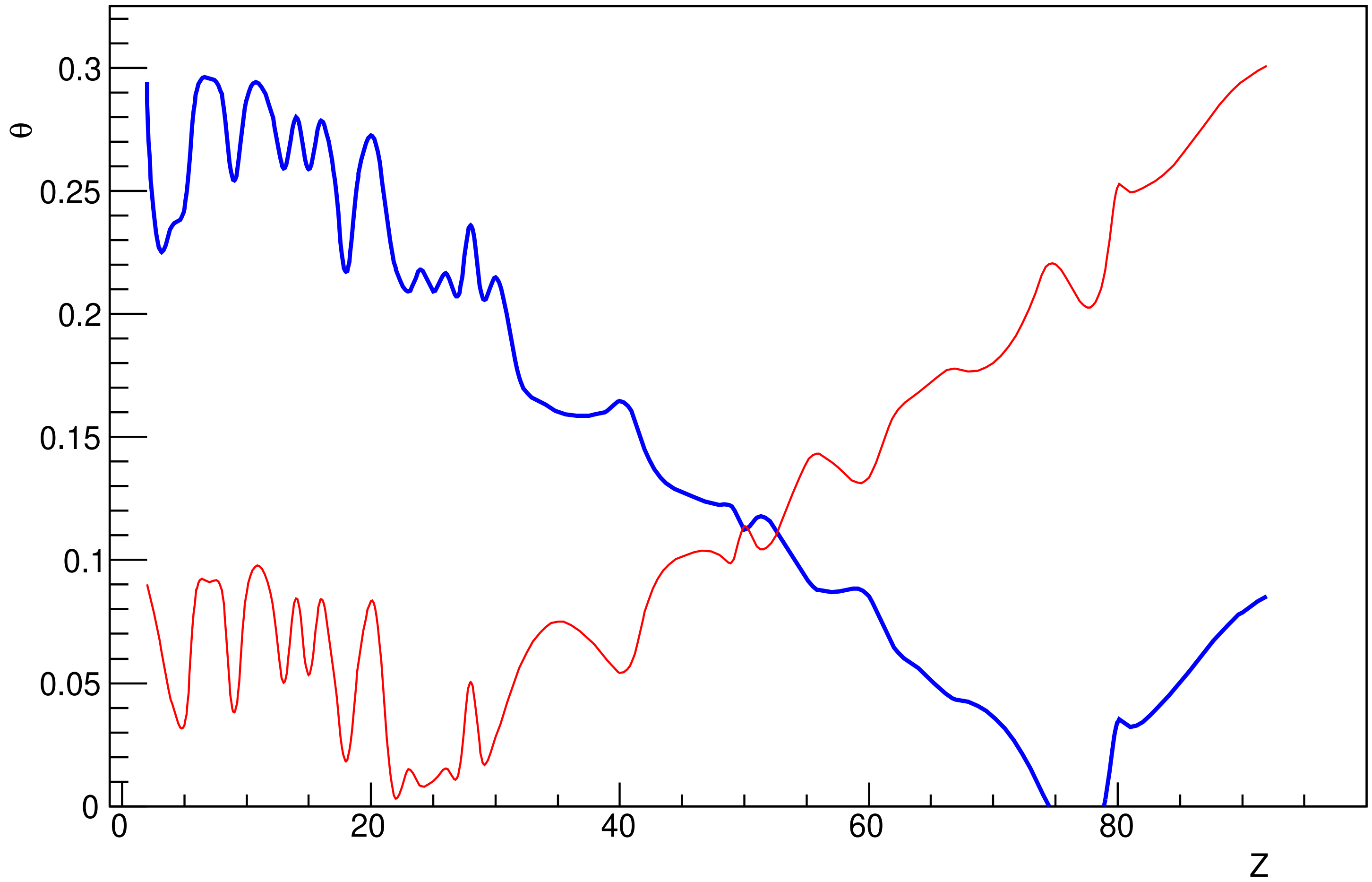




Current data+ theory uncertainty $\sim 10\%$**: two targets give** $\Delta\theta > 0.2$

$$\boldsymbol{BR(\mu Au \to e Au) \leq 7 \times 10^{-13}} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to e Ti) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

In the future...with a 5% theory uncertainty:

