Transition radiation from a Dirac particle wave packet traversing a mirror

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References (see for more)

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Outline

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Problem statement

Issue we are going to address

- Does a quantum charged particle with the wave function of a nontrivial form radiate as a charged fluid with the corresponding charged current?
- What are the manifestations of the wave function profile in radiation from charged particles?
- What are the quantum corrections to transition radiation due to quantum recoil?
- Whether the classical charged fluid picture is reproduced in the small quantum recoil limit, as one may suspect.

Outcomes

General results

1. The probability of radiation from a charged particle wave packet strongly depends on the measurement of the state of the escaping charged particle.
2. The inclusive probability of radiation from a charged particle wave packet cannot be described by the classical formula for radiation from the corresponding current even in the small quantum recoil limit. In the case of transition radiation, only the diagonal of the wave packet in the momentum space contributes to the inclusive radiation probability.
3. The classical radiation from the current constructed with the aid of the particle wave packet is reproduced in the case when:
   - The escaping charged particle is measured in the state that is obtained from the initial one by a free evolution;
   - The coherent radiation from the N-particle wave packet with identical one-particle wave function profiles is considered for $N \gg 1$.

Particular results

4. The inclusive probability to record a photon created in transition radiation from a one Dirac particle wave packet traversing an ideally conducting plate is derived in the leading order of perturbation theory. The anomalous magnetic moment of the Dirac particle is taken into account.
5. It is shown that the quantum corrections to transition radiation from an electrically charged particle give rise to production of photons with polarization vector orthogonal to the reaction plane. These corrections result from both the quantum recoil and the finite size of a wave packet.
6. As for transition radiation produced by a neutron falling normally onto the conducting plate, the probability to detect a photon with polarization vector lying in the reaction plane does not depend on the observation angle and the energy of the incident particle.
7. The transition radiation produced by the wave packet of one twisted Dirac particle is described.
8. The quantum formula for the inclusive probability to detect a photon radiated by the N-particle wave packet is derived.

Scheme

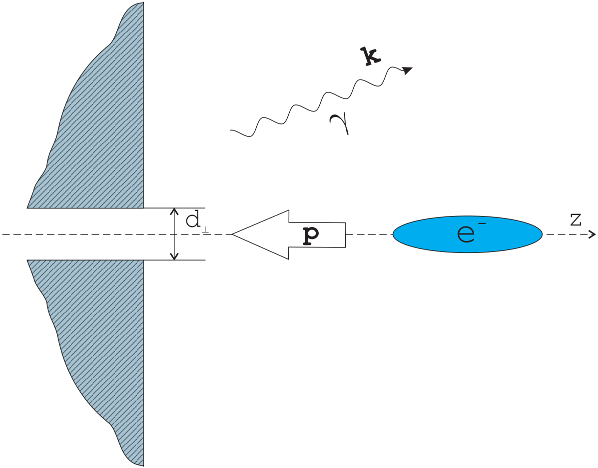

Figure : The schematic representation of the experimental installation for observation of transition radiation. The electron wave packet falls onto the conducting plate resulting in the production of photons. The conducting plate is perforated in order to suppress the effects of diffraction and bremsstrahlung. The case of normal falling is depicted.

Model

Lagrangian density of the system

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi - \frac{\mu_a}{2}F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi. \qquad (1)$$

μ_a is the anomalous magnetic moment.
e is the particle charge.
m is the particle mass.

Quantum electromagnetic field in the interaction picture

$$\hat{\mathbf{A}}(x) = \sum_\lambda \int_+ \frac{Vd\mathbf{k}}{(2\pi)^3}\frac{1}{\sqrt{2k_0V}}\big[\mathbf{a}_\lambda(k_3;z)e^{-ik_0x^0+i\mathbf{k}_\perp\mathbf{x}_\perp}\hat{c}_\lambda(\mathbf{k}) + \text{H.c.}\big]. \qquad (2)$$

$k_0 = |\mathbf{k}|$, V characterizes the normalization volume. $\lambda = \pm 1$.
The integration region is restricted by the inequality $k^3 \geqslant 0$.

Mode functions of the electromagnetic field

$$\mathbf{a}_\lambda(k_3;z) = \sum_{r=\pm1}\mathbf{f}_r^{(\lambda)}(\mathbf{k})e^{irk^3z}, \quad \mathbf{f}_r^{(\lambda)}(\mathbf{k}) = r\mathbf{f}^{(\lambda)}(\mathbf{k}) + (1-r)\mathbf{e}_3f^{3(\lambda)}(\mathbf{k}), \qquad \text{for } z>0;$$
$$\mathbf{a}_\lambda(k_3;z) = 0, \qquad \text{for } z<0; \qquad (3)$$

$\mathbf{e}_3$ is the unit vector along the z axis.

