Hadronic contributions to the muon anomalous magnetic moment

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20th Lomonosov Conference on Elementary Particle Physics
Moscow State University/Zoom

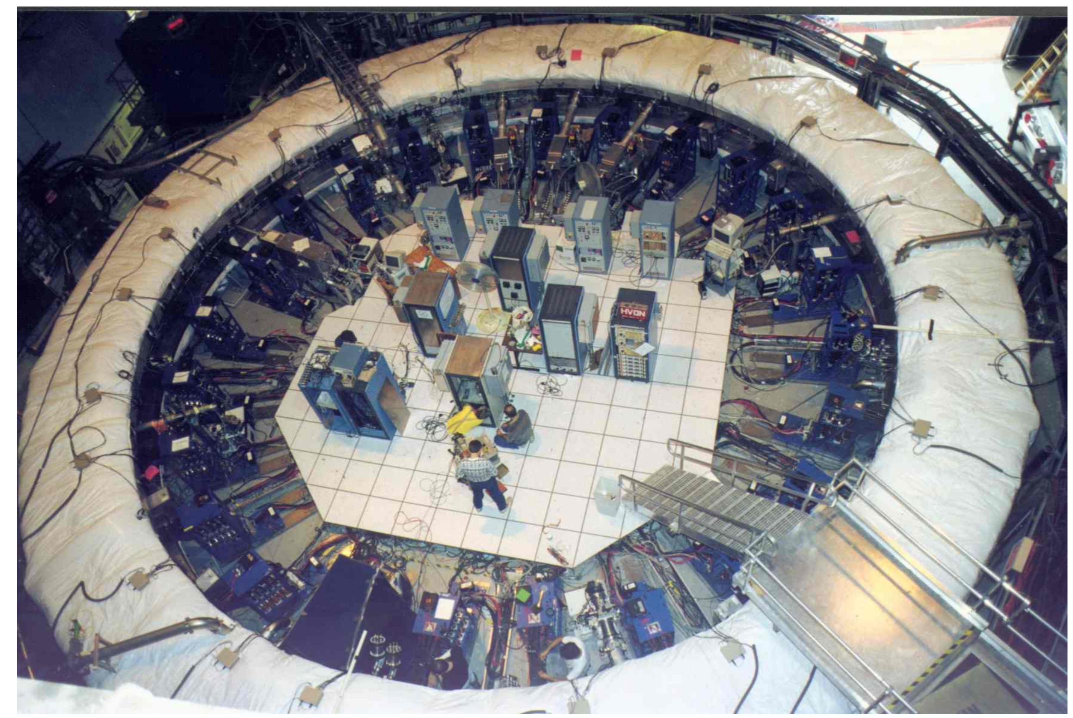
August 21, 2021

[Anomalous moment for the muon, Cern Courier May/June 2021]

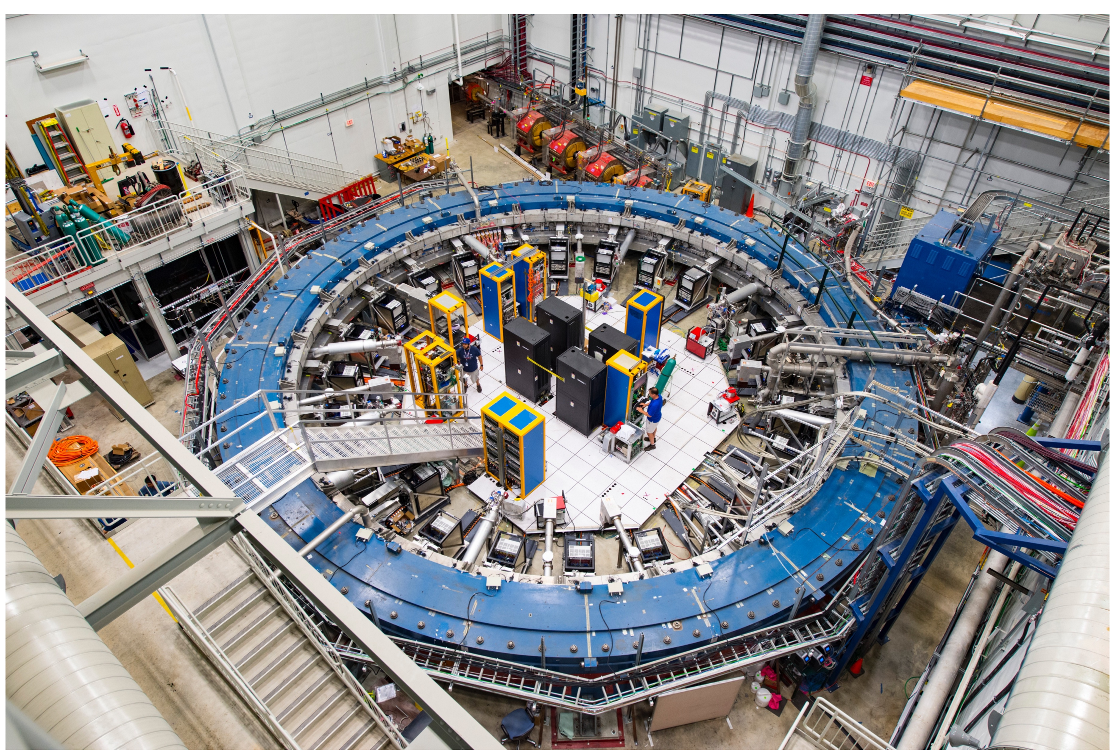
Credit: Mark Rayner/CERN

Muon g-2 experiments

CERN (1959-1979)



BNL E821(1997-2001)

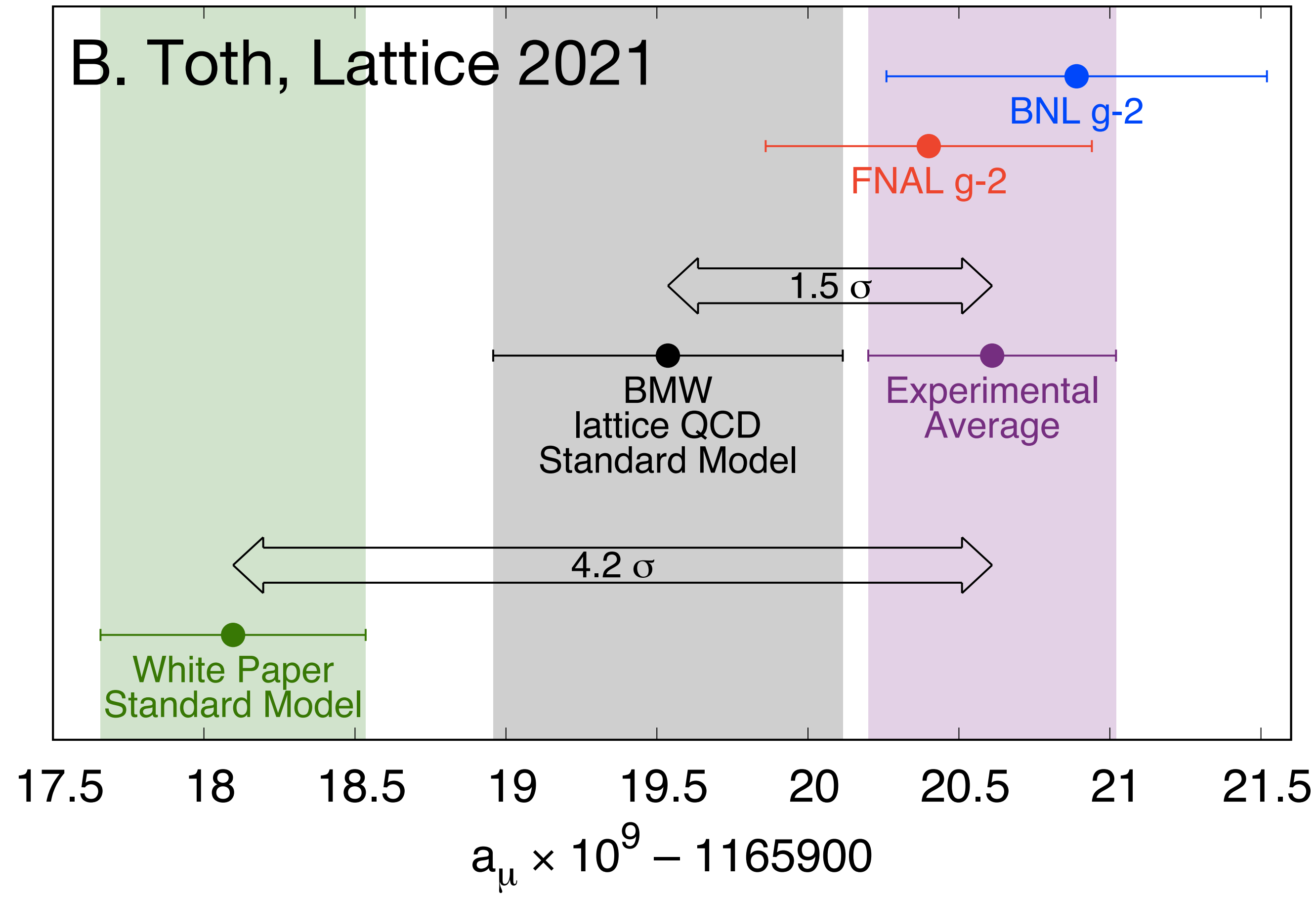


FNAL E989 (2018-)

