EXPLANATION OF THE NATURE OF LEPTON DOUBLETS OF THE STANDARD MODEL WITHIN THE FRAMEWORK OF A NON-HERMITIAN THEORY WITH A FUNDAMENTAL MASS

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Questions, that are not customary to ask in the SM:

1) Why do leptons in the SM form doublets - neutrinos of the corresponding type and a charged lepton?

2) Why are charged leptons chirally symmetric in mass, but there are no chirally symmetric partners in mass among the neutrinos?

3) Why are the mass of left-handed neutrinos very small on the lepton mass scale, however for the right-handed neutrino masses are very large (if they exist at all)? Can this be explained this phenomenon without fine-tuning in the mass sector?

4) How is the mass difference between the chiral components related to the electric charge of the particle?
The answers to these questions (simple and natural) are given by a small non-Hermitian extension of the SM

Its essence is best expressed by the modified Dirac equation

\[ (i\partial_\mu \gamma_\mu - m_1 \mp \gamma_5 m_2)\Psi(\vec{r}, t) = 0 \]

the last term here is anti-Hermitian \[ \pm \gamma_5 m_2 \]
We will use linear superposition of these mass components for particles with a certain chirality

\[ m_R = \frac{1}{2}(m_1 + m_2) \]

\[ m_L = \frac{1}{2}(m_1 - m_2) \]

Two modern directions of development of non-Hermitian theory

- no restrictions on mass components
- with the additional condition

\[ m_2 = \frac{m_1^2}{2M} \]

\[ M - \text{fundamental mass} \]
Following V.G. Kadyshhevsky, we adhere to the second direction with a fundamental mass. There are a number of reasons for this. The main reason is the requirements of the principle of equivalence. Some of its violations in non-Hermitian theories are inevitable, and the magnitude of the fundamental mass $M$ limits the scale of these violations.
The role of the mass corresponding to the threshold for the production of a chiral symmetric particle is played by the following combination of chiral components

\[ m = 2\sqrt{m_R m_L} \]

It is usually associated with gravitational mass.

The role of the dynamic (inert) mass, which determines the mobility of particles in external fields, is played by another combination of chiral components

\[ m^* = \left( \frac{\sqrt{m_R} + \sqrt{m_L}}{2} \right)^2 \geq m \]
There are two different ways of forming the Hermitian and non-Hermitian components of the mass, and, accordingly, two ways of forming the observed mass of particles. Introducing the dimensionless "observable mass"

\[ \nu \equiv m/M \]

they can be described by the following expressions

\[ \begin{align*}
\nu_1 &= \sqrt{2} \mu_-; \\
\nu_2 &= \mu_-; \\
\nu_3 &= \sqrt{2} \mu_+; \\
\nu_4 &= \mu_+
\end{align*} \]

\[ \mu_{\mp} = \sqrt{\left[ 1 \mp (1 - \nu^2)^{1/2} \right]/2} \]
Here are the relevant graphs

Very important: in the Dirac limit $M \to \infty$
only the components of the first type (lower curves) and the particles composed of them survive.
V.G. Kadyshevsky called such particles, which are "compatible" with the SM and the ordinary Hermitian theory of Dirac, **ORDINARY.** Their main feature is the close chiral components at $\nu \ll 1 \quad m_L \approx m_R$
Particles of the second type, disappearing in the Dirac limit $M \to \infty$, V.G. Kadyshevsky called "EXOTIC". They are incompatible with SM and Hermitian theory. Their main feature is the huge difference in the masses of the chiral components at $\nu \ll 1$, $m_L \ll m_R$. 

![Graph showing the relationship between $\nu$, $\nu_1$, and $\nu_2$.]
The **main idea** of our report: neutrinos are "exotic Kadyshhevsky particles". It is this point of view that makes it easy and natural to answer the questions named at the beginning of the report.
This behavior of the masses of elementary particles is not typical for the SM. But it is in many ways analogous to the behavior of particles in Solid State Physics. For example, the phonon dispersion curve

\[ \frac{\omega}{\Omega} = \sqrt{1 \pm \sqrt{1 - \frac{4M_1M_2 \cdot \sin^2(Kd)}{(M_1+M_2)^2}}} \]

Compare with the formula for the mass constituents of non-Hermitian theory

\[ \mu = \sqrt{[1 + (1 - v^2)^{1/2}]/2} \]
Dispersion curve of phonons

\[ \left[ 2C \left( \frac{1}{M_1} + \frac{1}{M_2} \right) \right]^{1/2} \]

- Optical phonons
- Acoustic phonons

\[ M_1 > M_2 \]

\[ \left( \frac{2C}{M_2} \right)^{1/2} \]

\[ \left( \frac{2C}{M_1} \right)^{1/2} \]

\[ \frac{\pi}{a} \]

\[ k \]

compare with slide 8
Very important: in the Dirac limit \( M \rightarrow \infty \) exotic particles disappear, and the mass spectrum of ordinary particles grows indefinitely. In the limit of a continuous string (lattice constant \( d \rightarrow 0 \)) the acoustic branch grows indefinitely, while the optical branch disappears.

In the limit of a continuous string \( d = 0 \), it is impossible to understand the nature of optical phonons. In our opinion, in the same way in the Hermitian SM it is impossible to understand the true nature of exotic particles - neutrinos.
Let's summarize. Answers on questions

Why do SM leptons form doublets consisting of one neutrino and one charged particle? Because the mass of a lepton can be formed in two ways - from very close chiral components \( v \ll 1 \quad m_L \approx m_R \) and from very distant from each other \( m_R \gg m_L \).

The first possibility corresponds to an ordinary charged lepton, the second - to an exotic neutrino particle.
Why is the mass of left-handed neutrinos so small on the lepton mass scale, and why is the right-handed neutrino mass very large? Because the mass of the left-handed neutrino is related to the mass of the corresponding lepton as

\[ m_{\nu_L} = \frac{m_i^2}{8M} \]

It is small on the lepton scale because it is suppressed by the factor \( m_i/M \).

No fine tuning is required for this. The mass of right-handed neutrinos is of the order of the fundamental mass, which explains its very high value.
Why can only an ordinary particle in a doublet have an electric charge? Because when interacting with photons (massless vector bosons), a lepton should easily change its chirality. Ordinary particles can do this, exotic ones cannot. Therefore, exotic particles can only interact with much more massive vector bosons.