Polarization vector of a photon with helicity λ

$$\mathbf{f}^{(\lambda)}(\mathbf{k}) = (\cos\phi\cos\theta - i\lambda\sin\phi, \sin\phi\cos\theta + i\lambda\cos\phi, -\sin\theta)/\sqrt{2}, \tag{4}$$

$k_\perp = |k^1 + ik^2|$, $\phi = \arg(k^1 + ik^2)$, $\sin\theta := k_\perp/k_0 \equiv n_\perp$, $\cos\theta := k^3/k_0 \equiv n^3$:

$$\mathbf{k} = k_0\mathbf{n} = k_0(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta). \tag{5}$$

State of the Dirac particle

$$|\varphi\rangle := \sum_\alpha \sqrt{\frac{(2\pi)^3}{V}}\tilde{\varphi}_\alpha|\alpha\rangle, \qquad \sum_\alpha \frac{(2\pi)^3}{V}|\tilde{\varphi}_\alpha|^2 = \sum_s \int d\mathbf{p}|\tilde{\varphi}_s(\mathbf{p})|^2 = 1. \tag{6}$$

- $\tilde{\varphi}_s(\mathbf{p})$ is concentrated at $p^3 < 0$.

Density matrix

$$R_{s,\bar{s}}(\mathbf{p},\bar{\mathbf{p}}) := \varphi_s(\mathbf{p})\varphi^*_{\bar{s}}(\bar{\mathbf{p}}). \tag{7}$$

Spin density matrix

$$\rho_{s,\bar{s}}(\mathbf{p},\bar{\mathbf{p}}) := R_{s,\bar{s}}(\mathbf{p},\bar{\mathbf{p}})/c(\mathbf{p}), \qquad c(\mathbf{p}) := \sum_s R_{s,s}(\mathbf{p},\mathbf{p}), \qquad \int d\mathbf{p}c(\mathbf{p}) = 1. \tag{8}$$

Model

Spin vector

$$s^\mu=\Big(\frac{\boldsymbol{\zeta}\mathbf{p}}{m},\boldsymbol{\zeta}+\frac{\mathbf{p}(\boldsymbol{\zeta}\mathbf{p})}{m(p_0+m)}\Big),\qquad s^\mu p_\mu=0,\qquad s^2=-\boldsymbol{\zeta}^2. \tag{9}$$

For pure states $\boldsymbol{\zeta}^2=1$.

General form of a pure state

$$\varphi_s(\mathbf{p})=\frac{\varphi_0(\mathbf{p})}{\sqrt{2\cosh\kappa}}e^{s(\kappa-i\psi)/2},\qquad \kappa,\ \psi\in\mathbb{R}. \tag{10}$$

$\kappa=\kappa(\mathbf{p})$ and $\psi=\psi(\mathbf{p})$.

Diagonal of the density matrix and the spin density matrix

$$c(\mathbf{p})=|\varphi_0(\mathbf{p})|^2,\qquad \rho_{s\bar{s}}=\frac{1}{2\cosh\kappa}e^{\kappa(s+\bar{s})/2-i\psi(s-\bar{s})/2}. \tag{11}$$

Rest frame spin vector

$$\boldsymbol{\zeta}=\boldsymbol{\tau}\tanh\kappa+\boldsymbol{\zeta}_0/\cosh\kappa,\qquad \boldsymbol{\zeta}^2=1. \tag{12}$$

$$\boldsymbol{\zeta}_0^2=1,\qquad \boldsymbol{\zeta}_0\boldsymbol{\tau}=0,\qquad \frac{d}{d\psi}\boldsymbol{\zeta}_0=[\boldsymbol{\tau},\boldsymbol{\zeta}_0]. \tag{13}$$