Conclusions

- We have determined a_μ to an unprecedented 460 ppb precision!
- The Run 1 result
  - 6% of ultimate data sample
  - 15% smaller error than BNL
  - 3.3σ tension with SM

$$a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$$

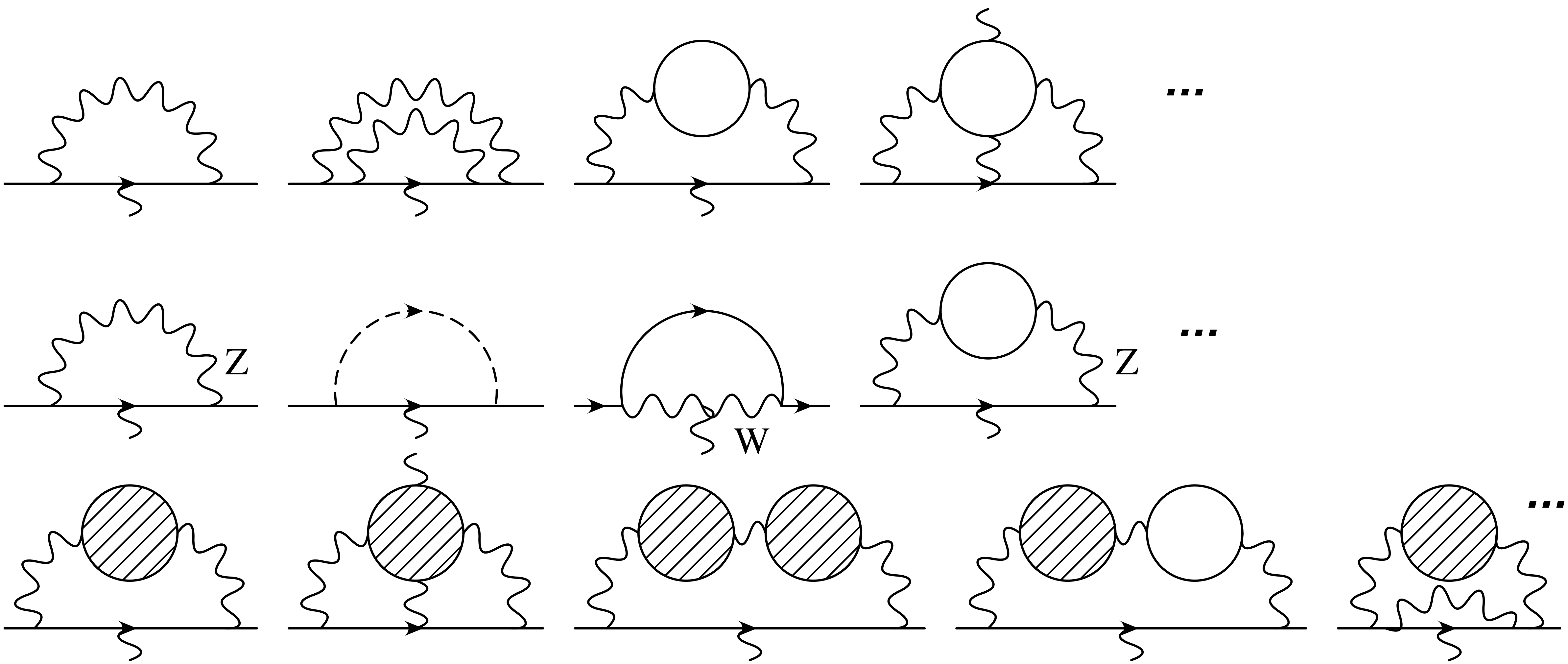


- After 20 years, we confirm the BNL experimental results!
- Combining BNL/FNAL and comparing to theory → 4.2σ tension

Standard Model Theory: QED+EW+QCD

$$\langle \mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle = -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

$$a_\mu \equiv (g-2)/2 = F_2(0) \qquad (q = p' - p)$$



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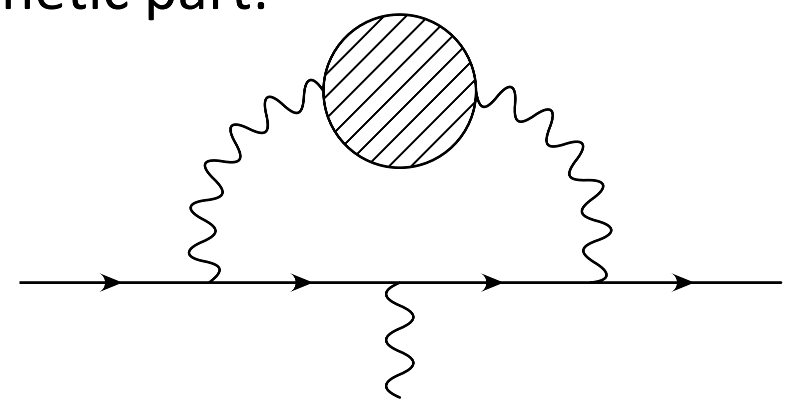
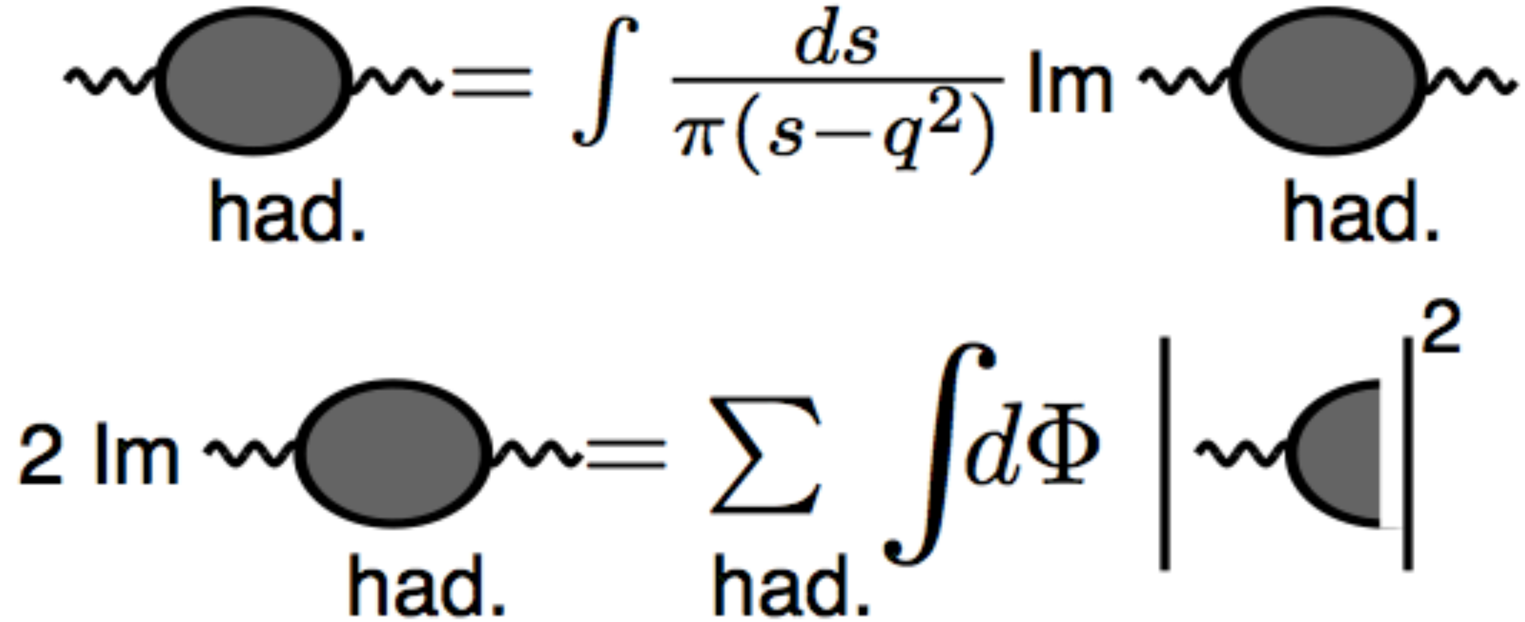
The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C.M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[14,*], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[18,*], C.T.H. Davies[19], M. Della Morte[20], S.I. Eidelman[21,22,*], A.X. El-Khadra[23,24,*], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D.W. Hertzog[33], A. Hoecker[34], M. Hoferichter[14,35,*], B.-L. Hoid[36], R.J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[26,37,*], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M.K. Marinković[46,47], P. Masjuan[48,49], A.S. Meyer[37], H.B. Meyer[12,13], T. Mibe[1,*], K. Miura[12,13,3], S.E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[12,*], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C.F. Redmer[12], B.L. Roberts[57,*], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[60,*], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18], M.N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C.A. Dominguez[67], A.E. Dorokhov[68], V.P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C.S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J.R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A.S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P.B. Mackenzie[24], W.J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E.T. Neil[77], A.V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A.E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E.P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A.S. Zhevlakov[78]

Hadronic vacuum polarization (HVP) I: data driven (e^+e^-)

