# Some Recent Results on Physics Beyond the Standard Model

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# Outline

- Some constraints on sterile neutrinos and related charged lepton flavor violation
- $n - \bar{n}$ oscillations: general formalism and current experimental limits
- Models in which $n - \bar{n}$ oscillations are dominant manifestation of baryon number violation
- Conclusions

# Some Constraints on Sterile Neutrinos and Related Charged Lepton Flavor Violation

Neutrino masses and lepton mixing have been established by $\nu$ oscillation experiments and are of great importance as physics beyond the original Standard Model (SM).

The active neutrino weak eigenstates $\nu_\ell$ are expressed in terms of mass eigenstates as $|\nu_\ell\rangle = \sum_{j=1}^{3+n_s} U_{\ell j} |\nu_j\rangle$, where $\ell = e, \ \mu, \ \tau$, and $n_s$ refers to possible additional mass eigenstates occurring as small admixtures in $|\nu_\ell\rangle$, and $U$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix. Denote the SM extended to include $\nu$ masses as the $\nu$SM, allow for $n_s$ sterile $\nu$ eigenstates.

Neutrino masses lead to a number of effects besides neutrino oscillations, e.g., decays with charged lepton flavor violation (CLFV) such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$, etc. However, in the minimal $\nu$SM, the rates are much too small to be observable:

$$BR(\mu \to e\gamma) = \frac{3\alpha_{em}}{32\pi} \Big| \sum_{j=1}^{3+n_s} U^*_{ej} U_{\mu j} \Big(\frac{m^2_{\nu_j}}{m^2_W}\Big) \Big|^2$$

(Marciano-Sanda 1977, Petcov 1977, B. W. Lee, Pakvasa, Sugawara, RS 1977). In the minimal $\nu$SM, $BR(\mu \to e\gamma) \sim 10^{-52}$, far below observable levels, and similarly with $BR(\mu \to ee\bar{e})$ and CLFV $\tau$ decays such as $\tau \to \ell\gamma$.

The suppression arises due to a leptonic GIM mechanism operating at the one-loop level; necessary and sufficient conditions are that leptons of the same chirality and charge have the same weak isospin $I$ and $I_3$ (B. W. Lee and RS, PRD16, 1444 (1977)).

With a mostly sterile neutrino, denoted $\nu_4$, having, e.g., $m_{\nu_4} \simeq 350$ MeV, using upper bounds $|U_{e4}|^2 \lesssim 10^{-9}$ and $|U_{\mu 4}|^2 \lesssim 10^{-12}$ from NA62, $BR(\mu \to e\gamma) \lesssim 10^{-32}$, still far below detectable BR; similar comments for CLFV $\tau$ decays.

A massive Dirac neutrino has a magnetic moment (Fujikawa and RS, 1980)

$$\mu_{\nu_i} = \frac{3eG_F m_{\nu_i}}{8\pi^2\sqrt{2}} = (3.2 \times 10^{-19})\Big(\frac{m_{\nu_i}}{1 \text{ eV}}\Big)\,\mu_B$$

where $\mu_B = e/(2m_e)$. Current limits from astrophysics and laboratory exps. $\mu_{\nu_i} \lesssim 10^{-11} - 10^{-12}$ (recent reviews by Giunti and Studenikin, 2015; Balantekin and Kayser, 2018)

Recent progress on obtaining a more sensitive search for $\mu_\nu$ using coherent elastic neutrino-atom scattering using $\bar{\nu}_e$ from tritium decay and a liquid He target (Cadeddu, Dordei, Giunti, Kouzakov, Picciau, Studenikin, PRD 100, 073014 (2019); talks here.

Tests for massive neutrino emission, via lepton mixing, in nuclear and particle decays (RS, PLB 96, 159 (1980); re neutrino mass limits, RS, in PDG, Rev. Mod. Phys. 52, S63 (1980)). Among these tests is the search for heavy neutrinos emitted in two-body leptonic decays of charged pseudoscalar mesons $M^+ \to \ell^+ \nu_\ell$, where $\ell = e, \ \mu$ and $M^+ = \pi^+, \ K^+, \ D^+, \ D_s^+, \ B^+$; one searches for a peak in $dN/dE_\ell$ at $E_\ell = (m_M^2 + m_\ell^2 - m_{\nu_4}^2)/(2m_M)$ due to the $\ell$ recoiling opposite a massive $\nu_4$.

The sensitivity of this test stems mainly from the (a) monochromatic signal, (b) the removal of helicity suppression for decay to a heavy neutrino in the $M^+ \to e^+ \nu_e$ decays, which involves a relative enhancement factor up to $\sim 10^4$ in $\pi^+_{e2}$ decay and $\sim 10^5$ in $K^+_{e2}$ decay with little phase space suppression for moderate $\nu_4$ masses $m_{\nu_4} < m_M - m_\ell$. (We consider $m_{\nu_4} >> $ eV scale discussed w.r.t. LSND/miniBooNE and reactor data.)

This "peak search" test was applied retroactively to data in 1980 to obtain first upper bounds in the PLB paper. It has been applied in a series of dedicated experiments on

- $\pi^+_{\ell 2}$ decay at IUCF, TRIUMF and SIN/PSI
- $K^+_{\ell 2}$ decay at KEK, Serpukhov, BNL, and CERN
- $B^\pm_{\ell 2}$ decay at Belle

to set very stringent upper bounds on $|U_{ej}|^2$ and $|U_{\mu j}|^2$ as function of $m_{\nu_j}$ for a heavy neutrino $\nu_j$.