Inclusive radiation probability

$$dP(\lambda,\mathbf{k};\varphi)=\frac{d\mathbf{k}}{32\pi^3k_0}\int\frac{d\mathbf{p}c(\mathbf{p})}{|p'_3p_3|}\sum_{r,r'}\frac{f^{(\lambda)}_{ri}(\mathbf{k})f^{*(\lambda)}_{r'j}(\mathbf{k})}{(p'_3-p_3+rk_3)(p'_3-p_3+r'k_3)}\times$$
$$\times\Big\{e^2\big(p^{(i}p'^{j)}+\eta^{ij}q^2/2+imq_\mu s_\nu\varepsilon^{\mu\nu ij}\big)+$$
$$+e\mu_a\big(2mq^2\eta^{ij}-2mq^iq^j+i[2m^2q_\mu s_\nu+$$
$$+(qs)p_\mu q_\nu+q^2q_\mu s_\nu/2]\varepsilon^{\mu\nu ij}\big)+$$
$$+\mu_a^2\big(q^2[2m^2\eta^{ij}-(p^i+p'^i)(p^j+p'^j)/2]-2m^2q^iq^j+$$
$$+2im[(qs)p_\mu q_\nu+q^2q_\mu s_\nu/2]\varepsilon^{\mu\nu ij}\big)\Big\}. \tag{14}$$

$$\varepsilon^{0123}=1,\qquad \eta^{ij}=-\delta_{ij},\qquad q_\mu:=p_\mu-p'_\mu,$$
$$q^2=2(m^2-pp')=k_3^2-(p_3-p'_3)^2<0,\qquad q^\mu=k^\mu_r+\delta^\mu_3(q^3-rk^3). \tag{15}$$

k^μ_r possesses the same components as k^μ but with $k^3\to rk^3$, $r=\pm1$.

Some properties of the general formula (14)

1. The expression in the curly brackets in (14) contracted with the polarization vectors can be written in an explicitly Lorentz-invariant form.
2. The classical transition radiation from a point charged particle without magnetic moment is described by the first term in the curly brackets standing at e^2 in (14) provided one replaces p'^i by p^i and employs the small quantum recoil approximation.
3. The terms depending on the effective particle spin contribute only to a circular polarization of radiated photons. They vanish when the polarization vector $\mathbf{f}^{(\lambda)}(\mathbf{k})$ is real and so corresponds to a certain linear polarization.
4. Only the momentum space diagonal of the density matrix contributes to the inclusive radiation probability. Hence, the inclusive radiation probability is not determined by the current constructed with the aid of the particle wave function even in the small quantum recoil limit.

Probability of transition radiation

Polarization vector

$$\mathbf{f} \sim [\mathbf{e}_3, \mathbf{k}] \tag{16}$$

- The polarization vector (16) is normal to the reaction plane spanned by the vectors $\mathbf{p}$ and $\mathbf{k}$ when the normal falling of the charged particle onto the conducting plate is considered.
- It is known that transition radiation from a classical point particle with charge e is linearly polarized in the plane spanned by the vectors $\mathbf{p}$ and $\mathbf{k}$.

Inclusive radiation probability of photons with polarization vector (16) for normal falling

$$dP(\lambda, \mathbf{k}; \varphi) = \frac{(e + 2m\mu_a)^2}{m^2} \int \frac{d\mathbf{p} c(\mathbf{p})}{n_3^2 + n_\perp^2 \gamma^2(\mathbf{p})} \frac{n_3^2 d\mathbf{k}}{16\pi^3 k_0}. \tag{17}$$

- The contribution (17) is entirely due to quantum effects (quantum recoil) and it is not overlapped by the classical contribution which is zero.
- The analogous effect exists in Vavilov-Cherenkov radiation [A. A. Sokolov, Iu. M. Loskutov, Zh. Eksp. Teor. Fiz. **32**, 630 (1957)].

Wave packet

$$\varphi_s(\mathbf{p}) = \delta_{s\sigma}\varphi_0(\mathbf{p}), \qquad \varphi_0(\mathbf{p}) = kp_\perp^{|l|}\exp\Big(-\frac{(p_3-p)^2}{4\sigma_3^2} - \frac{\mathbf{p}_\perp^2}{4\sigma_\perp^2} + il\psi\Big), \qquad \psi := \arg(p^1+ip^2), \qquad k^{-2} = (2\pi)^{3/2}\sigma_3\sigma_\perp^2(2\sigma_\perp^2)^{|l|}|l|!, \qquad l\in\mathbb{Z}. \tag{18}$$

$\sigma = \pm 1$ characterizes the spin projection of the state onto the z axis.