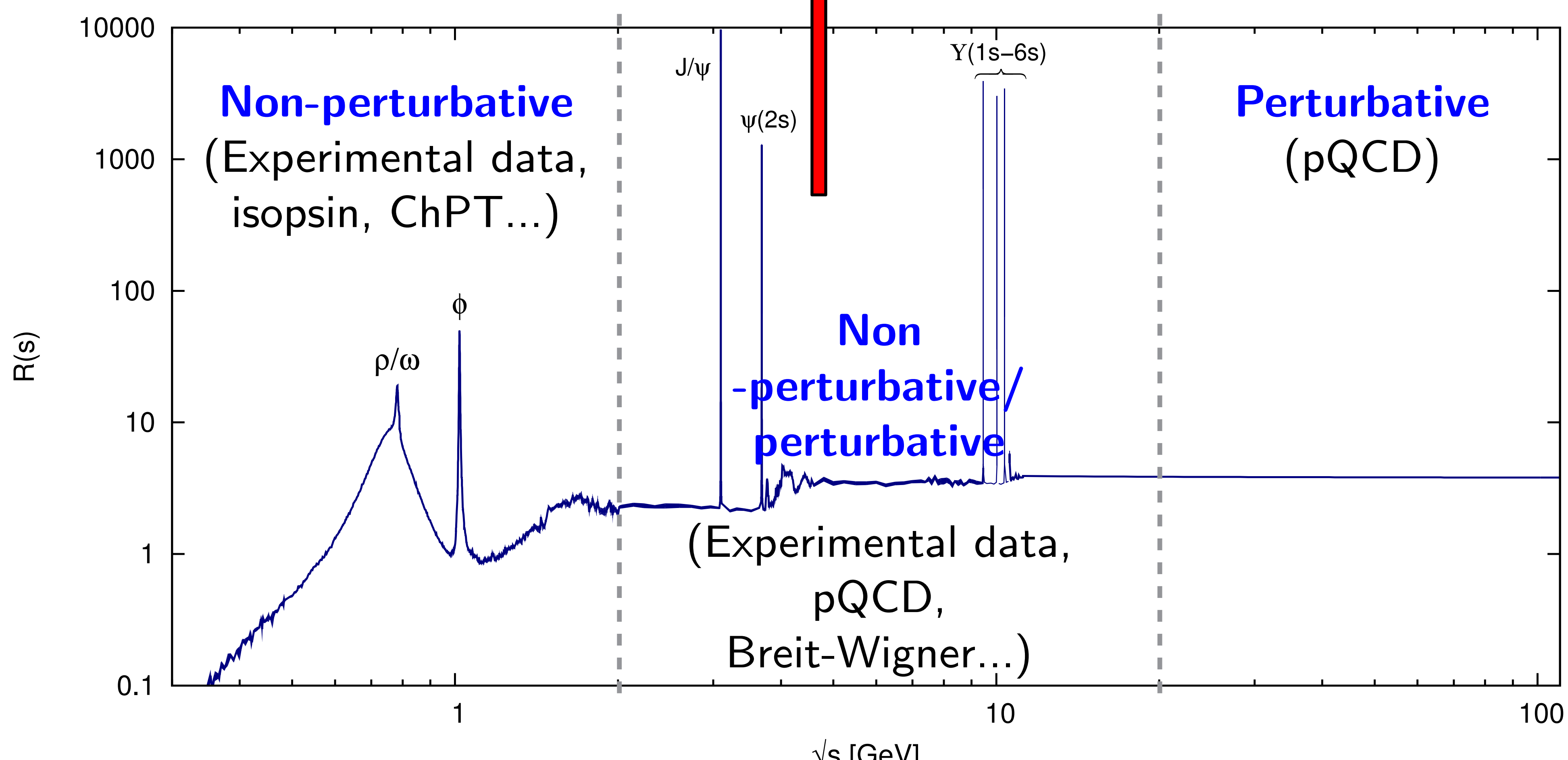
Need magnetic part:





Credit: T. Teubner

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\,R(s)K(s),\ \text{where}\ R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



Credit: T. Teubner

Data-driven a_μ-HVP

$$a_\mu^{\mathrm{HVP,\ LO}} = 693.1(2.8)_{\mathrm{exp}}(2.8)_{\mathrm{sys}}(0.7)_{\mathrm{DV+QCD}} \times 10^{\ \ 10}$$

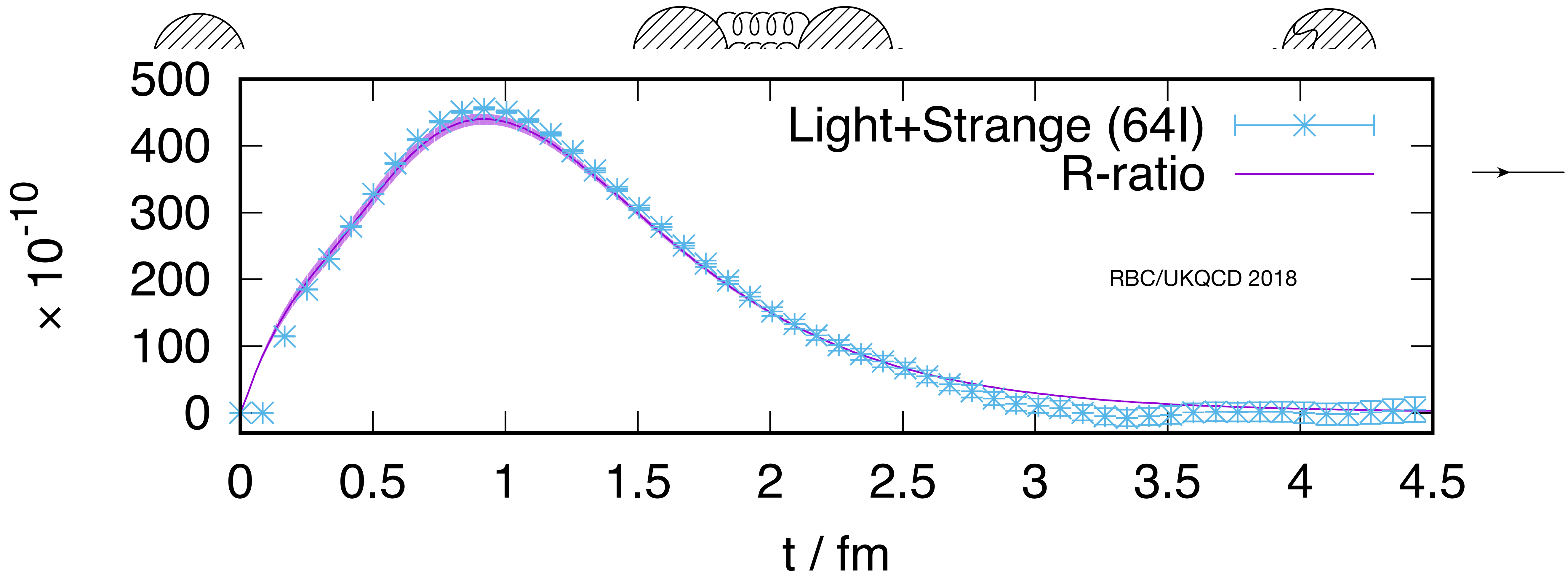
More precise than lattice determination. Total error larger than DHMZ and KNT separately.

Data from BABAR, BESIII, CMD-2, KLOE, SND

“Merged” value from DHMZ, KNT, and CHHKS (simple average in each channel for central value, conservative combination of errors). Errors statistical and systematic

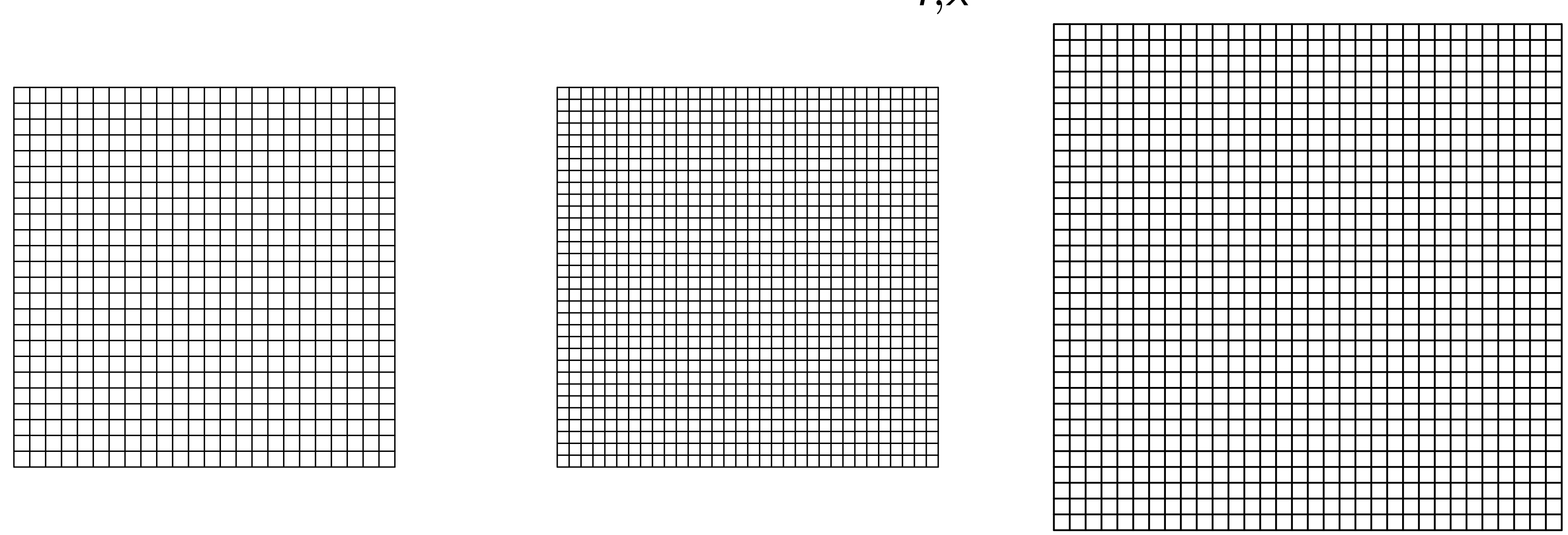
Hadronic vacuum polarization II: Lattice QCD