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

$$\hat{v}_{Al} \approx \frac{1}{2}(1,1,1,1)$$

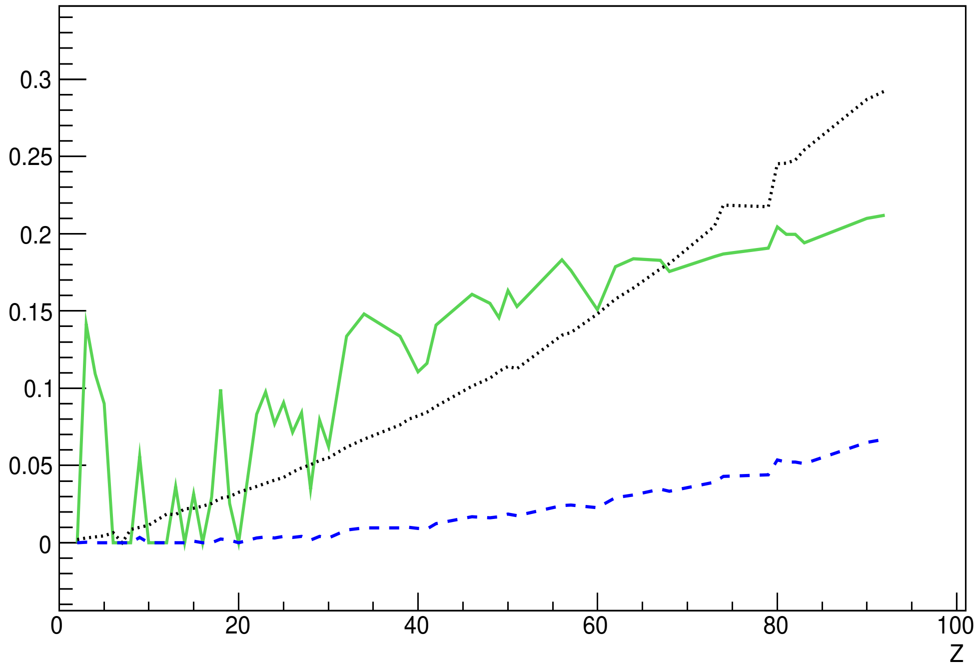
(recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

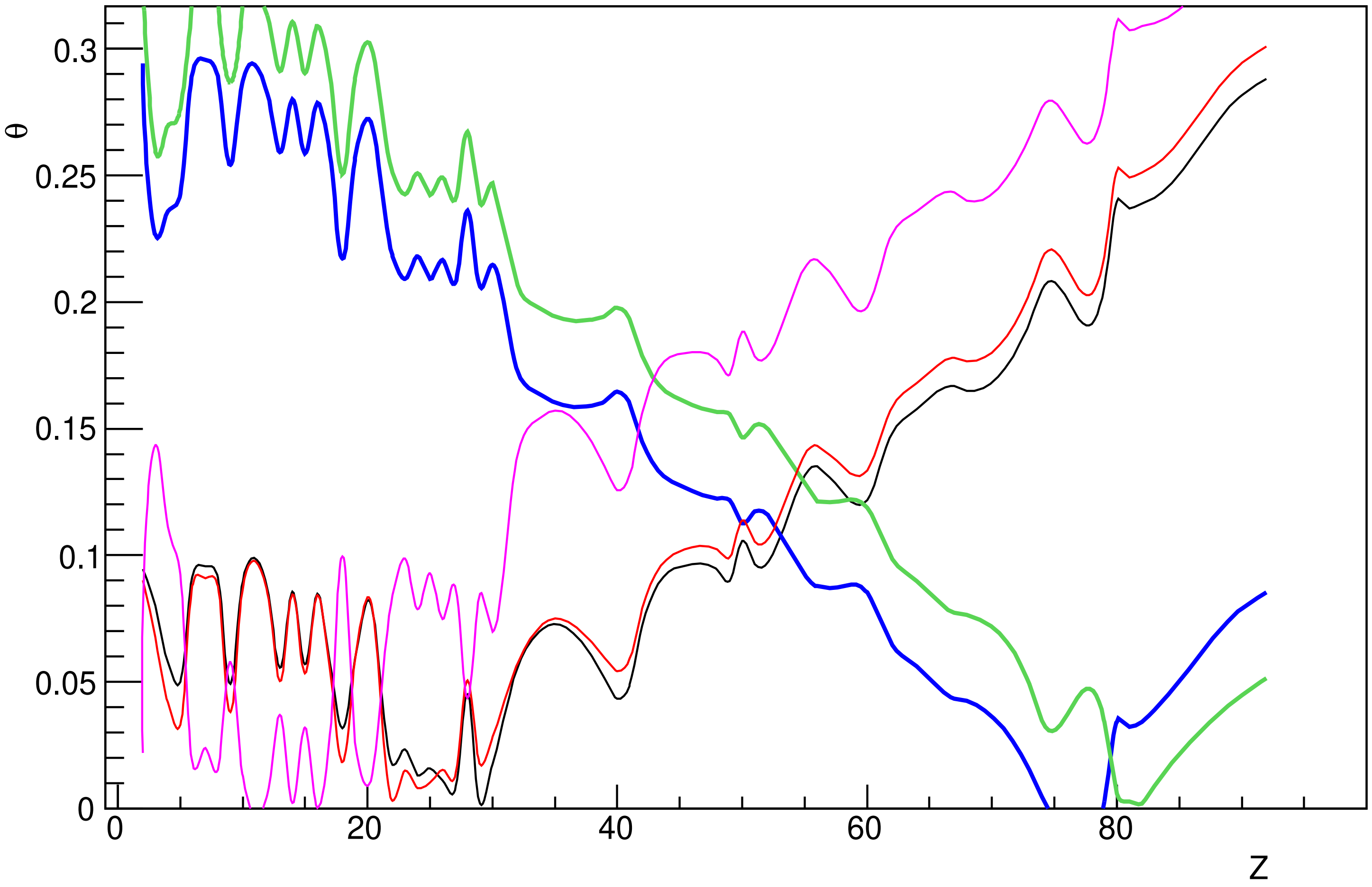
$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$