Recent results from:

PIENU experiment at TRIUMF (Bryman et al.) in PRD 97, 072012 (2018) ($\pi^+_{e2}$); PLB 798, 134980 (2019) ($\pi^+_{\mu 2}$)

NA62 experiment at CERN (including Lazzeroni, Goudzovski, Duk, Ceccucci, Bryman, Kudenko...) in PLB 772, 712 (2017); PLB 778, 137 (2018); PLB 807, 135599 (2020); PLB 816, 136259 (2021) ($K^+_{\mu 2}$ and $K^+_{e2}$).

Massive neutrino emission would also change

$$R^{(\pi)}_{e/\mu} = \frac{BR(\pi^+ \to e^+ \nu_e)}{BR(\pi^+ \to \mu^+ \nu_\mu)}$$

$R^{(K)}_{e/\mu}$, $R^{(D_s)}_{e/\tau}$, and $R^{(D_s)}_{\mu/\tau}$ from their SM values, so the agreement of the measured ratios $R^{(\pi)}_{e/\mu}$, $R^{(K)}_{e/\mu}$, and $R^{(D_s)}_{\mu/\tau}$ and the consistency of the upper limit on $R^{(D_s)}_{e/\tau}$ with SM predictions provide further constraints.

In the SM

$$\Gamma(M^+ \to \ell^+ \nu_\ell)_{SM} = \frac{G_F^2 |V_{ab}|^2 f_M^2 m_M m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_M^2}\right)^2$$

where $V_{ab} = V_{ud}$ for $M^+ = \pi^+$, $V_{ab} = V_{us}$ for $M^+ = K^+$, etc.

For the decay to a $\nu_4$ of non-negligible mass,

$$\Gamma(M^+ \to \ell^+ \nu_4) = \frac{G_F^2 |V_{ab}|^2 |U_{\ell 4}|^2 f_M^2 m_M^3}{8\pi} \rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) \ ,$$

where

$$\rho(x,y) = [x + y - (x-y)^2]\,[1 + (x-y)^2 - 2(x+y)]^{1/2} \ ,$$

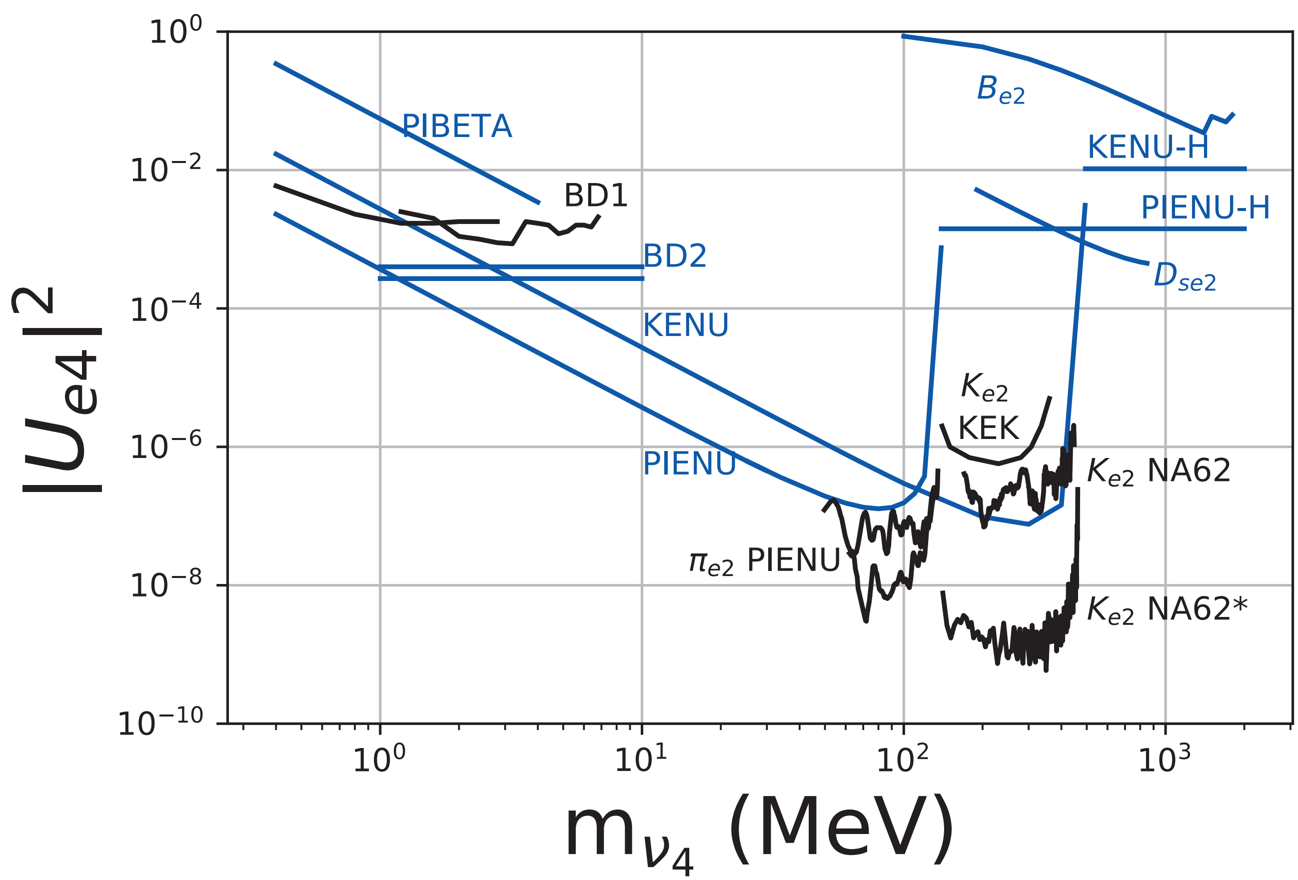
and

$$\delta_\ell^{(M)} = \frac{m_\ell^2}{m_M^2} \ , \quad \delta_{\nu_4}^{(M)} = \frac{m_{\nu_4}^2}{m_M^2} \ ,$$