Blum 2002



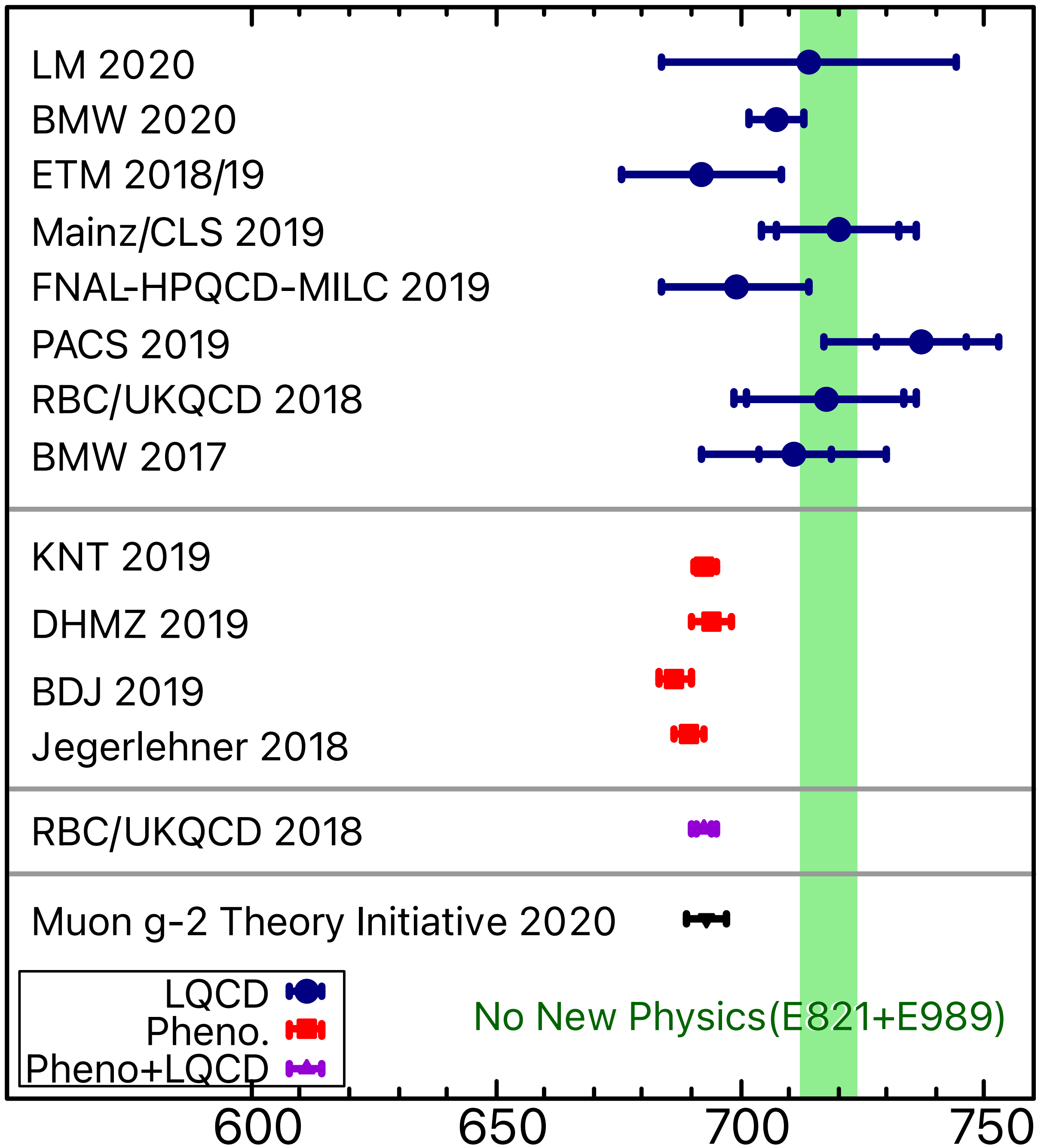
$$a_\mu^{\mathrm{HVP}} = \sum_t w(t)C(t), \qquad C(t) = \frac{1}{3}\sum_{i,\vec{x}} \langle j_i(\vec{x},t)j_i(0)\rangle$$

Calculate $C(t)$ on 4d
Euclidean space-time lattice



24D: $24^3 \times 64$, $4.8\,\mathrm{fm}$ box   32Dfine: $32^3 \times 64$, $4.8\,\mathrm{fm}$ box   32D: $32^3 \times 64$, $6.4\,\mathrm{fm}$ box

a_μ-HVP Summary



(0.75%)

HVP (BMW-20): $a_\mu = 707.5\ (5.5)$ x 10^{-10}

(2.6%)

HVP (Lattice): $a_\mu = 711.6\ (18.4)$ x 10^{-10}

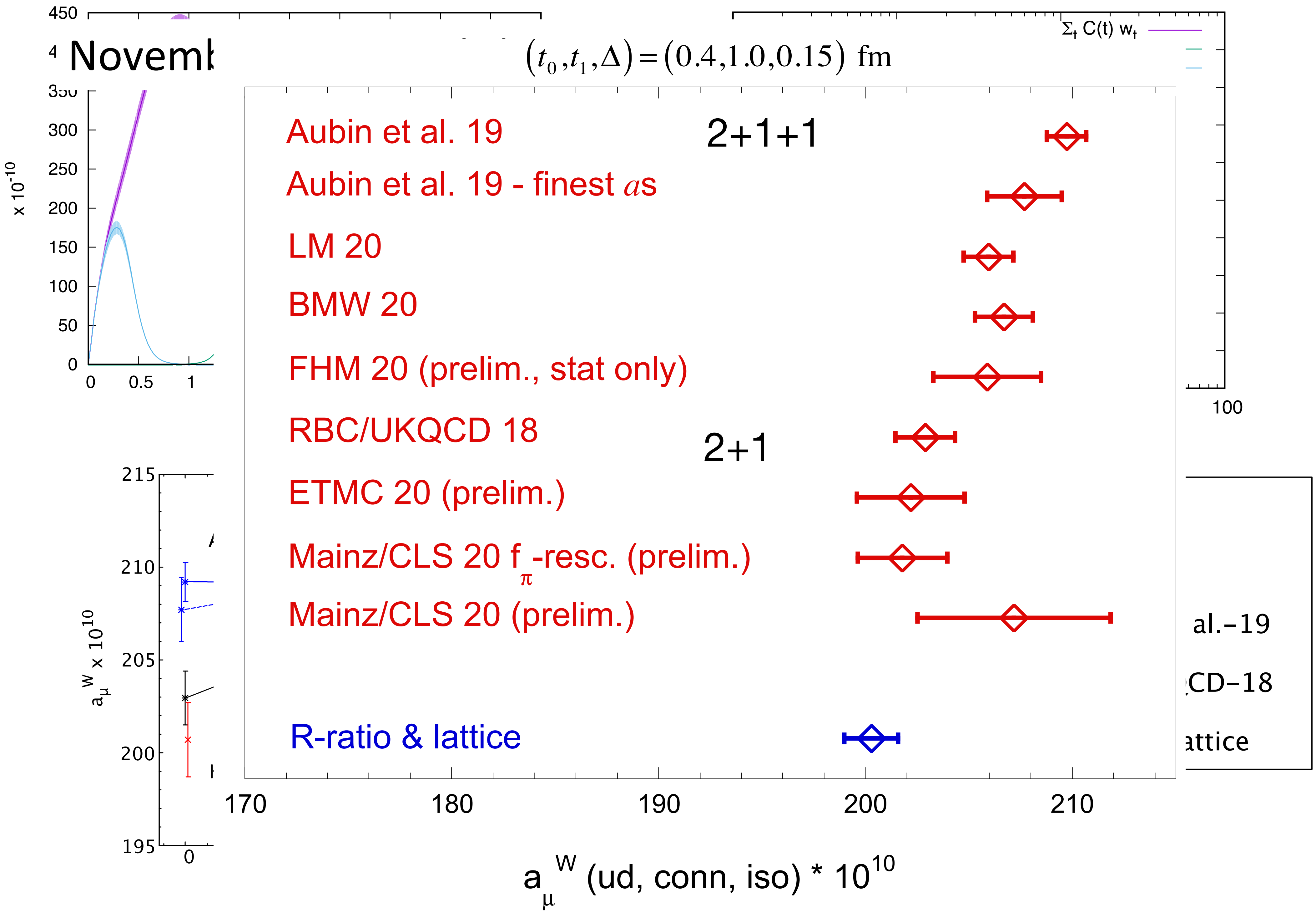
(0.58%)

HVP (pheno): $a_\mu = 693.1\ (4.0)$ x 10^{-10}

Lattice – pheno $\approx 18.5\ (18.8)$

BMW-20 – pheno $\approx 14.4\ (6.8)$

Towards precise comparisons: the window method [RBC/UKQCD-18]

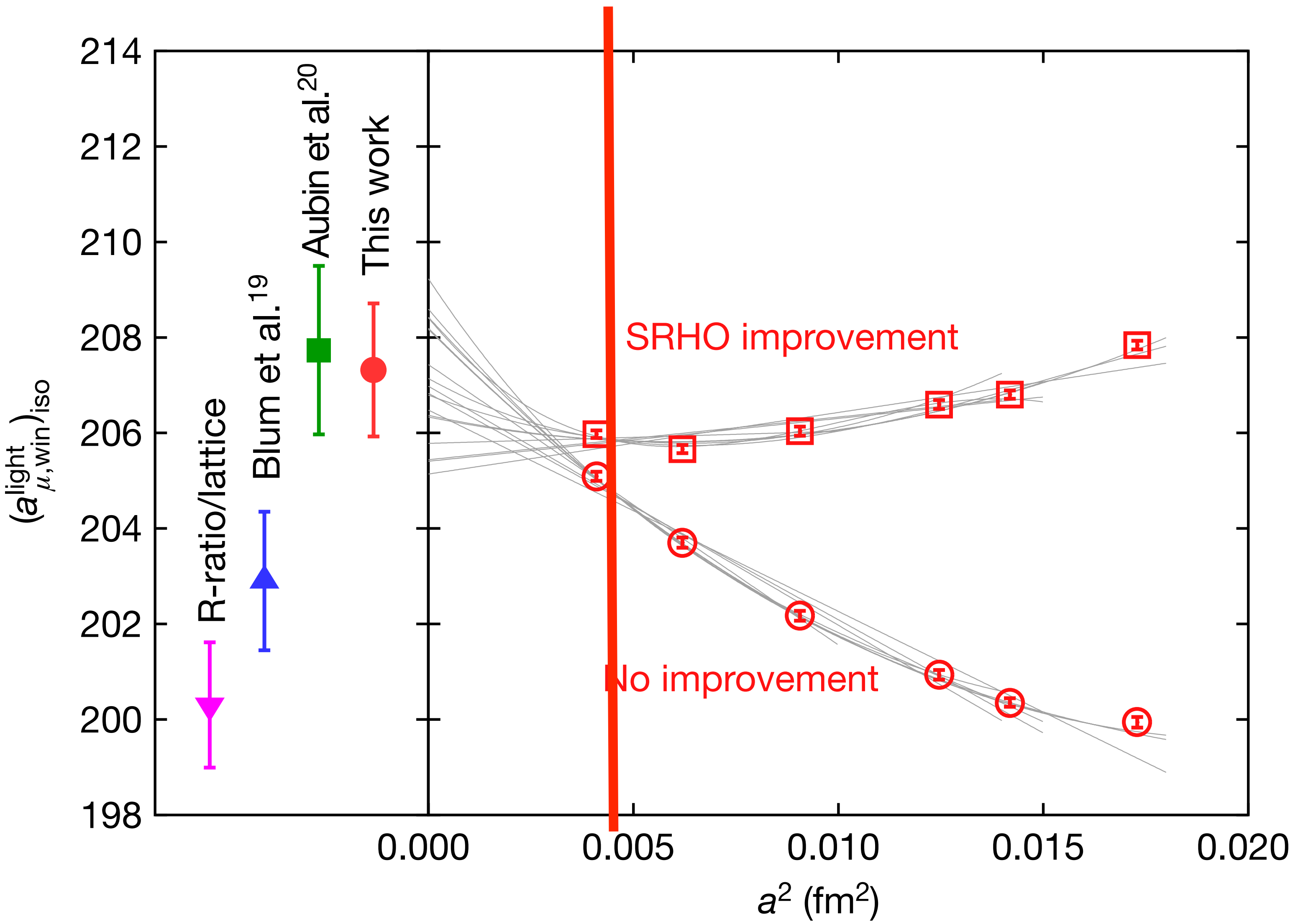


(ud connected, 0.4 – 1.0 fm window)

Towards precise comparisons: the window method [RBC/UKQCD-18]

Continuum extrapolation - What lattice spacing is fine enough?