probe 3 combinations of SI coeffs

All current data... $\boldsymbol{BR(\mu Au \to eAu) \leq 7 \times 10^{-13}}$ $(Au : Z = 79)$

$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12}$ $(Ti : Z = 22)$



$\boldsymbol{BR(\mu Pb \to ePb) \leq 4.6 \times 10^{-11}}$

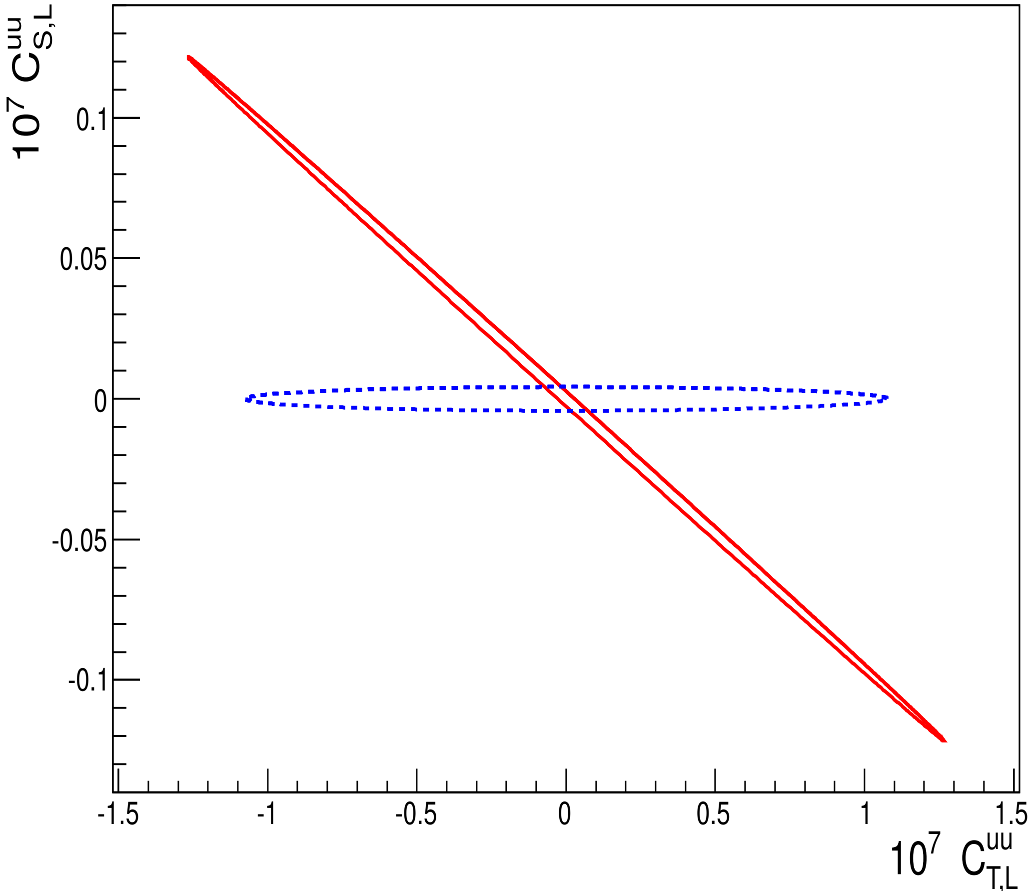
$BR(\mu S \to eS) \leq 7 \times 10^{-11}$ S = Sulpher, Z = 16

$BR(\mu Cu \to eCu) \leq 1.6 \times 10^{-8}$ Cu = Copper, Z = 29

sensitivity *vs* constraint

Suppose that $BR(\mu Al \to eAl) \lesssim 10^{-14}$, and :

$$\delta\mathcal{L}(m_W) = C_T^{uu}(\overline{e}\sigma P_Y \mu)(\overline{u}\sigma u) + C_S^{uu}(\overline{e}P_Y \mu)(\overline{u}u)$$



C_T^{uu}, C_S^{uu} constrained to live inside blue (red) ellipse at exptal scale (at m_W): sensitivity to C_S^{uu} = cut ellipse @ $C_T^{uu} = 0$; constraint = live in projection of ellipse onto C_S^{uu} axis.