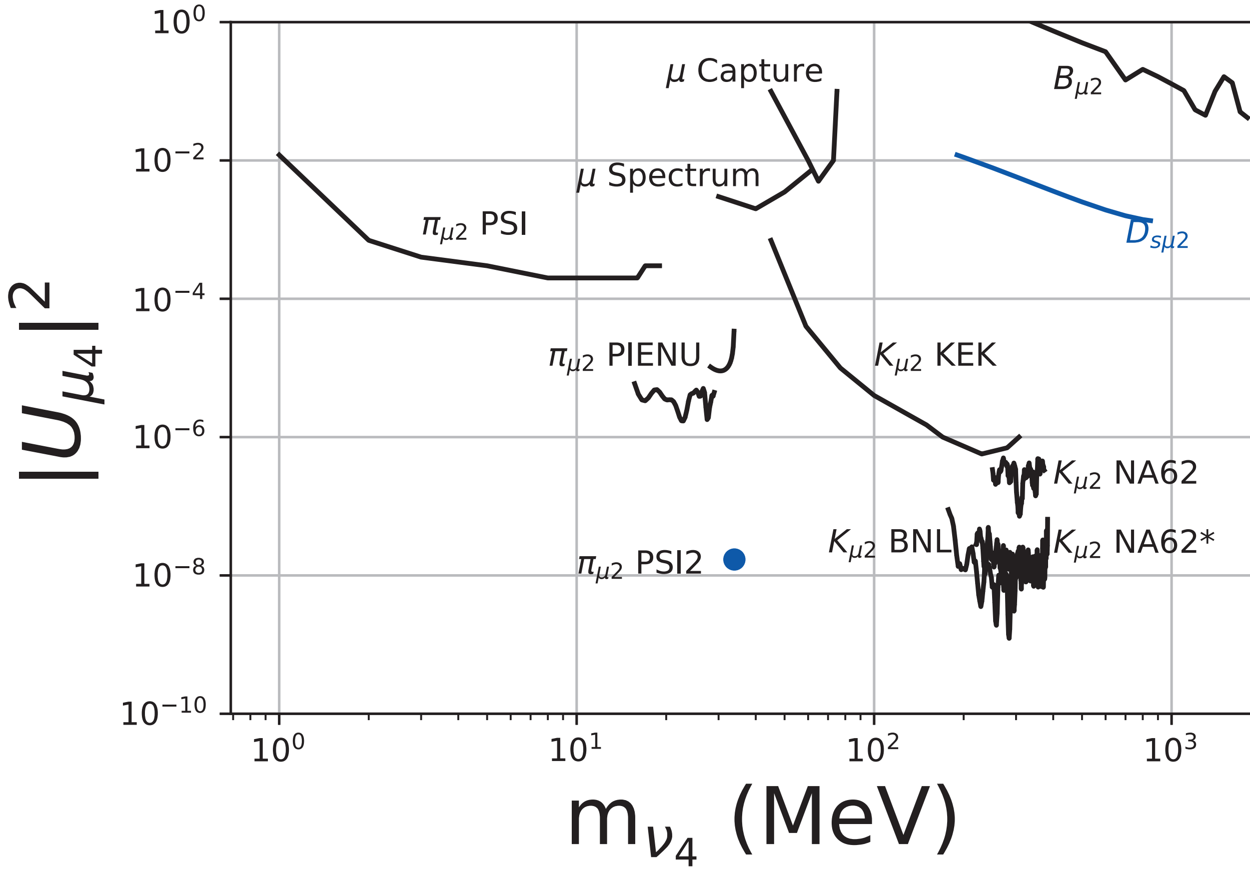
For a massless neutrino, $\rho(x,0) = x(1-x)^2$ with $x = \delta_\ell^{(\pi)}$. Thus,

$$\frac{\Gamma(M^+ \to \ell^+ \nu_4)}{\Gamma(M^+ \to \ell^+ \nu_\ell)_{SM}} = |U_{\ell 4}|^2 \bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$$

where $\bar{\rho}(x,y) = \rho(x,y)/\rho(x,0) = \rho(x,y)/[x(1-x)^2]$. Recent analysis: Bryman and RS, Phys. Rev. D 100, 053006, 073011 (2019):



$|U_{e4}|^2$
$10^{0}$
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
$m_{\nu_4}$ (MeV)
PIBETA
BD1
BD2
KENU
PIENU
$\pi_{e2}$ PIENU
$B_{e2}$
KENU-H
PIENU-H
$D_{se2}$
$K_{e2}$
KEK
$K_{e2}$ NA62
$K_{e2}$ NA62*



$10^0$
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^0$
$10^1$
$10^2$
$10^3$
$|U_{\mu_4}|^2$
$m_{\nu_4}$ (MeV)
$\mu$ Capture
$B_{\mu 2}$
$\mu$ Spectrum
$\pi_{\mu 2}$ PSI
$D_{s\mu 2}$
$\pi_{\mu 2}$ PIENU
$K_{\mu 2}$ KEK
$K_{\mu 2}$ NA62
$K_{\mu 2}$ BNL
$K_{\mu 2}$ NA62*
$\pi_{\mu 2}$ PSI2

These are direct kinematic limits. Other bounds can be obtained from searches for production and decay of heavy neutrinos in fixed-target exps. and at colliders (dependent on $\nu$ decay channels).