BMW 20 - light quark window



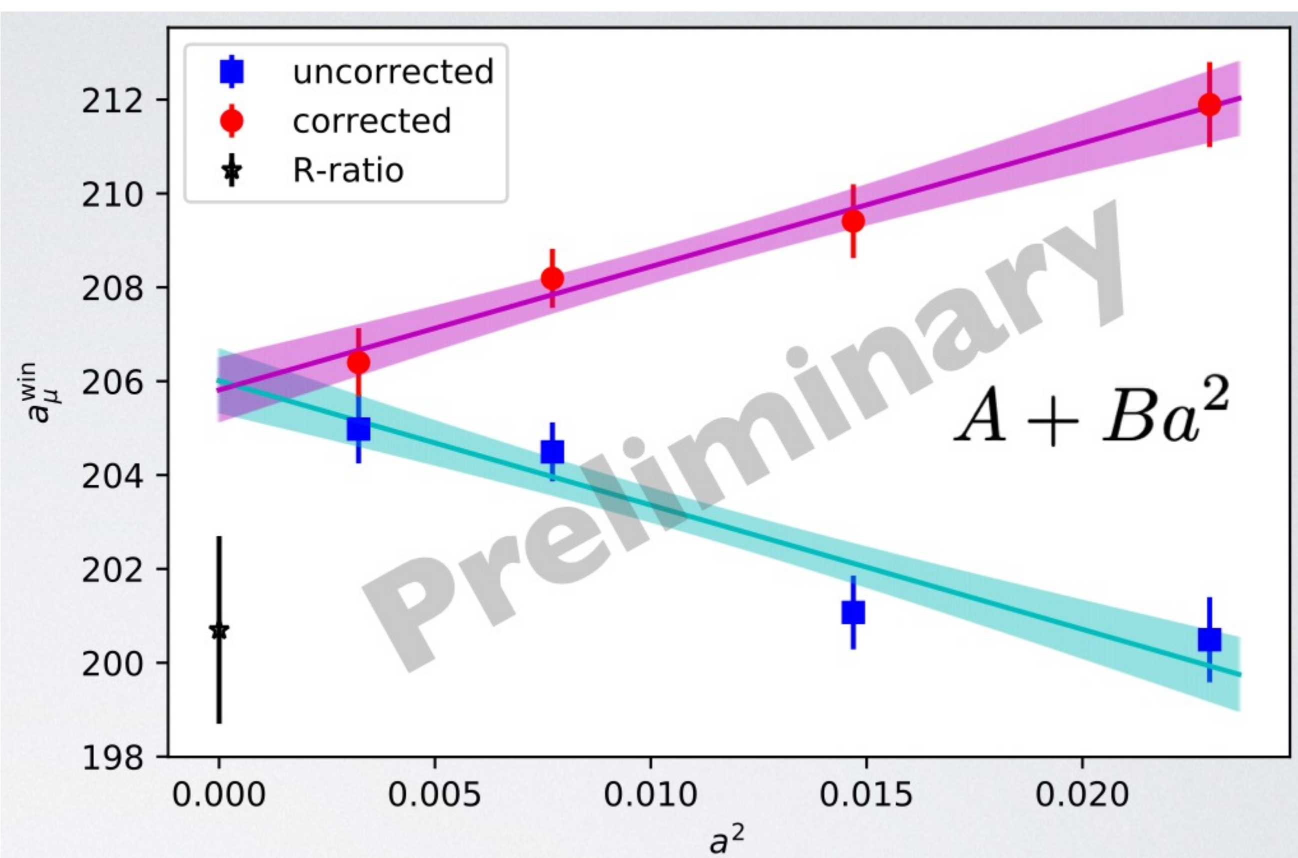
3.7σ tension between BMW20 and R-ratio for Window!

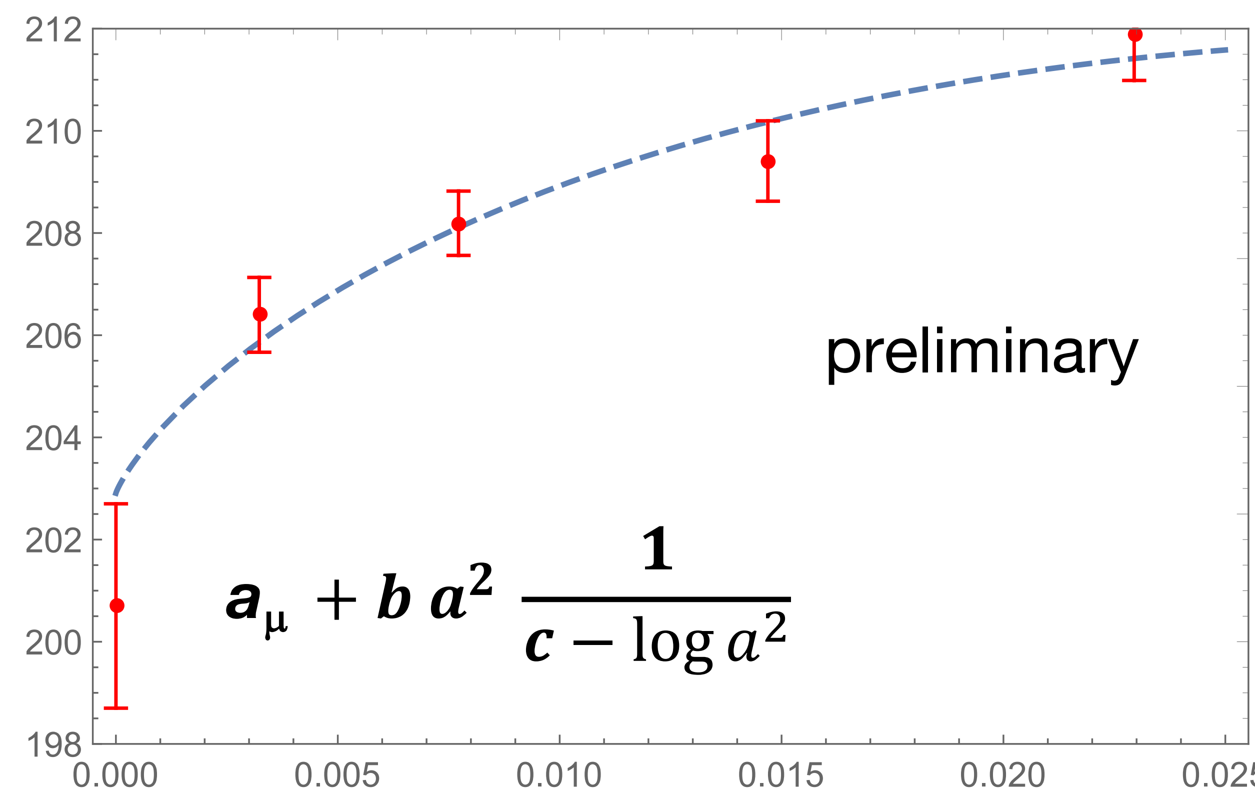
Red line for comparison with next slides

Slide credit: Christoph Lehner

Towards precise comparisons: the window method [RBC/UKQCD-18]

- Is lattice continuum limit incompatible with R-ratio result?
- Or do we need even finer lattices?
- Fits to ud connected, 0.4 – 1.0 fm window:

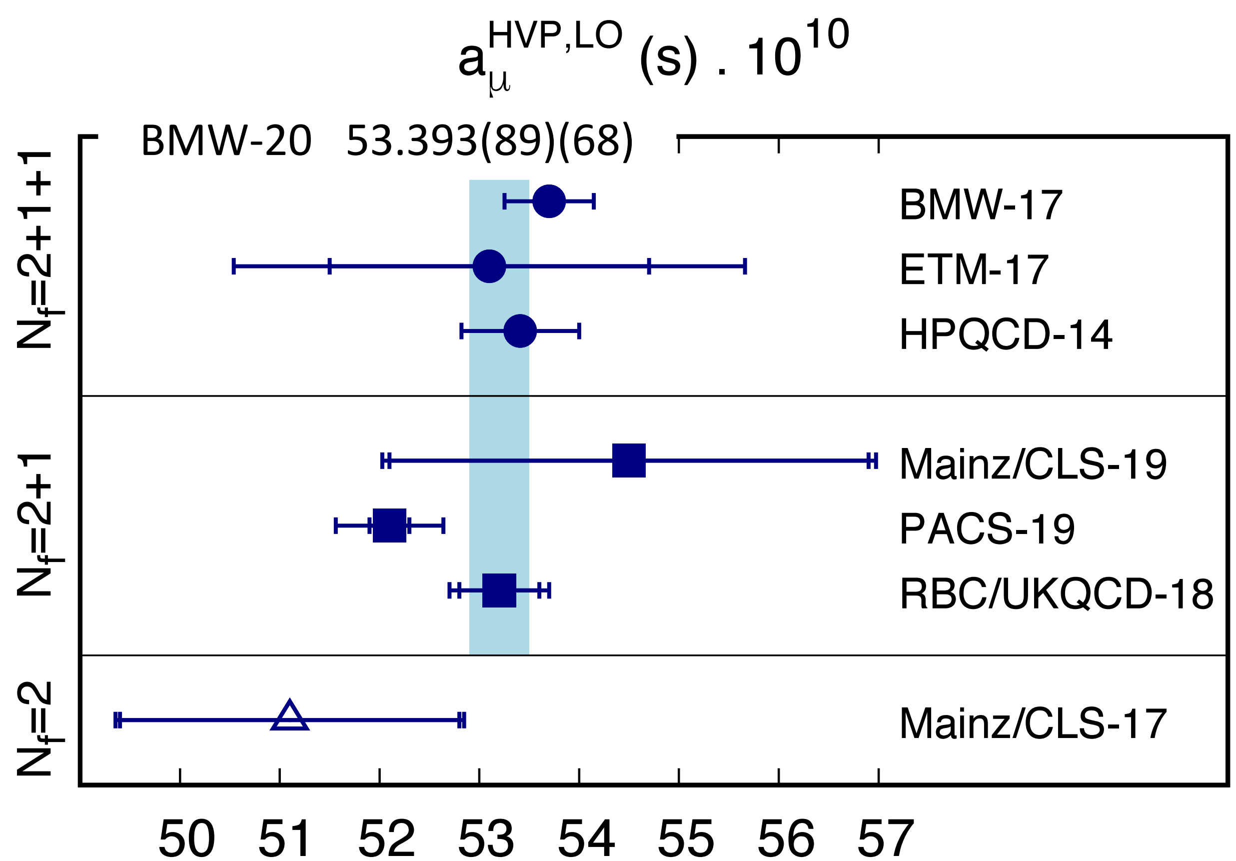


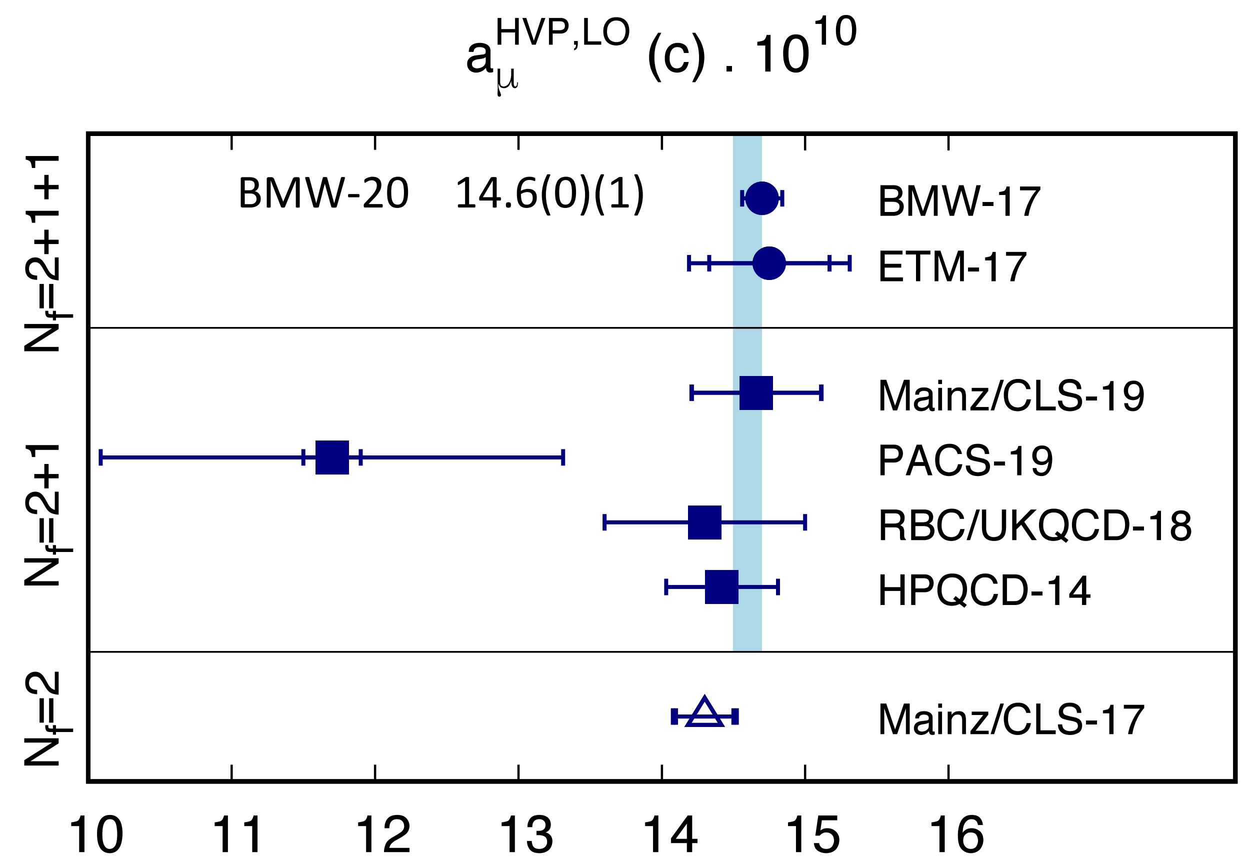


Aubin *et al.*

Strange and charm contributions

Seem to be in good shape

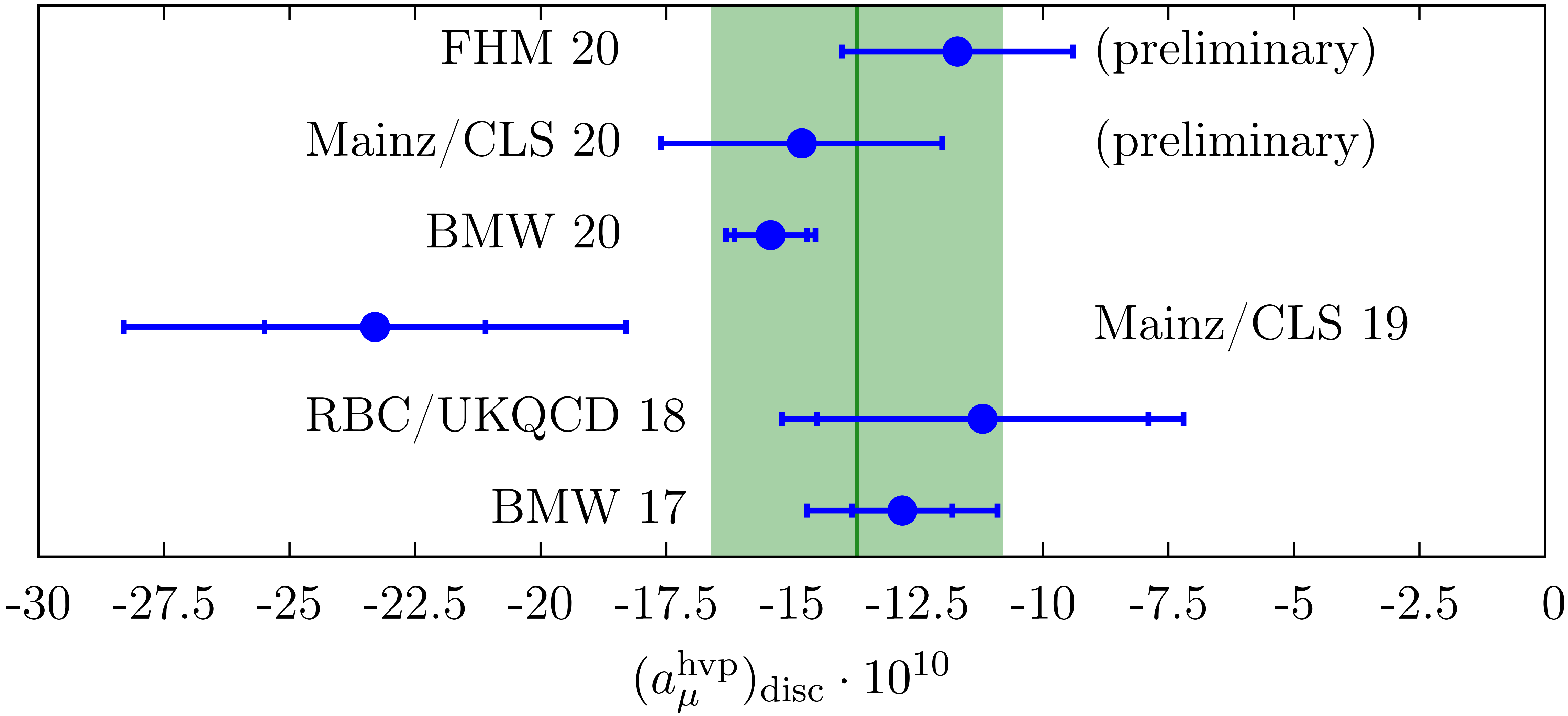




Disconnected contributions

Seem to be in good shape

H. Wittig @ Lattice HVP workshop

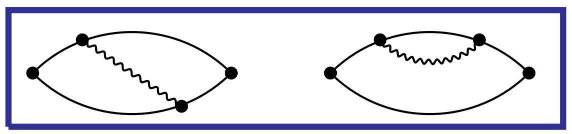


Isospin-breaking contributions

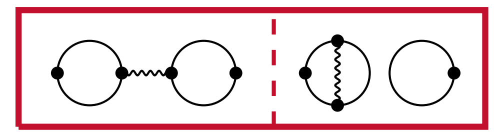
Needs further study, improvement

V. Gülpers @ Lattice HVP workshop

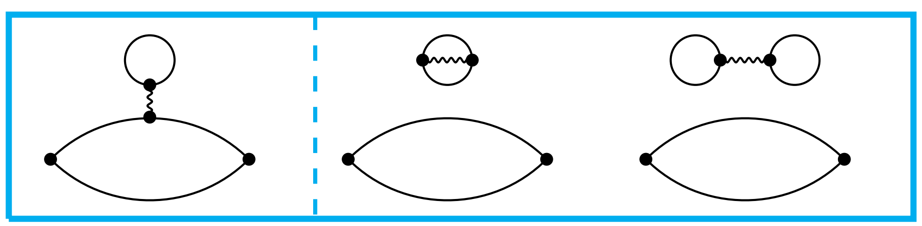
Overview of published results - contributions to $a_\mu \times 10^{10}$