Properties of the twisted states (18)

- The state (18) is the eigenvector of the projection of the total angular momentum operator on the z axis with the eigenvalue $l+\sigma/2$. The case $l=0$ describes a cylindrically symmetric Gaussian wave packet.
- The momentum space diagonal of the density matrix

$$c(\mathbf{p}) = k^2 p_\perp^{2|l|}\exp\Big(-\frac{(p_3-p)^2}{2\sigma_3^2} - \frac{\mathbf{p}_\perp^2}{2\sigma_\perp^2}\Big). \tag{19}$$

- As is seen from the general formula (14) for the inclusive radiation probability, this probability does not depend on the phase of the wave function (18).

Transition radiation from twisted particles

Polarization vectors

$$\mathbf{f}^{(1)} = (\cos\theta, 0, -\sin\theta), \qquad \mathbf{f}^{(2)} = (0, 1, 0). \tag{20}$$

$\mathbf{n} = (\sin\theta, 0, \cos\theta)$.

Inclusive radiation probability for $\mu_a = 0$

$$\begin{aligned} dP(1,\mathbf{k};\varphi) \approx \frac{e^2p^2}{(m^2+n_\perp^2p^2)^2}\Big\{ & n_\perp^2(m^2+p^2)+ \\ & + \frac{(|l|+1)\sigma_\perp^2}{(m^2+n_\perp^2p^2)^2}\big[m^4(1-10n_\perp^2+10n_\perp^4) \\ & - 2m^2p^2n_\perp^2(5-7n_\perp^2+n_\perp^4)+p^4n_\perp^4\big]\Big\}\frac{d\mathbf{k}}{4\pi^3k_0^3}. \end{aligned} \tag{21}$$

$$dP(2,\mathbf{k};\varphi) \approx e^2\frac{p^2(|l|+1)\sigma_\perp^2}{(m^2+n_\perp^2p^2)^2}\frac{n_3^2 d\mathbf{k}}{4\pi^3k_0^3}. \tag{22}$$

Properties of the inclusive radiation probability (21), (22)

- The leading contribution to (21) is the standard expression for the probability of transition radiation from the mirror [V. L. Ginzburg, V. N. Tsytovich, *Transition Radiation and Transition Scattering* (Hilger, Bristol, 1990)]. The correction proportional to $\sigma_\perp^2$ is a non-paraxial quantum correction stemming from the wave packet profile.
- The non-paraxial quantum correction is the leading contribution to (22). The non-paraxial corrections proportional to $(|l|+1)\sigma_\perp^2/m^2$ are typical for the processes with twisted particles [see, e.g., D. Karlovets, Phys. Rev. A **98**, 012137 (2018)]. These corrections are enhanced for the states with large projection of the orbital angular momentum.
- Thus there are two quantum effects leading to the radiation of photons with polarization vector $\mathbf{f}^{(2)}$ lying out of the reaction plane. They are the quantum recoil (17) and the divergence of momenta in the wave packet (22).

Inclusive radiation probability for $e = 0$ (twisted neutron)

$$dP(1,\mathbf{k};\varphi) = \mu_a^2\Big(1 - \frac{(|l|+1)\sigma_\perp^2 n_3^2}{m^2 + n_\perp^2 p^2}\Big)\frac{d\mathbf{k}}{4\pi^3 k_0}, \qquad (23)$$

$$dP(2,\mathbf{k};\varphi) = \frac{\mu_a^2}{m^2+n_\perp^2 p^2}\Big\{m^2 - \frac{(|l|+1)\sigma_\perp^2}{(m^2+n_\perp^2 p^2)^2}\big[m^4(1-3n_\perp^2) - \\ - m^2 p^2 n_\perp^2(4-n_\perp^2) - p^4 n_\perp^4\big]\Big\}\frac{n_3^2 d\mathbf{k}}{4\pi^3 k_0}. \qquad (24)$$

Properties of the inclusive radiation probability (23), (24)

- The leading contribution to (23) does not depend on the form of the wave packet provided $\sigma_\perp$ is sufficiently small. For example, this contribution has the same form for the state (18) with two Gaussian humps with respect to the variable p_3. The radiation described by this contribution is isotropic.
- In the ultrarelativistic limit, the expressions for the leading contributions to (23) and (24) integrated over the azimuthal angle turn into classical formulas of [V. L. Ginzburg, V. N. Tsytovich, *Transition Radiation...*] where one should take $\theta_0 = \pi/2$. In other words, in this limit the radiation is the same as from a classical magnetic moment μ_a directed perpendicularly to the particle momentum.