Given the Pontecorvo-Maki-Nakagawa-Sakata lepton mixing, it follows that massive neutrinos can also be emitted, via lepton mixing, in nuclear beta decay; with one $\nu_4$,

$$\frac{dN}{dE} = C\Big[(1-|U_{e4}|^2)pE(E_0-E)^2+|U_{e4}|^2pE(E_0-E)\Big[(E_0-E)^2-m^2_{\nu_4}\Big]^{1/2}\theta(E_0-E-m_{\nu_4})\Big],$$

where $E = e$ energy; $C = G_F^2|V_{ud}|^2F|\mathcal{M}|^2/(2\pi^3)$ ($F =$ Fermi function, $\mathcal{M} =$ nuclear matrix element).

Signature: kink in the Kurie plot at the endpoint of the massive $\nu_4$ distribution; suggestion to search for such kinks, and initial bounds obtained from retroactive data analysis (RS,PLB,1980).

This kink search has been performed by many exps.with $^3$H (Mainz, Troitsk), $^{20}$F, $^{35}$S, $^{63}$Ni, $^{64}$Cu, $^{187}$Re, etc., getting upper limits $|U_{e4}|^2$ ranging from $\sim 10^{-2}$ from $\sim 10^{-3}$ for $m_{\nu_4}$ from $\sim 0.3$ keV eV to $\sim 2$ MeV.

The kink search test can be performed in future in KATRIN (looking away from endpoint) (e.g., Drewes, Lasserre, Merle, Mertens, et al., arXiv:1602.04816).

Another method is to measure both electron and recoil ion in nuclear beta decay (e.g., Cook et al., 1992; Finocchiaro, RS, 1992); used with $^{38m}$K $\beta^+$ decay by Trinczek et al. (2003), obtaining $|U_{e4}|^2 \lesssim 10^{-2}$ for $0.5 < m_{\nu_4} < 3$ MeV.

Analysis of recoil ion spectrum in $e$-capture in $^7$Be (Likhovid and Pantuev, 2021) yielding $|U_{e4}|^2 \lesssim \mathrm{few} \times 10^{-4}$ for 300-750 keV.

R+D work for an exp. to measure ion recoil after $e$-capture in $^{131}$Cs (Martoff, Meyers, Smith et al.)

Set of superallowed Fermi nuclear beta decays have very good mutual agreement on $\mathcal{F}t$ rate values, with $Q$ values ranging over several MeV. Emission of a massive $\nu_4$ would change rates and remove this mutual agreement. This agreement can thus be used to set an upper limit $|U_{e4}|^2 \lesssim \mathrm{few} \times 10^{-4}$ for this mass range (Bryman and RS, 2019).

Further bounds can be obtained from cosmology, e.g., with assumptions about early universe history, agreement of primordial nucleosynthesis predictions for H, He abundances suggest lifetime $\tau_{\nu_4} \lesssim O(1)$ sec, yielding lower bounds on $|U_{\ell 4}|^2$, depending on $m_{\nu_4}$. Thus, cosmology disfavors some regions of $m_{\nu_4}$ and $|U_{\ell 4}|^2$.

Considerable recent interest in heavy neutrinos by theorists as well as experimentalists, e.g. Asaka and Shaposhnikov 2005; Gorbunov and Shaposhnikov 2007; Kusenko, Pascoli, and Semikoz 2008; Kusenko 2009; Boyarsky et al. 2009; Helo, Kovalenko, Schmidt 2011; Abada et al. 2014; SHIP Proposal at CERN (Alekhin et al., 2016) de Gouvêa and Kobach 2016; Fernandez-Martinez et al. 2016; Drewes and Garbrecht 2017; Batell et al. 2018; Bondarenko, Boyarsky, Gorbunov, Ruchayskiy 2018; Coloma et al. 2020; Bondarenko et al. 2021; and many other theoretical papers.

Current upper bounds on some CLFV $\mu$ and $\tau$ decays:

- $BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$ (MEG at PSI)
- $BR(\mu \to ee\bar{e}) < 1.0 \times 10^{-12}$ (SINDRUM II at SIN/PSI)
- $BR(\tau \to e\gamma) < 3.3 \times 10^{-8}$ (limits on CLFV $\tau$ decays from Belle and BABAR)
- $BR(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$
- $BR(\tau \to ee\bar{e}) < 2.7 \times 10^{-8}$
- $BR(\tau \to e\mu\bar{\mu}) < 2.7 \times 10^{-8}$
- $BR(\tau \to \mu e\bar{e}) < 1.8 \times 10^{-8}$
- $BR(\tau \to \mu\mu\bar{\mu}) < 2.1 \times 10^{-8}$

BSM physics that could yield CLFV at observable levels includes supersymmetry, $Z'$ vector bosons with flavor-nondiagonal couplings, etc.

So searches for CLFV processes such as decays and $\mu - e$ conversion in the field of a nucleus are interesting as probes of BSM physics.

Some current/future exps. on CLFV processes

- MEG II (PSI), planning to achieve sensitivity to $BR(\mu^+ \to e^+\gamma) \sim 10^{-14}$ (EPJC 78, 380 (2018); arXiv:1912.08656)
- Mu3e (PSI), planning to achieve sensitivity to $BR(\mu^+ \to e^+e^+e^-) \sim 10^{-14}$ (arXiv:2009.11690),
- Belle II (KEK) improving upper limits on CLFV $\tau$ decays (arXiv:1011.0352)
- Mu2e (Fermilab) searching for $\mu^- + (Z, A) \to e^- + (Z, A)$ (talks by Miller and Di Falco)

An analysis of BABAR data from search for $\tau \to \ell\gamma$ yields the first upper bounds on two other CLFV decays: $BR(\tau \to e\gamma\gamma) < 2.4 \times 10^{-4}$ and $BR(\tau \to \mu\gamma\gamma) < 5.8 \times 10^{-4}$ (Bryman, Ito, RS, arXiv:2106.02451).