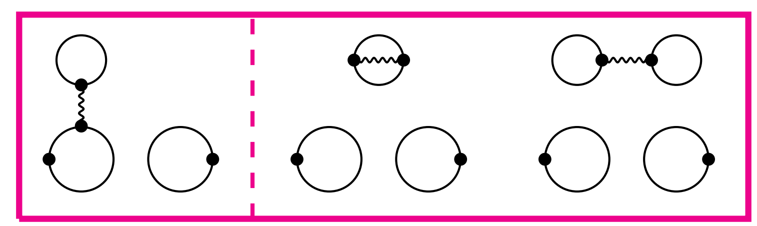
BMW	$-1.27(40)(33)$
RBC/UKQCD	$5.9(5.7)(1.7)$
ETM	$1.1(1.0)$



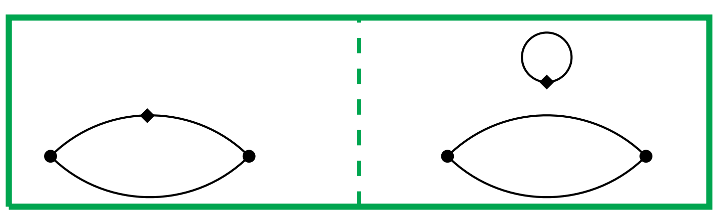
$-0.55(15)(11)$	BMW
$-6.9(2.1)(2.0)$	RBC/UKQCD



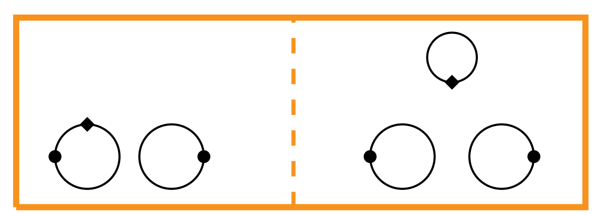
$-0.0095(86)(99)$ $0.42(20)(19)$ BMW



$0.011(24)(14)$ $-0.047(33)(23)$ BMW



$6.59(63)(53)$		BMW
$10.6(4.3)(6.8)$		RBC/UKQCD
$6.0(2.3)$		ETM
$7.7(3.7)$	$9.0(2.3)$	FHM
$9.0(0.8)(1.2)$		LM



$-4.63(54)(69)$ BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]

HVP: to reach desired precision (2-5 per-mil)

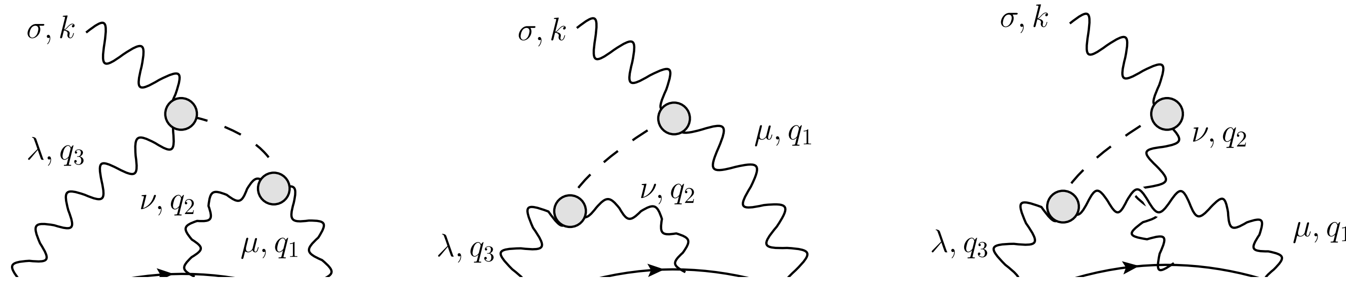
- Strange, charm, (bottom) contributions in good shape
  (will not resolve issues)
- FV corrections $(L > 6\ \mathrm{fm})$ reliable (NNLO χPT, LLGS, HP)
  Important to have a big box (BMW, PACS use L = 10 fm)
- Statistical precision top priority for DW, TM, Wilson (in progress)
  Improved bounding method, low-lying states for long distance tail
- Physical masses
  (most groups already)
- More, more precise disconnected and IB calculations needed
- Continuum limit and scale setting (per-mil) are crucial

HVP updates soon

- RBC/UKQCD
  - 3rd lattice spacing, a^{-1}=2.7 GeV, 6 fm box
  - Window continuum limit w/3 lattice spacings by Fall
  - 2+1 $\rightarrow$ 2+1+1 correction (expected to be small)
  - Full result w/3 lattice spacings by end of year
  - Error on total below 1%
- Aubin *et al.*
  - NNLO FV and taste breaking (discretization) corrections
  - Improved window and full connected results by end of year

a_μ-HLbL from data and models

T. Aoyama, N. Asmussen, M. Benayoun et al. *Physics Reports 887 2020) 1–166*



Good agreement between data/dispersive and lattice approaches

T. Aoyama, N. Asmussen, M. Benayoun et al. *Physics Reports 887 2020) 1–166*

$$a_\mu^{\pi^0\text{-pole}}(\text{disp}) = 63.0^{+2.7}_{\ 2.1} \times 10^{\ 11},$$

$$a_\mu^{\pi^0\text{-pole}}(\text{CA}) = 63.6(2.7) \times 10^{\ 11}$$

$$a_\mu^{\pi^0\text{-pole}}(\text{lattice}) = 62.3(2.3) \times 10^{\ 11}$$

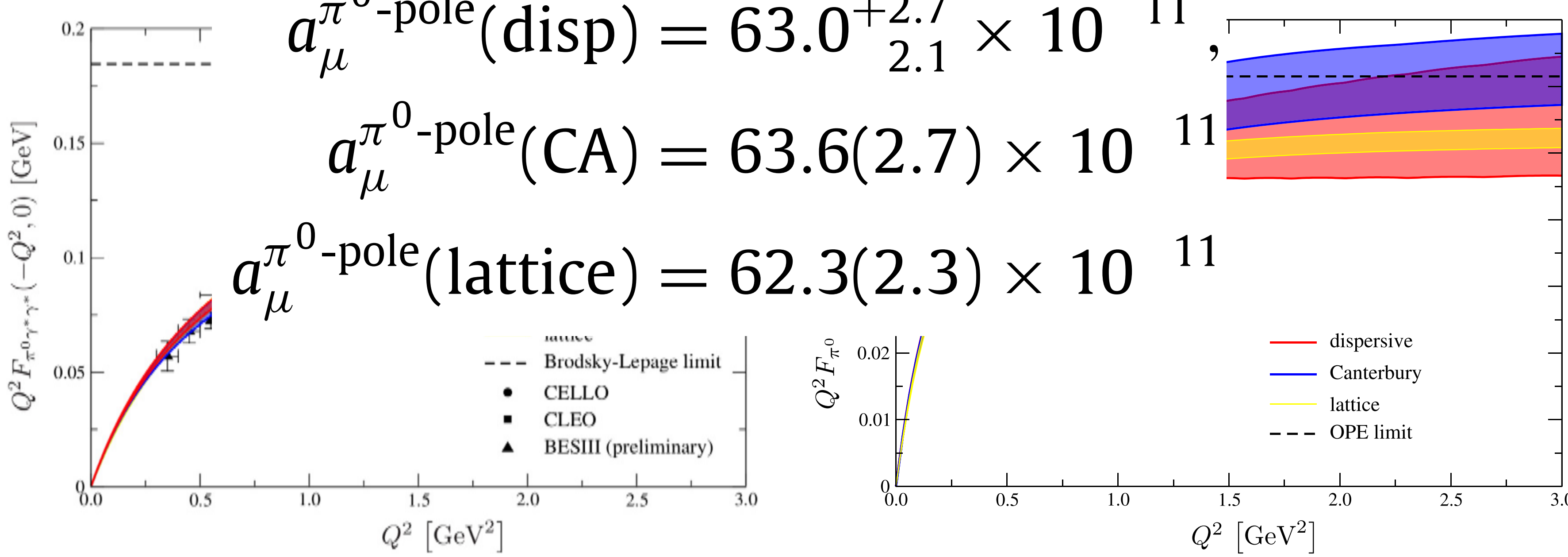


Fig. 60. Comparison of the π^0 TFF from dispersion theory [21,497] (red), CA [19] (blue), and lattice QCD [22] (yellow). We show both the singly- (left) and the doubly-virtual (right) form factors.

a_μ-HLbL from data and models

T. Aoyama, N. Asmussen, M. Benayoun et al. *Physics Reports 887 2020) 1–166*

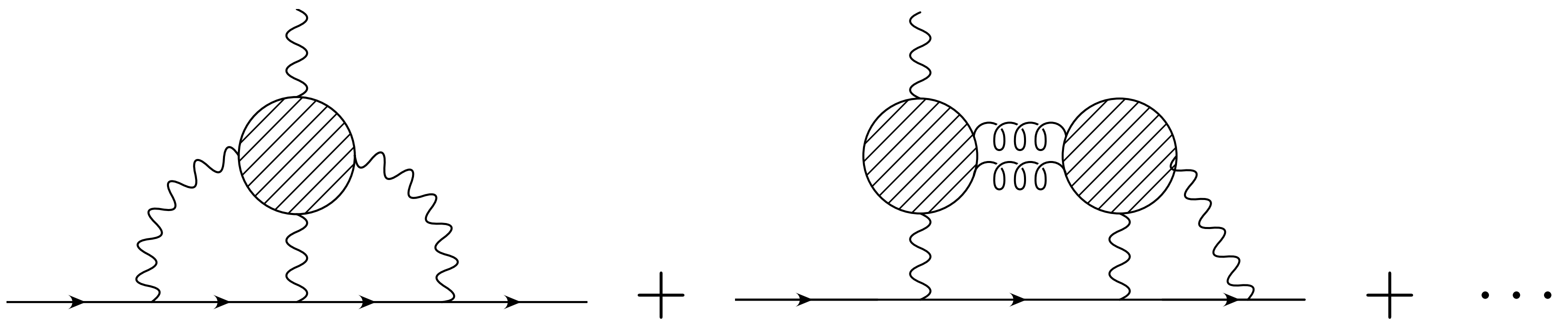
Table 15
Comparison of two frequently used compilations for HLbL in units of 10^{11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