Radiation from N-particle wave packet

Initial state of the system

$$c\varphi^1_{\text{in}\,\alpha_1}\cdots\varphi^N_{\text{in}\,\alpha_N}a^\dagger_{\alpha_1}\cdots a^\dagger_{\alpha_N}|0\rangle,$$
$$|c|^{-2}=\det(\bar\varphi^i_{\text{in}}\varphi^j_{\text{in}}). \tag{25}$$

Inclusive radiation probability in the leading order of perturbation theory

$$P(\gamma)=|c|^2\sum_{k,l=1}^N d_{lk},$$
$$d_{lk}:=\det\begin{bmatrix}\bar\varphi^1_{\text{in}}\varphi^1_{\text{in}} & \cdots & \bar\varphi^k_{\text{in}}A^\dagger\varphi^1_{\text{in}} & \cdots & \bar\varphi^N_{\text{in}}\varphi^1_{\text{in}}\\ \vdots & & \vdots & & \vdots\\ \bar\varphi^1_{\text{in}}A\varphi^l_{\text{in}} & \cdots & \bar\varphi^k_{\text{in}}A^\dagger A\varphi^l_{\text{in}} & \cdots & \bar\varphi^N_{\text{in}}A\varphi^l_{\text{in}}\\ \vdots & & \vdots & & \vdots\\ \bar\varphi^1_{\text{in}}\varphi^N_{\text{in}} & \cdots & \bar\varphi^k_{\text{in}}A^\dagger\varphi^N_{\text{in}} & \cdots & \bar\varphi^N_{\text{in}}\varphi^N_{\text{in}}\end{bmatrix}. \tag{26}$$

$A_{\alpha'\alpha}$ is the one-particle transition amplitude.

Approximation

$$(\bar{\varphi}^i_{\text{in}}\varphi^j_{\text{in}}) \approx \delta_{ij}, \qquad i,j=\overline{1,N}. \tag{27}$$

Inclusive radiation probability

$$P(\gamma) \approx \sum_{k=1}^{N} \bar{\varphi}^k_{\text{in}} A^\dagger A \varphi^k_{\text{in}} + {\sum_{k,l=1}^{N}}' \big[(\bar{\varphi}^k_{\text{in}} A^\dagger \varphi^k_{\text{in}})(\bar{\varphi}^l_{\text{in}} A \varphi^l_{\text{in}}) - (\bar{\varphi}^k_{\text{in}} A^\dagger \varphi^l_{\text{in}})(\bar{\varphi}^l_{\text{in}} A \varphi^k_{\text{in}})\big],$$

$${\sum_{k,l=1}^{N}}' (\bar{\varphi}^k_{\text{in}} A^\dagger \varphi^k_{\text{in}})(\bar{\varphi}^l_{\text{in}} A \varphi^l_{\text{in}}) = \Big|\sum_{l=1}^{N} \bar{\varphi}^l_{\text{in}} A \varphi^l_{\text{in}}\Big|^2 - \sum_{l=1}^{N} \big|\bar{\varphi}^l_{\text{in}} A \varphi^l_{\text{in}}\big|^2. \tag{28}$$

Interpretation of the terms in (28)

- The first term in (28) is the incoherent sum of the quantum contributions to radiation from the wave packets φ^k.
- The second term in (28) is the classical contribution to radiation produced by the sum of the classical currents corresponding to the wave packets φ^k, the incoherent contribution to the classical radiation should be excluded.
- The third term in (28) is the exchange term.

Neglecting the exchange term, we have

$$P(\gamma) \approx \sum_{k=1}^{N} \left(\bar{\varphi}^k_{\text{in}} A^\dagger A \varphi^k_{\text{in}} - \left|\bar{\varphi}^k_{\text{in}} A \varphi^k_{\text{in}}\right|^2\right) + \left|\sum_{k=1}^{N} \bar{\varphi}^k_{\text{in}} A \varphi^k_{\text{in}}\right|^2. \tag{29}$$

Some properties of (29)

- The classical contribution with the total current of N wave packets determines the inclusive probability of coherent radiation from the N-particle wave packet.
- This term dominates provided the classical radiation amplitudes corresponding to the wave packets φ^k add up coherently and N is large.
- For example, the transition radiation from a bunch train of twisted electrons with the wave functions obtained from each other by a parallel transport contains coherent harmonics. The radiation at these harmonics is approximately the coherent radiation from point charged magnetic moments with $\mu \approx l\mu_B$, where l is a large projection of the orbital angular momentum of a one twisted electron [I. P. Ivanov, D. V. Karlovets, Phys. Rev. Lett. **110**, 264801 (2013)].