There is also interest in processes violating total lepton number, $L$. Upper limits on neutrinoless double beta decay $(Z, A) \to (Z+2, A) + 2e^-$ are very sensitive probes of $|\Delta L_e| = 2$ interactions, and $K$ decays yield complementary limits for modes involving $\mu$, e.g., $K^+ \to \pi^- \mu^+ \mu^+$ (see fig.) and $K^+ \to \pi^- \mu^+ e^+$ .

From retroactive data analysis, first upper limit on $BR(K^+ \to \pi^- \mu^+ \mu^+)$ set in 1992 (Littenberg, RS); dedicated search by BNL E865 (2000, 2005) obtained

- $BR(K^+ \to \pi^- \mu^+ \mu^+) < 3.0 \times 10^{-9}$
- $BR(K^+ \to \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}$

By using the upper limit on $\mu^- + (Z, A) \to e^+ + (Z-2, A)$ with estimate for the nuclear matrix element, one can get an indrect upper limit $BR(K^+ \to \pi^- \mu^+ e^+) \lesssim O(10^{-11})$ (Littenberg, RS, 2000).

Recent progress by CERN NA62 (2019, 2021):

- $BR(K^+ \to \pi^- \mu^+ \mu^+) < 4.2 \times 10^{-11}$
- $BR(K^+ \to \pi^- e^+ \mu^+) < 4.2 \times 10^{-11}$

# $n - \bar{n}$ Oscillations: General Formalism

Motivations:

Producing the observed baryon asymmetry in the universe requires baryon-number-violating (BNV) interactions (as well as CP violation and deviation from thermal equilibrium) (Sakharov, 1967).

Suggestion of $n - \bar{n}$ transitions as a mechanism involved in generating baryon asymmetry in the universe (Kuzmin, 1970).

Standard Model (SM) conserves $B$ perturbatively. SU(2)$_L$ instantons produce nonperturbative violation of $B$ and $L$, while conserving $B - L$ ('t Hooft, 1976), but this is negligible (exponentially small) at temperatures $T$ low compared with the electroweak scale (finite-$T$ baryogenesis: Kuzmin, Rubakov, Shaposhnikov, 1985).

Grand unified theories (GUT's) also predict violation of $B$ and $L$. Besides proton decay ($\Delta B = -1$ BNV), $n - \bar{n}$ oscillations ($|\Delta B| = 2$ BNV) can occur.

There are good motivations for new experimental searches for $n - \bar{n}$ oscillations and associated $\Delta B = -2$ dinucleon decays as well as proton decay.

# General Formalism for $n-\bar{n}$ Oscillations

$n-\bar{n}$ Oscillations in Field-Free Vacuum:

CPT: $\langle n|H_{eff}|n\rangle = \langle\bar{n}|H_{eff}|\bar{n}\rangle = m_n - i\lambda_n/2$, where $H_{eff}$ denotes relevant Hamiltonian and $\lambda_n^{-1} = \tau_n = 0.88 \times 10^3$ sec. $H_{eff}$ may also mediate $n \leftrightarrow \bar{n}$ transitions: $\langle\bar{n}|H_{eff}|n\rangle \equiv \delta m$. Consider the matrix in $(n, \bar{n})$ basis:

$$\mathcal{M} = \begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix}$$

Diagonalizing $\mathcal{M}$ yields mass eigenstates

$$|n_\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm |\bar{n}\rangle)$$

with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda_n/2$.

So if start with pure $|n\rangle$ state at $t=0$, then there is a finite probability $P$ for it to be an $|\bar{n}\rangle$ at $t \neq 0$:

$$P(n(t)=\bar{n}) = |\langle\bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda_n t}$$

where $\tau_{n\bar{n}} = 1/|\delta m|$.

Most sensitive reactor $n-\bar{n}$ exp. done with ILL High Flux Reactor (HFR) at Grenoble (Baldo-Ceolin, Fidecaro,.., 1985-1994, obtaining limit $\tau_{n\bar{n}} \geq 0.86 \times 10^8$ sec.

A future $n-\bar{n}$ search experiment could significantly improve this limit or observe a signal.

Plan for $n-\bar{n}$ search exp. at European Spallation Source, ESS, under construction in Lund, Sweden; white paper: J. Phys. G 48, 070501 (2021) [arXiv:2006.04907].

R+D for improved sensitivity with ultra-cold neutrons (UCNs), e.g., Fomin, Serebrov et al., Kurchatov Inst. PNPI, Gatchina.

$n-\bar{n}$ oscillations lead to matter instability ( $m.i.$) via annihilation of $\bar{n}$ with neighboring $n$ or $p$ in a nucleus (producing mainly multipion final state); resultant lifetime $\tau_{m.i.}$.

This matter instability can be searched for with large proton decay detectors; current bound from Super-K: $\tau_{m.i.} > 3.6 \times 10^{32}$ yrs, giving $\tau_{n\bar{n}} > 4.7 \times 10^8$ sec (90 % CL) (Abe et al., PRD 103, 012008 (2021)).

Question: are there theoretical models that provide motivation for a new $n - \bar{n}$ search experiment?

Answer: Yes; we proceed to discuss such models.

references: S. Girmohanta and RS, PLB 803, 135296 (2020) [arXiv:1910.08356]; PRD 101, 015017 (2020) [arXiv:1911.05102]; PRD 101, 095012 (2020) [arXiv:1911.05102]; S. Nussinov and RS, PRD 102, 035003 (2020) [arXiv:2005.12493]; S. Girmohanta, R. Mohapatra, RS, PRD 103, 015021 (2021) [arXiv:2011.01237].