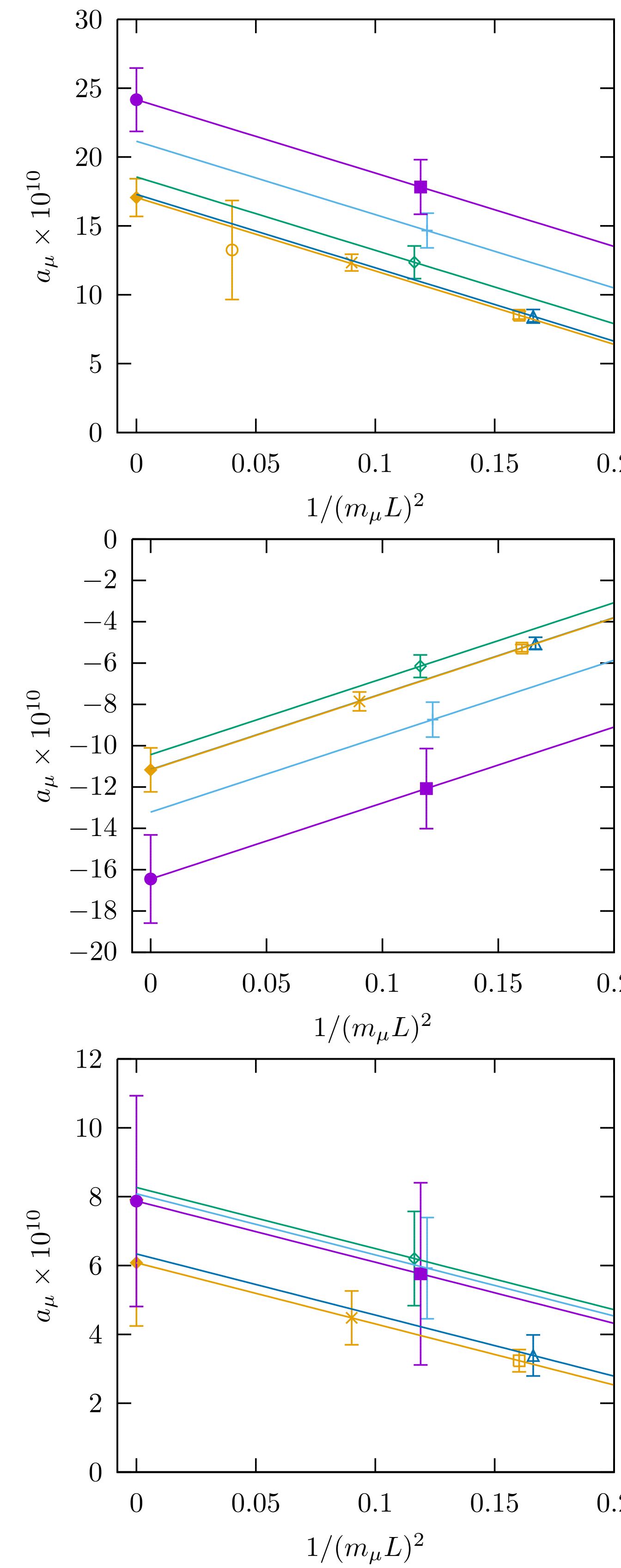
Contribution	PdRV(09) [475]	N/JN(09) [476,596]	J(17) [27]	Our estimate
π^0, η, η'-poles	[illegible]	[illegible]	[illegible]	93.8(4.0)
π, K-loops/boxes				16.4(2)
S-wave $\pi\pi$ rescatter				8(1)
subtotal				69.4(4.1)
scalars				1(3)
tensors				
axial vectors				6(6)
u, d, s-loops / short-c				15(10)
c-loop	2.3		2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

- Huge improvement in pole contributions
- All contributions computed or estimated
- Errors added in quad for dispersive results
- Errors added linearly for model-dependent results

Lattice HLbL

Blum, *et al.* (RBC) PRL 124 (2020)
Editor's Suggestion



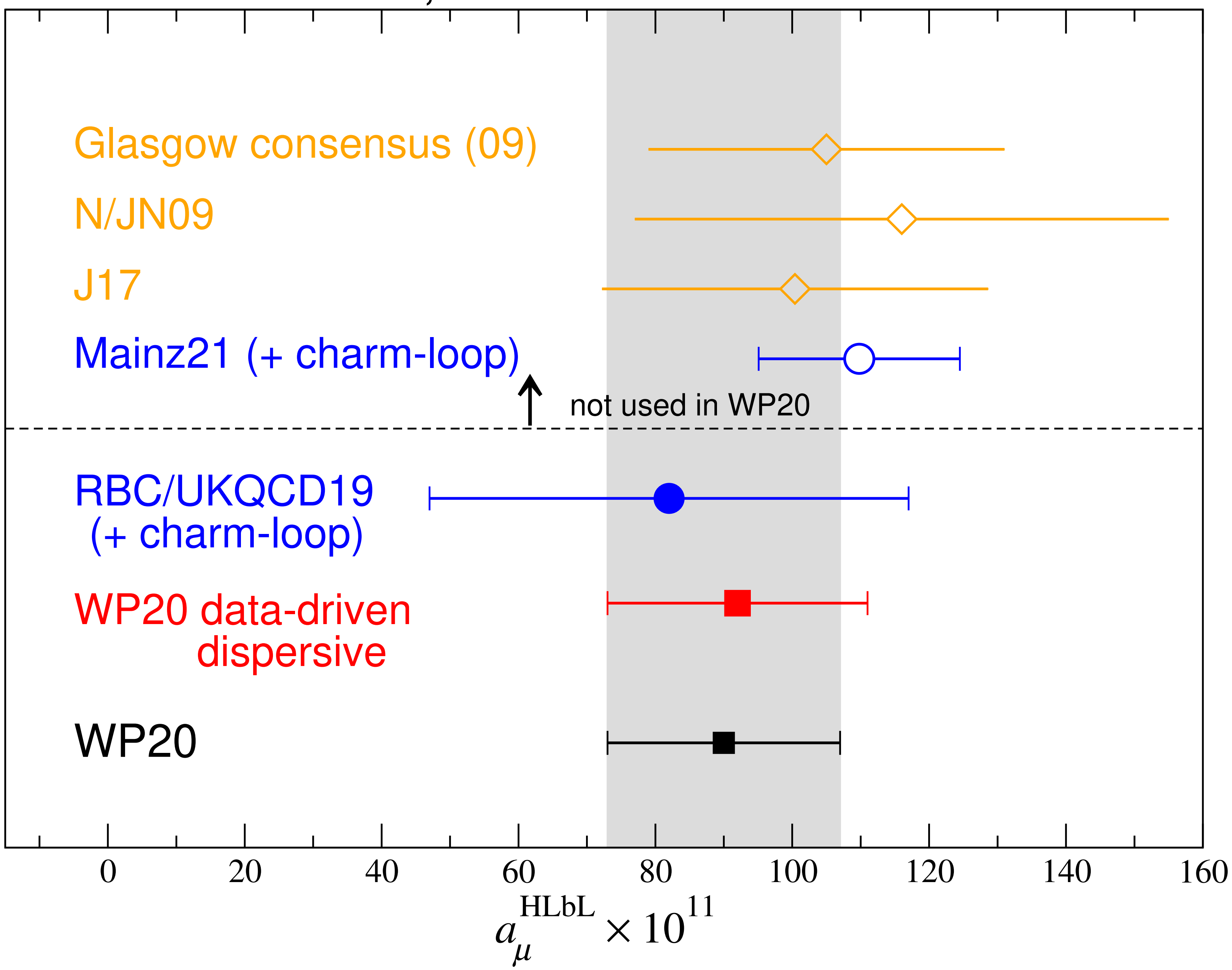


$$a_\mu^{\mathrm{HLBL}} = 7.87 \pm 3.06 \pm 1.77 \times 10^{-10}$$

- RBC: first lattice calculation with all errors controlled.
- 1 G core-hours on ALCF's Mira (BG/Q).
- 1st HLbL calculation was done on USQCD resources (Blum, *et al*., PRL 114 (2015))
- Crucial for Standard Model Comparison
- Included in Muon g-2 Theory Initiative average
    - 92(19)x 10^{-11} (phenomenology)
    - 90(17)x 10^{-11} (phenomenology+lattice)
- Unlikely to explain discrepancy with experiment

HLbL lattice-pheno comparision

Credit: A. El-Khadra, Lattice 2021



models of QCD +EFT, large N_c

Lattice QCD + QED

data + dispersive approach

a_μ-HLbL outlook

- More data for data-driven approach
- RBC: QED∞ calculation, 10-20% accuracy (5 years)
- New result from Mainz group (arXive:2104.02632)
- Other lattice groups starting (BMW, FHM, …)
- Combined 10% result (or better) within 5 years possible

Outlook

- Standard Model remarkable success
- Combining BNL and Fermilab experiments and using muon g-2 theory initiative SM value, discrepancy grew from 3.7 to 4.2 σ
- Using BMW20 lattice value for HVP, discrepancy is 1.5 σ
- Consistency of data-driven and lattice HVP calculations crucial for discovery of new physics
- HLbL in good shape- can't rescue SM
- Improved lattice calculations in progress

Acknowledgements

- Work partially supported by US Department of Energy
- Computations done under ALCC at Argonne Large Computing Facility and XSEDE (NSF) at Texas Advanced Computing Center

Further viewing/reading

- [Trevor Noah: Physics may be a lie (2:09 minute mark)]
- [The Muon g–2 Anomaly Explained]
- [Fermilab E989 announces first results]
- [An anomalous moment for the muon]
- [Muon g-2 Theory Initiative white paper]