# $n - \bar{n}$ Oscillations in an Extra-Dimensional Model

We discuss models in which proton decay can easily be suppressed well below experimental limits while $n - \bar{n}$ oscillations can occur at a level comparable to existing limits (Girmohanta and RS, 2020, 2021; Nussinov and RS, PRL 88, 171601 (2002)).

Consider models in a $d = 4 + n$ dimensional spacetime, with $n$ extra spatial dimensions. Denote usual spacetime coords. as $x_\nu$, $\nu = 0, 1, 2, 3$ and consider $n$ extra compact coordinates, $y_\lambda$ with $0 \le y_\lambda \le L$, i.e., size of extra dimension(s) is $L$.

Each SM fermion $f$ has the form $\Psi_f(x,y) = \psi_f(x)\chi_f(y)$ with strong localization at a point $y_f$ in the extra dimensions, with a Gaussian profile of half-width $\sigma \equiv \mu^{-1} << L$:

$$\chi_f(y) = Ae^{-\mu^2\|y-y_f\|^2} = Ae^{-\|\eta-\eta_f\|^2}$$

where $\|y_f\| = (\sum_{\lambda=1}^n y_{f,\lambda}^2)^{1/2}$, $A$ is a normalization constant, and we define a convenient dimensionless variable $\eta_f = \mu y_f = y_f/\sigma$.

Such models are of interest partly because they can provide a mechanism for obtaining a generational hierarchy in fermion masses and quark mixing by placement of fermion wave function centers in extra dimensions (Arkani-Hamed + Schmaltz; Mirabelli+Schmaltz, 2000).

We use a low-energy effective field theory (EFT) approach with an ultraviolet cutoff $M_*$, where $M_* > \mu$ for self-consistency. It suffices here to consider the lowest Kaluza-Klein (KK) modes; effects of higher KK modes are discussed in Girmohanta, Mohapatra, RS, PRD 103, 015021 (2021). Gauge and Higgs fields have flat profiles in the extra dimensions.

Starting from the Lagrangian in the $d$-dimensional spacetime, one obtains the resultant low-energy EFT in 4D by integrating over the extra $n$ dimensions.

For canonical normalization of the 4D fermion kinetic term, $A = (2/\pi)^{n/4}\mu^{n/2}$.

Define $\Lambda_L = 1/L$; take $\Lambda_L \simeq 10^2$ TeV, i.e., $L \simeq 2 \times 10^{-19}$ cm, and $\sigma/L = 1/30$; this gives adequate separation of fermions while fitting in the compactification interval $[0, L]$, consistent with precision electroweak data, collider bounds, flavor-changing neutral current constraints.

A Yukawa interaction in the $d$-dimensional space with coefficients of order unity and moderate separation of localized fermion wavefunction centers yields a strong hierarchy in the low-energy 4D Yukawa interaction,

$$\int d^n y \, \bar{\chi}(y_{f_L})\chi(y_{f_R}) \sim \int d^n \eta \, e^{-\|\eta-\eta_{f_L}\|^2} e^{-\|\eta-\eta_{f_R}\|^2} \sim e^{-(1/2)\|\eta_{f_L}-\eta_{f_R}\|^2}$$

Resultant fermion masses $m_f$:

$$m_f \simeq h^{(f)} \frac{v}{\sqrt{2}} \exp\Big[ -\frac{1}{2}\|\eta_{f_L} - \eta_{f_R}\|^2 \Big] ,$$

where $v/\sqrt{2}$ is SM Higgs VEV. With $h^{(f)} \simeq 1$, produce fermion generational hierarchy via different separation distances $\|\eta_{f_L} - \eta_{f_R}\|$ for different generations.

Leading nucleon decay operators are of the form $qqq\ell$. Hence, one can suppress nucleon decay well below experimental limits by arranging that the wavefunction centers of the $u$ and $d$ quarks are separated far from those of the leptons.

Key point: this does not suppress $n - \bar{n}$ oscillations because the $n - \bar{n}$ transition operators do not involve leptons.

For example, one nucleon decay operator is (with $\ell = e,\ \mu$)

$$\mathcal{O}_1^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha\ T} C d_R^{\beta}][u_R^{\gamma\ T} C \ell_R]$$

where $\alpha,\ \beta,\ \gamma$ are SU(3)$_c$ color indices.

The product of $y$-dependent fermion wavefunctions in this operator is

$$A^4 \exp\left[-\left\{2\|\eta - \eta_{u_R}\|^2 + \|\eta - \eta_{d_R}\|^2 + \|\eta - \eta_{\ell_R}\|^2\right\}\right]$$

The integral over $y$ yields

$$I_1^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\left\{2\|\eta_{u_R} - \eta_{d_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell_R}\|^2 + \|\eta_{d_R} - \eta_{\ell_R}\|^2\right\}\right]$$

where $b_4 = (\mu/\sqrt{\pi})^n$.

One can guarantee that this is sufficiently small by taking the distances between wavefunction centers $\|\eta_{u_R} - \eta_{\ell_R}\|$ and/or $\|\eta_{d_R} - \eta_{\ell_R}\|^2$ sufficiently large.

Analyze $n-\bar{n}$ oscillations: with $H_{eff}^{(n\bar{n})} = \int d^3x\, \mathcal{H}^{(n\bar{n})}$, $\delta m = \langle \bar{n} | H_{eff}^{(n\bar{n})} | n \rangle$.

In $d=4$ dims., effective Lagrangian

$$\mathcal{L}_{eff}^{(n\bar{n})}(x) = \sum_r c_r^{(n\bar{n})}\, \mathcal{O}_r^{(n\bar{n})}(x) + h.c. \; .$$

Correspondingly, in $d = 4+n$ dimensions,

$$\mathcal{L}_{eff,4+n}^{(n\bar{n})}(x,y) = \sum_r \kappa_r^{(n\bar{n})} O_r^{(n\bar{n})}(x,y) + h.c. \; .$$

where the $\mathcal{O}_r^{(n\bar{n})}(x)$ and $O_r^{(n\bar{n})}(x,y)$ are 6-quark operators in $d=4$ and $d=4+n$ dims. Coeffs. $\kappa_r^{(n\bar{n})} = \bar{\kappa}_r^{(n\bar{n})}/M_{n\bar{n}}^{5+2n}$, where $M_{n\bar{n}}$ is an effective mass scale of physics producing the $n-\bar{n}$ oscillations.

Integration of fermion wavefunctions in the $O_r^{(n\bar{n})}(x,y)$ over $y$ yield the coeffs. $c_r^{(n\bar{n})}$ in terms of $\kappa_r^{(n\bar{n})}$

Relevant six-quark operators in SM EFT:

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_2^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}\epsilon_{km}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

where $Q_L = \binom{u}{d}_L$, $i, j...$ are SU(2)$_L$ indices, and color SU(3)$_c$ tensors are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

symmetries: $(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = T_{(\alpha\beta)(\gamma\delta)(\rho\sigma)}$; $(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = T_{[\alpha\beta][\gamma\delta](\rho\sigma)}$.

The integrals of these operators over $y$: operators $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ yield the integral

$$I_{1,2}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3}\|\eta_{u_R} - \eta_{d_R}\|^2\right],$$

$O_3^{(n\bar{n})}$ yields the integral

$$I_3^{(n\bar{n})} = b_6 \exp\left[-\frac{1}{6}\left\{2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2\right\}\right].$$

$O_4^{(n\bar{n})}$ yields the integral

$$I_4^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3}\|\eta_{Q_L} - \eta_{d_R}\|^2\right].$$

where $b_6 = (2 \cdot 3^{-1/2}\,\pi^{-1}\mu^2)^n$.

Then coefficients. $c_r^{(n\bar{n})} = \frac{\bar{\kappa}_r^{(n\bar{n})}}{(M_{n\bar{n}})^5}\, I_r^{(n\bar{n})}$

Consider, e.g., case $n = 2$: one can fit data on quark masses, mixing with

$$\|\eta_{Q_L} - \eta_{u_R}\| = 4.75; \quad \|\eta_{Q_L} - \eta_{d_R}\| \simeq 4.60; \quad \|\eta_{u_R} - \eta_{d_R}\| \simeq 7$$

We find $|c_r^{(n\bar{n})}|$ for $r = 1, 2, 3$ are $<<$ $|c_4^{(n\bar{n})}|$, and hence focus on $c_4^{(n\bar{n})}$. Then

$$\delta m \simeq c_4^{(n\bar{n})} \langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle \simeq \left( \frac{4\mu^4}{3\pi^2 M_{n\bar{n}}^9} \right) \left( \frac{2^{1/2} m_d}{v} \right)^{8/3} \langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle$$

Requiring that $\tau_{n\bar{n}} = 1/|\delta m|$ agree with the lower limit from Super-K, $\tau_{n\bar{n}} > 4.7 \times 10^8$ sec. yields the lower bound on the mass scale of $n - \bar{n}$ oscillations:

$$M_{n\bar{n}} > (47\,\mathrm{TeV}) \left( \frac{\tau_{n\bar{n}}}{4.7 \times 10^8\ \mathrm{sec}} \right)^{1/9} \left( \frac{\mu}{3 \times 10^3\ \mathrm{TeV}} \right)^{4/9} \left( \frac{|\langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle|}{\Lambda_{QCD}^6} \right)^{1/9} .$$

Hence, for relevant values of $M_{n\bar{n}}$ in this model, $n - \bar{n}$ oscillations could occur at a level that is close to the current limit.

This model was constructed using the SM gauge group $G_{SM} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. It is also of interest to analyze an extra-dimensional model using the extended gauge group

$$G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$$

in which $B - L$ is gauged.

We have carried out this analysis in Girmohanta and RS, PRD 101, 095012 (2020) and Girmohanta, Mohapatra, and RS, PRD 103, 015021 (2021) with similar results.

There are also other models that can predict $n - \bar{n}$ oscillations to occur near to current limits (reviewed in the white paper J. Phys. G J. Phys. G $48$, 070501 (2021) [arXiv:2006.04907]).

# Conclusions

- There is much progress in studies of neutrino masses and lepton mixing, and their effects, including charged lepton flavor-violating processes. We have obtained new constraints on massive neutrino emission via lepton mixing in particle decays, yielding upper bounds on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$, where $\nu_4$ is a heavy neutrino.

- We have discussed models that show how new physics beyond the SM can produce $n - \bar{n}$ oscillations at rates comparable with current limits. These models also show that $n - \bar{n}$ oscillations can be the main manifestation of baryon number violation with proton decay being strongly suppressed.

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## Backup Slides: General Formalism for $n - \bar{n}$ Oscillations

In the $(n, \bar{n})$ basis, write

$$\mathcal{M} = \begin{pmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{pmatrix}$$

Diagonalization yields mass eigenstates

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

where

$$\tan(2\theta) = \frac{2\delta m}{\Delta M}$$

and $\Delta M = M_{11} - M_{22}$. The energy eigenvalues are

$$E_{1,2} = \frac{1}{2}\left[ M_{11} + M_{22} \pm \sqrt{(\Delta M)^2 + 4(\delta m)^2} \right]$$

Let $\Delta E = E_1 - E_2 = \sqrt{(\Delta M)^2 + 4(\delta m)^2}$; transition probability:

$$P(n(t) \to \bar{n}) = |\langle \bar{n} | n(t) \rangle|^2 = \sin^2(2\theta)\ \sin^2[(\Delta E)t/2]\, e^{-\lambda_n t}$$

$$= \left[ \frac{(\delta m)^2}{(\Delta M/2)^2 + (\delta m)^2} \right] \sin^2 \left[ \sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t \right] e^{-\lambda_n t}$$

N.B.: if $\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t << 1$, then by expanding the sin, the quantity $(\Delta M/2)^2 + (\delta m)^2$ cancels, so

$$P(n(t) \to \bar{n}) \simeq [(\delta m)t]^2\, e^{-\lambda_n t} = (t/\tau_{n\bar{n}})^2\, e^{-\lambda_n t}$$

Although $\Delta M = 2\vec{\mu}_n \cdot \vec{B}$, where $\vec{B}$ is a small residual magnetic field in a reactor exp., this inequality enables exp. to be sensitive to $\delta m$.

## $n - \bar{n}$ Oscillations in Matter:

For $n - \bar{n}$ oscillations involving a neutron bound in a nucleus, consider

$$\mathcal{M} = \begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix}$$

with

$$m_{n,eff} = m_n + V_n \ , \quad m_{\bar{n},eff.} = m_n + V_{\bar{n}}$$

where the nuclear potential $V_n$ is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part representing the $\bar{n}N$ annihilation: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$ with $V_{nR}, \ V_{\bar{n}R}, \ V_{\bar{n}I} \sim O(100)$ MeV (Dover, Gal, Richard; Friedman; recently work by Barrow, Golubeva, Ladd, Paryev, Richard for $^{12}$C (ESS) and $^{40}$Ar (DUNE)).

Mixing is thus strongly suppressed; $\tan(2\theta)$ is determined by

$$\frac{2\delta m}{|m_{n,eff.} - m_{\bar{n},eff.}|} = \frac{2\delta m}{\sqrt{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}} << 1$$

Using the reactor exp. bound on $|\delta m|$, this gives $|\theta| \lesssim 10^{-31}$. This suppression in mixing is compensated for by the large number of nucleons in a nucleon decay detector, $\sim 10^{33}$ $n$'s in Super-K.

Eigenvalues:

$$m_{1,2} = \frac{1}{2}\left[m_{n,eff.} + m_{\bar{n},eff.} \pm \sqrt{(m_{n,eff.} - m_{\bar{n},eff.})^2 + 4(\delta m)^2}\;\right]$$

Expanding $m_1$ for the mostly $n$ mass eigenstate $|n_1\rangle \simeq |n\rangle$,

$$m_1 \simeq m_n + V_n - i\frac{(\delta m)^2\, V_{\bar{n}I}}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

Imaginary part leads to matter instability, mainly via $\bar{n}n,\ \bar{n}p \to \pi$'s, with rate

$$\Gamma_{m.i.} = \frac{1}{\tau_{m.i.}} = \frac{2(\delta m)^2 |V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

So $\tau_{m.i.} \propto (\delta m)^{-2} = \tau_{n\bar{n}}^2$.

Writing $\tau_{m.i.} = R\,\tau_{n\bar{n}}^2$, one has $R \sim O(100)\ \mathrm{MeV} \sim 10^{23}\ \mathrm{sec}$, dependent on nucleus.

On integration calculations for the $n - \bar{n}$ model: Given the localization of fermion wavefunctions on scale $L_\mu << L$, in the integration over the extra dimensions, can extend $\int_0^L \longrightarrow \int_{-\infty}^{\infty}$ to good approximation.

Integrals over extra dimensions have the general form (with $\int d^n\eta = \int_{-\infty}^{\infty} d^n\eta$)

$$\int d^n\eta \, \exp\Big[-\sum_{i=1}^{m} a_i \|\eta - \eta_{f_i}\|^2\Big] = \Big[\frac{\pi}{\sum_{i=1}^{m} a_i}\Big]^{n/2} \exp\Bigg[\frac{-\sum_{j,k=1;\ j<k}^{m} a_j a_k \|\eta_{f_j} - \eta_{f_k}\|^2}{\sum_{s=1}^{m} a_s}\Bigg].$$

For example, for $m = 3$,

$$\int d^n\eta \, \exp\Big[-\Big(a_1\|\eta - \eta_{f_1}\|^2 + a_2\|\eta - \eta_{f_2}\|^2 + a_3\|\eta - \eta_{f_3}\|^2\Big)\Big] =$$

$$= \Big[\frac{\pi}{a_1 + a_2 + a_3}\Big]^{n/2} \exp\Bigg[\frac{-\Big(a_1 a_2\|\eta_{f_1} - \eta_{f_2}\|^2 + a_2 a_3\|\eta_{f_2} - \eta_{f_3}\|^2 + a_3 a_1\|\eta_{f_3} - \eta_{f_1}\|^2\Big)}{a_1 + a_2 + a_3}\Bigg].$$

Recall result of calculation in $n-\bar{n}$ model:

$$M_{n\bar{n}} > (47\,\mathrm{TeV})\Big(\frac{\tau_{n\bar{n}}}{4.7\times 10^{8}\ \mathrm{sec}}\Big)^{1/9}\Big(\frac{\mu}{3\times 10^{3}\ \mathrm{TeV}}\Big)^{4/9}\left(\frac{|\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle|}{\Lambda_{QCD}^{6}}\right)^{1/9}.$$

where $\Lambda_{QCD} = 0.25$ GeV. This bound is not very sensitive to the precise size of $\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle$ because of the 1/9 power in the exponent. Lattice calculation: $|\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle| \simeq 2\Lambda_{QCD}^{6}$ (Buchoff et al., 2019); substituting this yields factor $2^{1/9} = 1.08$, so lower bound is (1.08)47 TeV = 51 TeV.